Computer Algebra Independent Integration Tests

Summer 2023 edition

5-Inverse-trig-functions/5.5-Inverse-secant/156-5.5.1-u-a+b-arcsec-c-x-^n

Nasser M. Abbasi

September 5, 2023 Compiled on September 5, 2023 at 11:55am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [174]. This is test number [156].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (174)	0.00 (0)
Mathematica	100.00 (174)	0.00 (0)
Maple	79.89 (139)	20.11 (35)
Fricas	71.26 (124)	28.74 (50)
Giac	55.75 (97)	44.25 (77)
Sympy	40.23 (70)	59.77 (104)
Maxima	35.63 (62)	64.37 (112)
Mupad	30.46 (53)	69.54 (121)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

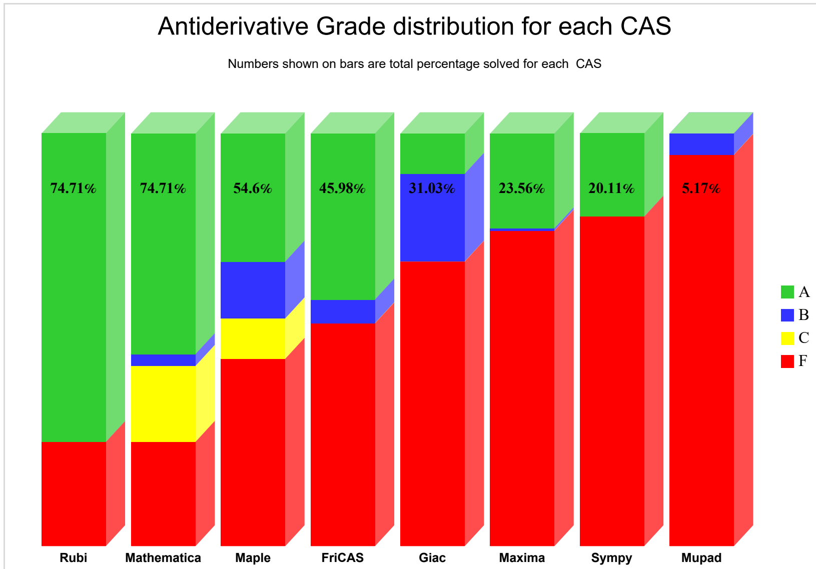
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

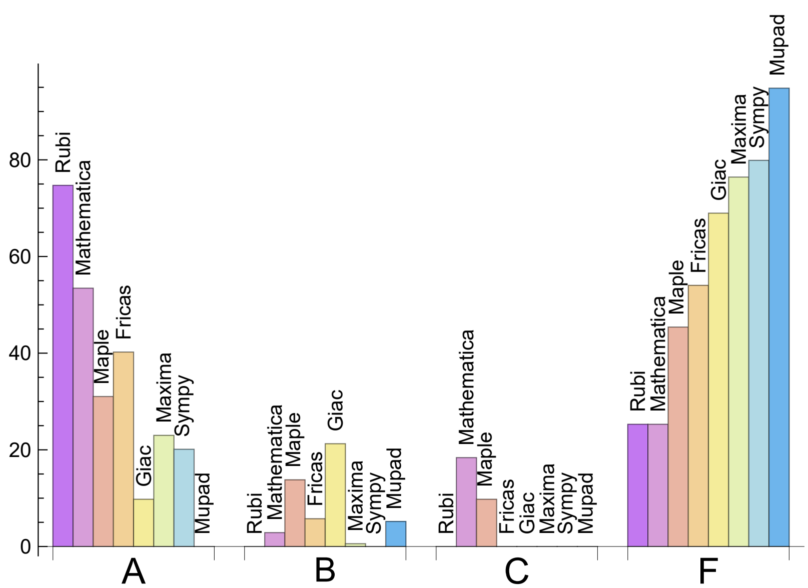
System	% A grade	% B grade	% C grade	% F grade
Rubi	74.713	0.000	0.000	25.287
Mathematica	53.448	2.874	18.391	25.287
Fricas	40.230	5.747	0.000	54.023
Maple	31.034	13.793	9.770	45.402
Maxima	22.989	0.575	0.000	76.437
Sympy	20.115	0.000	0.000	79.885
Giac	9.770	21.264	0.000	68.966
Mupad	0.000	5.172	0.000	94.828

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	35	100.00	0.00	0.00
Fricas	50	98.00	2.00	0.00
Giac	77	61.04	5.19	33.77
Maxima	112	46.43	0.00	53.57
Sympy	104	72.12	27.88	0.00
Mupad	121	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.28
Fricas	0.33
Mupad	1.11
Maxima	1.95
Mathematica	3.69
Giac	4.09
Maple	5.13
Sympy	20.67

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	31.77	1.25	27.00	1.17
Sympy	123.46	1.35	33.00	1.18
Rubi	202.76	1.00	129.50	1.00
Mathematica	249.16	1.13	124.00	1.09
Maxima	256.45	12.72	140.50	1.42
Fricas	256.83	1.90	93.50	1.23
Maple	294.71	1.46	138.00	1.14
Giac	1786.87	12.64	65.00	1.20

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

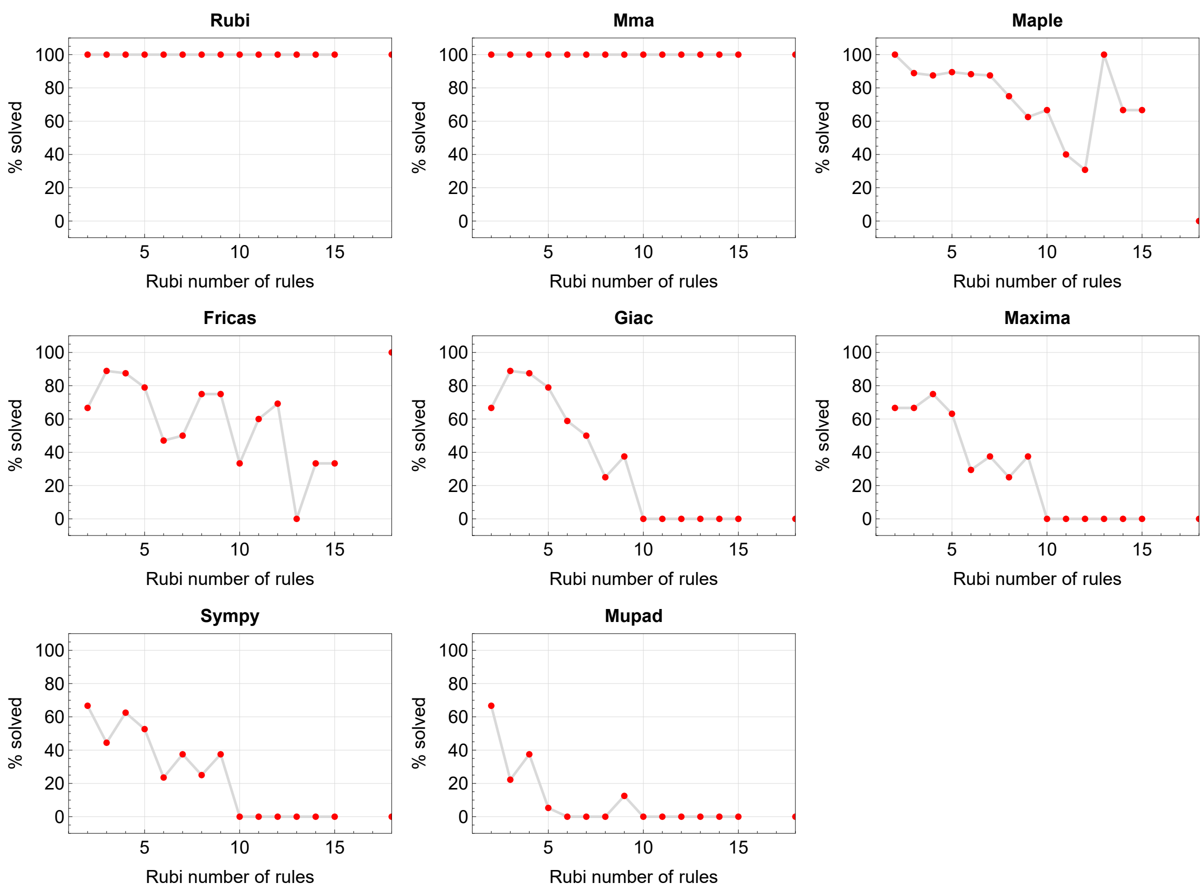


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

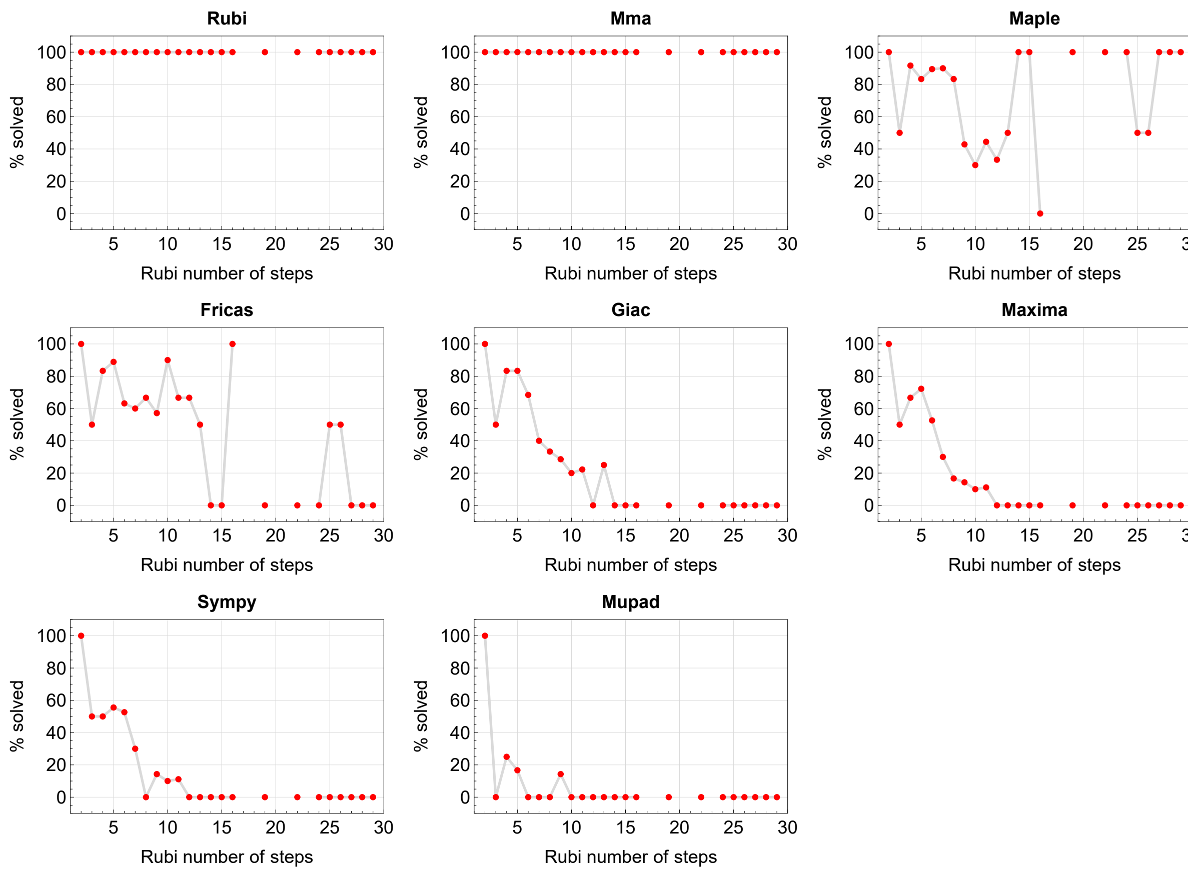


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

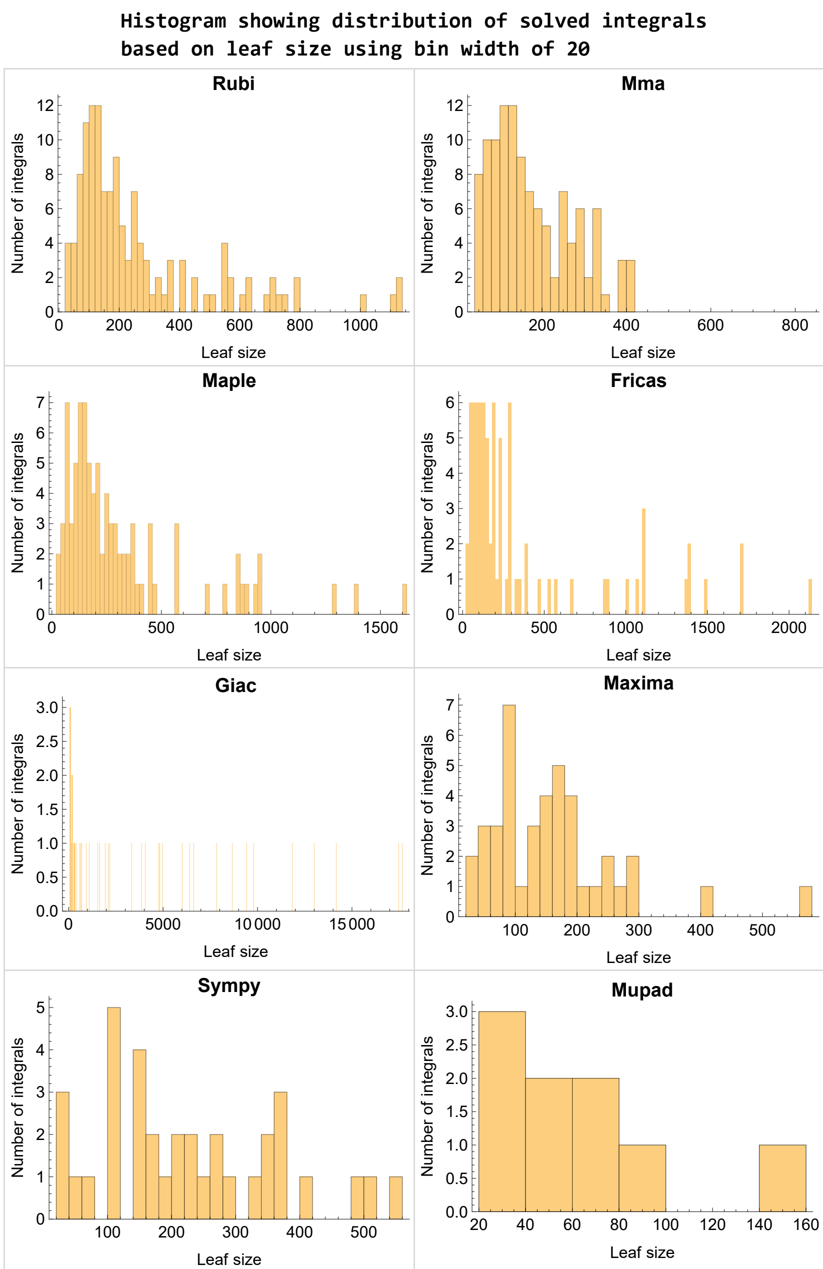


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

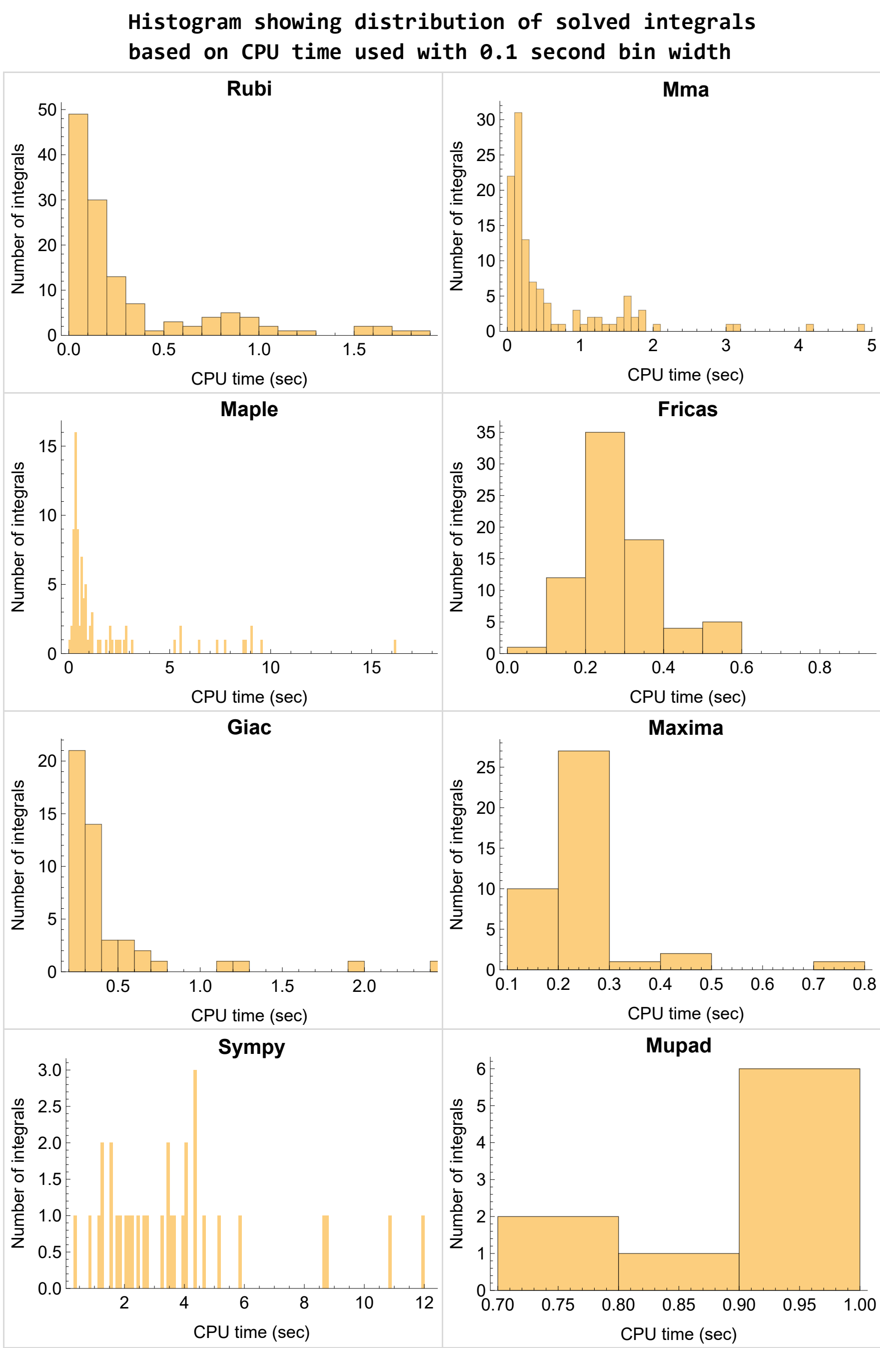


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

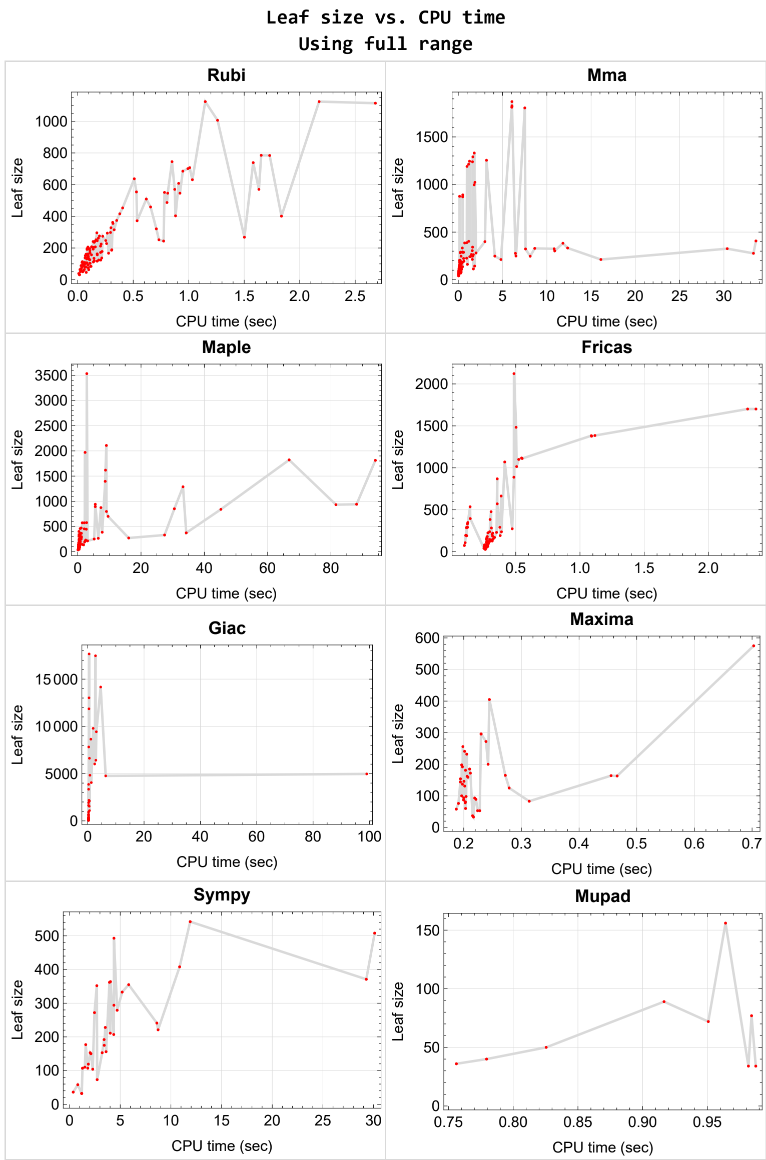


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{33, 34, 35, 39, 40, 41, 45, 46, 47, 51, 52, 54, 55, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137, 144, 145, 146, 147, 154, 155, 156, 157, 164, 165, 166, 167, 168, 169, 173, 174\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{91, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110\}$

Maple $\{91, 92, 94, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

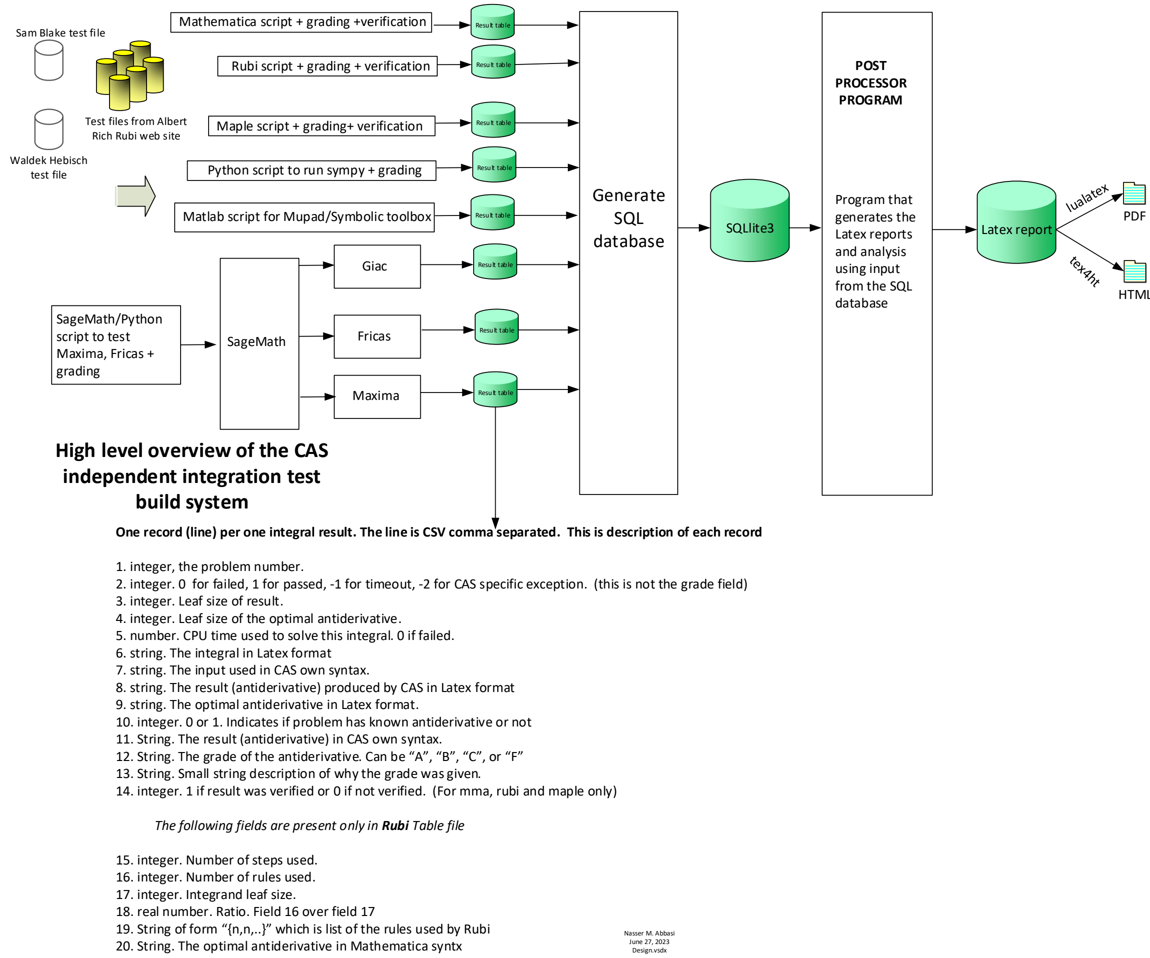
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 58, 59, 60, 61, 62, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 100, 101, 102, 103, 108, 109, 110, 138, 148, 158, 161, 162, 163, 170, 171, 172 }

B grade { 96, 97, 99, 104, 107 }

C grade { 63, 64, 65, 67, 68, 98, 105, 106, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 139, 140, 141, 142, 143, 149, 150, 151, 152, 153, 159, 160 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 18, 19, 22, 24, 26, 36, 37, 38, 42, 43, 44, 48, 49, 50, 58, 59, 60, 61, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 89, 90 }

B grade { 10, 12, 17, 20, 21, 23, 25, 28, 29, 30, 31, 32, 56, 57, 62, 63, 67, 68, 81, 82, 88, 98, 105, 106 }

C grade { 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110 }

F normal fail { 27, 53, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 29, 30, 31, 32, 56, 57, 58, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 98, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 158, 159, 160, 170, 171, 172 }

B grade { 7, 17, 59, 61, 62, 105, 106, 151, 152, 153 }

C grade { }

F normal fail { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 64, 65, 66, 67, 68, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 161, 162, 163 }

F(-1) timedout fail { 63 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 20, 22, 29, 56, 57, 58, 59, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88 }

B grade { 31 }

C grade { }

F normal fail { 8, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 30, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 61, 62, 79, 80, 89, 90, 92, 94, 96, 97, 98, 99, 104, 105, 106, 107, 113, 122, 133, 143, 158, 159, 161, 162, 163, 170, 171, 172 }

F(-1) timedout fail { }

F(-2) exception fail { 63, 64, 65, 66, 67, 68, 91, 93, 95, 100, 101, 102, 103, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160 }

Giac

A grade { 9, 10, 11, 12, 13, 14, 21, 22, 23, 36, 37, 38, 73, 74, 75, 85, 86 }

B grade { 1, 2, 3, 4, 5, 6, 7, 15, 17, 20, 29, 30, 31, 32, 42, 43, 44, 48, 49, 50, 56, 57, 58, 59, 69, 70, 71, 72, 76, 77, 78, 81, 82, 83, 84, 87, 88 }

C grade { }

F normal fail { 16, 18, 24, 25, 26, 27, 28, 53, 63, 64, 65, 66, 67, 68, 80, 90, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 172 }

F(-1) timedout fail { 96, 97, 100, 104 }

F(-2) exception fail { 8, 19, 45, 60, 61, 62, 79, 89, 91, 92, 93, 94, 95, 98, 99, 101, 102, 103, 105, 106, 107, 108, 109, 110, 170, 171 }

Mupad

A grade { }

B grade { 6, 7, 9, 10, 20, 29, 58, 59, 72 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 120, 121, 122, 129, 130, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 163, 170, 171, 172 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 56, 57, 58, 59, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88 }

B grade { }

C grade { }

F normal fail { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 42, 43, 44, 48, 49, 50, 53, 60, 61, 62, 63, 64, 65, 66, 67, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 98, 100, 101, 102, 103, 111, 112, 113, 119, 120, 122, 131, 132, 133, 138, 139, 140, 141, 142, 143, 148, 149, 151, 152, 153, 162, 163, 172 }

F(-1) timedout fail { 68, 97, 99, 104, 105, 106, 107, 108, 109, 110, 121, 125, 126, 129, 130, 145, 150, 154, 155, 156, 157, 158, 159, 160, 161, 165, 166, 170, 171 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	107	177	162	116	221	8644	0
N.S.	1	1.00	0.94	1.55	1.42	1.02	1.94	75.82	0.00
time (sec)	N/A	0.042	0.107	0.330	0.206	0.283	8.733	1.120	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	72	79	81	63	153	3862	0
N.S.	1	1.00	0.81	0.89	0.91	0.71	1.72	43.39	0.00
time (sec)	N/A	0.031	0.073	0.239	0.203	0.283	2.034	0.321	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	97	141	131	107	175	4828	0
N.S.	1	1.00	1.09	1.58	1.47	1.20	1.97	54.25	0.00
time (sec)	N/A	0.032	0.058	0.260	0.202	0.287	3.422	0.791	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	70	60	53	107	1926	0
N.S.	1	1.00	0.97	1.09	0.94	0.83	1.67	30.09	0.00
time (sec)	N/A	0.020	0.086	0.298	0.203	0.269	1.275	0.315	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	85	94	98	94	107	2101	0
N.S.	1	1.00	1.33	1.47	1.53	1.47	1.67	32.83	0.00
time (sec)	N/A	0.024	0.055	0.242	0.204	0.286	1.796	0.590	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	50	61	37	40	58	634	40
N.S.	1	1.00	1.28	1.56	0.95	1.03	1.49	16.26	1.03
time (sec)	N/A	0.009	0.025	0.247	0.215	0.269	0.810	0.287	0.779

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	59	38	53	63	32	63	34
N.S.	1	1.00	1.84	1.19	1.66	1.97	1.00	1.97	1.06
time (sec)	N/A	0.015	0.041	0.112	0.228	0.280	1.205	0.277	0.987

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	59	84	0	0	0	0	0
N.S.	1	1.00	0.92	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.020	0.618	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	40	58	33	27	36	43	36
N.S.	1	1.00	1.29	1.87	1.06	0.87	1.16	1.39	1.16
time (sec)	N/A	0.016	0.028	0.193	0.217	0.263	0.361	0.298	0.756

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	66	95	83	39	119	58	50
N.S.	1	1.00	1.29	1.86	1.63	0.76	2.33	1.14	0.98
time (sec)	N/A	0.025	0.036	0.243	0.314	0.254	1.854	0.280	0.825

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	71	58	40	110	65	0
N.S.	1	1.00	0.98	1.18	0.97	0.67	1.83	1.08	0.00
time (sec)	N/A	0.029	0.044	0.250	0.187	0.253	1.509	0.283	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	78	138	125	52	192	83	0
N.S.	1	1.00	1.03	1.82	1.64	0.68	2.53	1.09	0.00
time (sec)	N/A	0.036	0.056	0.240	0.279	0.258	3.407	0.284	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	79	76	51	156	87	0
N.S.	1	1.00	0.84	0.96	0.93	0.62	1.90	1.06	0.00
time (sec)	N/A	0.038	0.057	0.244	0.191	0.269	3.601	0.289	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	88	174	165	62	241	104	0
N.S.	1	1.00	0.87	1.72	1.63	0.61	2.39	1.03	0.00
time (sec)	N/A	0.047	0.068	0.313	0.272	0.254	8.615	0.287	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	124	170	163	146	0	6625	0
N.S.	1	1.00	1.16	1.59	1.52	1.36	0.00	61.92	0.00
time (sec)	N/A	0.082	0.151	0.710	0.466	0.298	0.000	0.608	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	226	285	0	0	0	0	0
N.S.	1	1.00	1.54	1.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.943	1.174	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	90	123	87	111	0	2181	0
N.S.	1	1.00	1.61	2.20	1.55	1.98	0.00	38.95	0.00
time (sec)	N/A	0.052	0.195	0.674	0.200	0.283	0.000	0.447	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	163	194	0	0	0	0	0
N.S.	1	1.00	1.77	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.179	0.500	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	129	206	0	0	0	0	0
N.S.	1	1.00	1.39	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.103	0.773	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	75	114	78	57	0	105	89
N.S.	1	1.00	1.50	2.28	1.56	1.14	0.00	2.10	1.78
time (sec)	N/A	0.045	0.097	0.555	0.203	0.274	0.000	0.293	0.917

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	102	177	0	82	0	147	0
N.S.	1	1.00	1.09	1.88	0.00	0.87	0.00	1.56	0.00
time (sec)	N/A	0.059	0.083	0.494	0.000	0.257	0.000	0.308	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	108	153	164	93	0	168	0
N.S.	1	1.00	1.06	1.50	1.61	0.91	0.00	1.65	0.00
time (sec)	N/A	0.070	0.126	0.903	0.455	0.276	0.000	0.299	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	148	265	0	120	0	215	0
N.S.	1	1.00	1.10	1.98	0.00	0.90	0.00	1.60	0.00
time (sec)	N/A	0.084	0.119	0.819	0.000	0.283	0.000	0.297	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	288	365	0	0	0	0	0
N.S.	1	1.00	1.39	1.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.595	1.158	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	403	574	0	0	0	0	0
N.S.	1	1.00	1.71	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	1.178	1.438	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	184	252	0	0	0	0	0
N.S.	1	1.00	1.46	2.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.478	1.096	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	289	0	0	0	0	0	0
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.247	0.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	204	364	0	0	0	0	0
N.S.	1	1.00	1.59	2.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.141	0.859	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	141	196	146	98	0	196	156
N.S.	1	1.00	1.76	2.45	1.82	1.22	0.00	2.45	1.95
time (sec)	N/A	0.063	0.133	0.685	0.200	0.286	0.000	0.311	0.964

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	185	293	0	150	0	278	0
N.S.	1	1.00	1.35	2.14	0.00	1.09	0.00	2.03	0.00
time (sec)	N/A	0.078	0.144	0.806	0.000	0.271	0.000	0.304	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	204	299	575	172	0	336	0
N.S.	1	1.00	1.20	1.76	3.38	1.01	0.00	1.98	0.00
time (sec)	N/A	0.110	0.206	1.156	0.703	0.275	0.000	0.324	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	283	470	0	225	0	427	0
N.S.	1	1.00	1.36	2.26	0.00	1.08	0.00	2.05	0.00
time (sec)	N/A	0.125	0.229	1.095	0.000	0.283	0.000	0.314	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	18
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.50
time (sec)	N/A	0.012	2.467	0.708	0.304	0.264	0.374	33.763	0.743

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	16
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.60
time (sec)	N/A	0.005	0.024	0.413	0.305	0.259	0.386	11.353	0.720

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	16	20
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	1.14	1.43
time (sec)	N/A	0.018	0.240	0.342	0.327	0.251	0.901	1.689	0.788

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	47	0	0	0	55	0
N.S.	1	1.00	0.93	1.02	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.077	0.067	0.451	0.000	0.000	0.000	0.286	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	58	0	0	0	95	0
N.S.	1	1.00	0.89	0.92	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.099	0.059	0.372	0.000	0.000	0.000	0.275	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	91	102	0	0	0	199	0
N.S.	1	1.00	0.78	0.87	0.00	0.00	0.00	1.70	0.00
time (sec)	N/A	0.175	0.148	0.368	0.000	0.000	0.000	0.288	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	583	28	12	14	18
N.S.	1	1.00	1.17	1.00	48.58	2.33	1.00	1.17	1.50
time (sec)	N/A	0.010	11.915	0.561	0.954	0.265	0.927	105.842	0.781

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	577	26	12	12	16
N.S.	1	1.00	1.20	1.00	57.70	2.60	1.20	1.20	1.60
time (sec)	N/A	0.005	23.964	0.504	0.951	0.254	0.999	23.495	0.743

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	560	30	14	16	20
N.S.	1	1.00	1.14	1.00	40.00	2.14	1.00	1.14	1.43
time (sec)	N/A	0.018	3.274	0.400	0.826	0.277	1.839	4.042	0.822

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	69	78	0	0	0	226	0
N.S.	1	1.00	0.92	1.04	0.00	0.00	0.00	3.01	0.00
time (sec)	N/A	0.095	0.263	0.408	0.000	0.000	0.000	0.281	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	80	77	0	0	0	357	0
N.S.	1	1.00	0.95	0.92	0.00	0.00	0.00	4.25	0.00
time (sec)	N/A	0.120	0.326	0.371	0.000	0.000	0.000	0.272	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	223	153	0	0	0	694	0
N.S.	1	1.00	1.25	0.86	0.00	0.00	0.00	3.90	0.00
time (sec)	N/A	0.219	0.599	0.373	0.000	0.000	0.000	0.304	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	1790	42	12	0	18
N.S.	1	1.00	1.17	1.00	149.17	3.50	1.00	0.00	1.50
time (sec)	N/A	0.011	2.736	0.603	29.312	0.253	1.942	0.000	0.788

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	1744	40	12	12	16
N.S.	1	1.00	1.20	1.00	174.40	4.00	1.20	1.20	1.60
time (sec)	N/A	0.005	9.570	0.575	28.499	0.251	2.032	52.479	0.754

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	1568	45	14	16	20
N.S.	1	1.00	1.14	1.00	112.00	3.21	1.00	1.14	1.43
time (sec)	N/A	0.018	1.368	0.507	21.865	0.256	3.576	7.963	0.833

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	88	154	0	0	0	580	0
N.S.	1	1.00	0.85	1.50	0.00	0.00	0.00	5.63	0.00
time (sec)	N/A	0.111	0.299	0.468	0.000	0.000	0.000	0.311	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	114	157	0	0	0	929	0
N.S.	1	1.00	1.02	1.40	0.00	0.00	0.00	8.29	0.00
time (sec)	N/A	0.142	0.315	0.398	0.000	0.000	0.000	0.298	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	169	307	0	0	0	1640	0
N.S.	1	1.00	0.74	1.35	0.00	0.00	0.00	7.19	0.00
time (sec)	N/A	0.260	0.371	0.393	0.000	0.000	0.000	0.306	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	991	44	15	18	22
N.S.	1	1.00	1.12	1.00	61.94	2.75	0.94	1.12	1.38
time (sec)	N/A	0.016	4.374	0.760	14.782	0.287	22.090	0.803	1.069

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	512	30	15	18	22
N.S.	1	1.00	1.12	1.00	32.00	1.88	0.94	1.12	1.38
time (sec)	N/A	0.015	2.770	0.612	6.847	0.274	10.072	0.577	1.075

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	82	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.38
time (sec)	N/A	0.020	0.725	1.937	0.359	0.249	1.121	0.940	0.723

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	643	32	15	18	22
N.S.	1	1.00	1.12	1.00	40.19	2.00	0.94	1.12	1.38
time (sec)	N/A	0.018	1.430	1.370	1.900	0.258	5.312	1.654	0.716

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	166	402	272	291	362	9430	0
N.S.	1	1.00	0.99	2.41	1.63	1.74	2.17	56.47	0.00
time (sec)	N/A	0.278	0.181	0.391	0.239	0.376	3.941	3.068	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	124	306	200	209	228	6416	0
N.S.	1	1.00	1.00	2.47	1.61	1.69	1.84	51.74	0.00
time (sec)	N/A	0.189	0.113	0.385	0.242	0.316	3.528	2.894	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	114	110	93	130	104	1547	77
N.S.	1	1.00	1.36	1.31	1.11	1.55	1.24	18.42	0.92
time (sec)	N/A	0.115	0.146	0.316	0.219	0.315	2.286	0.528	0.984

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	59	38	53	63	32	63	34
N.S.	1	1.00	1.84	1.19	1.66	1.97	1.00	1.97	1.06
time (sec)	N/A	0.016	0.029	0.090	0.224	0.275	1.188	0.285	0.982

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	333	453	0	0	0	0	0
N.S.	1	1.00	1.35	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.453	2.082	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	142	193	0	477	0	0	0
N.S.	1	1.00	1.37	1.86	0.00	4.59	0.00	0.00	0.00
time (sec)	N/A	0.108	0.139	2.190	0.000	0.308	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	247	574	0	1117	0	0	0
N.S.	1	1.00	1.44	3.34	0.00	6.49	0.00	0.00	0.00
time (sec)	N/A	0.206	0.316	2.069	0.000	0.546	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	333	798	0	0	0	0	0
N.S.	1	1.00	0.90	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.535	12.350	9.077	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	277	386	0	0	0	0	0
N.S.	1	1.00	0.88	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	33.340	7.735	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	212	252	0	0	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	16.108	5.207	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	124	215	0	0	0	0	0
N.S.	1	1.00	1.04	1.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	0.212	3.125	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	326	875	0	0	0	0	0
N.S.	1	1.00	1.09	2.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	30.378	7.329	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	637	407	1618	0	0	0	0	0
N.S.	1	1.18	0.75	3.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.510	33.621	8.780	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	141	328	296	192	408	17474	0
N.S.	1	1.00	0.68	1.59	1.44	0.93	1.98	84.83	0.00
time (sec)	N/A	0.092	0.176	0.658	0.230	0.379	10.861	2.764	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	123	254	232	170	294	9792	0
N.S.	1	1.00	0.76	1.58	1.44	1.06	1.83	60.82	0.00
time (sec)	N/A	0.075	0.131	0.603	0.205	0.335	4.376	1.928	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	150	132	154	141	153	4051	0
N.S.	1	1.00	1.38	1.21	1.41	1.29	1.40	37.17	0.00
time (sec)	N/A	0.039	0.180	0.313	0.194	0.309	3.220	1.280	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	104	116	89	123	73	1088	72
N.S.	1	1.00	1.20	1.33	1.02	1.41	0.84	12.51	0.83
time (sec)	N/A	0.045	0.100	0.326	0.221	0.298	2.723	0.631	0.951

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	69	108	94	67	150	113	0
N.S.	1	1.00	0.66	1.03	0.90	0.64	1.43	1.08	0.00
time (sec)	N/A	0.054	0.086	0.308	0.200	0.273	2.108	0.286	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	94	127	137	89	279	158	0
N.S.	1	1.00	0.62	0.84	0.90	0.59	1.84	1.04	0.00
time (sec)	N/A	0.069	0.101	0.310	0.198	0.281	4.696	0.288	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	110	145	172	110	371	202	0
N.S.	1	1.00	0.56	0.74	0.87	0.56	1.88	1.03	0.00
time (sec)	N/A	0.085	0.128	0.329	0.212	0.271	29.280	0.292	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	118	139	185	128	364	13018	0
N.S.	1	1.00	0.60	0.71	0.94	0.65	1.86	66.42	0.00
time (sec)	N/A	0.108	0.190	0.622	0.210	0.317	4.036	0.470	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	98	121	144	107	272	7820	0
N.S.	1	1.00	0.64	0.79	0.94	0.70	1.78	51.11	0.00
time (sec)	N/A	0.090	0.212	0.639	0.194	0.276	2.459	0.367	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	79	217	100	86	177	3346	0
N.S.	1	1.00	0.57	1.57	0.72	0.62	1.28	24.25	0.00
time (sec)	N/A	0.067	0.089	0.722	0.196	0.283	1.595	0.334	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	115	137	0	0	0	0	0
N.S.	1	1.00	0.93	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	0.074	1.855	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	132	143	0	0	0	0	0
N.S.	1	1.00	0.96	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.090	1.528	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	186	459	405	273	542	4760	0
N.S.	1	1.00	0.74	1.82	1.61	1.08	2.15	18.89	0.00
time (sec)	N/A	0.165	0.226	0.773	0.244	0.473	11.903	6.385	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	153	340	296	237	355	14166	0
N.S.	1	1.00	0.80	1.78	1.55	1.24	1.86	74.17	0.00
time (sec)	N/A	0.084	0.145	0.421	0.230	0.389	5.834	4.602	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	136	249	198	230	207	6018	0
N.S.	1	1.00	0.84	1.54	1.22	1.42	1.28	37.15	0.00
time (sec)	N/A	0.091	0.129	0.447	0.196	0.351	4.358	2.472	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	127	231	159	221	211	4968	0
N.S.	1	1.00	0.80	1.46	1.01	1.40	1.34	31.44	0.00
time (sec)	N/A	0.094	0.144	0.421	0.207	0.321	4.017	99.042	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	127	175	181	127	333	222	0
N.S.	1	1.00	0.69	0.96	0.99	0.69	1.82	1.21	0.00
time (sec)	N/A	0.109	0.157	0.456	0.203	0.295	5.191	0.306	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	153	207	241	159	508	293	0
N.S.	1	1.00	0.63	0.86	1.00	0.66	2.11	1.22	0.00
time (sec)	N/A	0.139	0.174	0.488	0.202	0.278	30.110	0.301	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	162	198	256	187	493	17666	0
N.S.	1	1.00	0.67	0.82	1.06	0.77	2.04	73.00	0.00
time (sec)	N/A	0.154	0.188	0.824	0.199	0.319	4.378	0.502	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	125	352	192	153	352	11858	0
N.S.	1	1.00	0.64	1.81	0.98	0.78	1.81	60.81	0.00
time (sec)	N/A	0.100	0.192	0.819	0.198	0.325	2.689	0.446	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	160	231	0	0	0	0	0
N.S.	1	1.00	0.86	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.254	2.524	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	194	216	0	0	0	0	0
N.S.	1	1.00	1.03	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.649	2.418	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	546	546	1023	374	0	0	0	0	0
N.S.	1	1.00	1.87	0.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.921	1.883	34.293	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	487	487	891	446	0	0	0	0	0
N.S.	1	1.00	1.83	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.804	0.508	2.713	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	871	272	0	0	0	0	0
N.S.	1	1.00	1.71	0.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	0.498	16.106	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	459	459	876	1970	0	0	0	0	0
N.S.	1	1.00	1.91	4.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.656	0.147	2.329	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	997	331	0	0	0	0	0
N.S.	1	1.00	1.81	0.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.782	1.792	27.443	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	608	608	1255	701	0	0	0	0	0
N.S.	1	1.00	2.06	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.909	3.185	9.589	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	570	570	1213	577	0	0	0	0	0
N.S.	1	1.00	2.13	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.874	1.169	2.806	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	286	265	0	384	0	0	0
N.S.	1	1.00	2.18	2.02	0.00	2.93	0.00	0.00	0.00
time (sec)	N/A	0.081	0.520	6.497	0.000	0.300	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	546	546	1190	2108	0	0	0	0	0
N.S.	1	1.00	2.18	3.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.811	0.985	9.061	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	784	784	1331	941	0	0	0	0	0
N.S.	1	1.00	1.70	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.729	1.804	88.106	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	745	745	1245	852	0	0	0	0	0
N.S.	1	1.00	1.67	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.849	1.270	30.533	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	739	739	1239	840	0	0	0	0	0
N.S.	1	1.00	1.68	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.581	1.601	45.227	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	785	785	1291	933	0	0	0	0	0
N.S.	1	1.00	1.64	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.653	1.649	81.579	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	707	707	1805	1396	0	0	0	0	0
N.S.	1	1.00	2.55	1.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.008	7.513	8.687	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	389	941	0	1015	0	0	0
N.S.	1	1.00	2.48	5.99	0.00	6.46	0.00	0.00	0.00
time (sec)	N/A	0.124	0.959	5.551	0.000	0.507	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	386	894	0	888	0	0	0
N.S.	1	1.00	2.00	4.63	0.00	4.60	0.00	0.00	0.00
time (sec)	N/A	0.139	0.723	5.549	0.000	0.485	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	685	685	1871	3533	0	0	0	0	0
N.S.	1	1.00	2.73	5.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.947	6.060	2.870	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1124	1124	1819	1822	0	0	0	0	0
N.S.	1	1.00	1.62	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.149	6.074	66.801	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1124	1124	1827	1288	0	0	0	0	0
N.S.	1	1.00	1.63	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.175	6.067	33.262	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1114	1114	1812	1812	0	0	0	0	0
N.S.	1	1.00	1.63	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.682	6.045	94.037	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	403	403	342	0	0	1701	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	4.22	0.00	0.00	0.00
time (sec)	N/A	0.881	1.615	0.000	0.000	2.370	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	263	0	0	1383	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	4.70	0.00	0.00	0.00
time (sec)	N/A	0.267	1.699	0.000	0.000	1.088	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	213	0	0	1100	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	5.64	0.00	0.00	0.00
time (sec)	N/A	0.137	1.589	0.000	0.000	0.523	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.87	1.00	1.17
time (sec)	N/A	0.067	7.660	0.429	0.000	0.273	10.312	0.366	1.184

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.068	4.875	0.738	0.000	0.269	14.239	0.354	1.308

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.064	12.605	0.387	0.000	0.264	116.687	0.331	1.287

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.023	16.513	0.414	0.000	0.256	47.320	0.355	1.169

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.057	1.554	0.344	0.000	0.265	7.258	0.360	1.447

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	328	247	0	0	194	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.299	8.120	0.000	0.000	0.111	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	453	453	325	0	0	290	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.405	10.831	0.000	0.000	0.118	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	374	374	305	0	0	1701	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.351	1.579	0.000	0.000	2.305	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	247	0	0	1377	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	5.26	0.00	0.00	0.00
time (sec)	N/A	0.186	1.625	0.000	0.000	1.090	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.87	1.00	1.17
time (sec)	N/A	0.080	7.849	2.136	0.000	0.279	71.460	0.493	1.139

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.081	6.268	2.031	0.000	0.268	61.958	0.376	1.427

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	43	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.87	0.00	1.00	1.17
time (sec)	N/A	0.078	12.552	1.086	0.000	0.257	0.000	0.361	1.403

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	37	0	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.85	0.00	1.00	1.20
time (sec)	N/A	0.031	17.073	1.093	0.000	0.272	0.000	0.358	1.195

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.070	32.885	2.246	0.000	0.265	99.479	0.405	1.540

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.074	8.273	0.376	0.000	0.262	67.227	0.362	1.400

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	416	303	0	0	288	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.379	10.892	0.000	0.000	0.121	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	554	554	383	0	0	395	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.529	11.820	0.000	0.000	0.146	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	321	281	0	0	1385	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	4.31	0.00	0.00	0.00
time (sec)	N/A	0.708	2.005	0.000	0.000	1.116	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	225	225	242	0	0	1111	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	4.94	0.00	0.00	0.00
time (sec)	N/A	0.211	1.364	0.000	0.000	0.550	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	108	0	0	869	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	6.58	0.00	0.00	0.00
time (sec)	N/A	0.100	0.364	0.000	0.000	0.356	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	31	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.35	0.87	1.00	1.17
time (sec)	N/A	0.063	1.144	0.427	0.000	0.253	7.816	0.349	1.263

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	33	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.43	0.96	1.00	1.17
time (sec)	N/A	0.074	4.734	0.817	0.000	0.292	27.725	0.361	1.361

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.065	32.378	0.452	0.000	0.261	54.463	0.350	1.475

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.021	0.630	0.473	0.000	0.284	15.389	0.356	1.167

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	246	246	143	0	0	107	0	0	0
N.S.	1	1.00	0.58	0.00	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.176	1.849	0.000	0.000	0.106	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	362	249	0	0	196	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.315	6.517	0.000	0.000	0.112	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1006	1006	329	0	0	291	0	0	0
N.S.	1	1.00	0.33	0.00	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	1.260	8.637	0.000	0.000	0.125	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	265	0	0	1483	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	5.88	0.00	0.00	0.00
time (sec)	N/A	0.732	1.480	0.000	0.000	0.503	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	157	161	0	0	1070	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	6.82	0.00	0.00	0.00
time (sec)	N/A	0.176	1.030	0.000	0.000	0.414	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	0	283	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	3.54	0.00	0.00	0.00
time (sec)	N/A	0.074	0.239	0.000	0.000	0.310	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	42	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.83	0.87	1.00	1.17
time (sec)	N/A	0.083	8.180	1.748	0.000	0.278	82.808	0.363	1.343

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	44	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.91	0.00	1.00	1.17
time (sec)	N/A	0.091	10.616	2.420	0.000	0.268	0.000	0.360	1.571

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.079	14.410	1.352	0.000	0.260	144.523	0.394	1.502

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.070	5.264	1.476	0.000	0.261	35.071	0.428	1.242

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	113	0	0	74	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.063	1.705	0.000	0.000	0.100	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	212	0	0	190	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.225	4.817	0.000	0.000	0.118	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	701	701	277	0	0	331	0	0	0
N.S.	1	1.00	0.40	0.00	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.993	6.453	0.000	0.000	0.124	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	244	242	0	0	2123	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	8.70	0.00	0.00	0.00
time (sec)	N/A	0.774	1.300	0.000	0.000	0.486	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	137	0	0	664	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	0.175	0.464	0.000	0.000	0.386	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	132	0	0	571	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	4.14	0.00	0.00	0.00
time (sec)	N/A	0.095	0.446	0.000	0.000	0.356	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	53	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.30	0.00	1.00	1.17
time (sec)	N/A	0.085	14.073	1.816	0.000	0.267	0.000	0.379	1.284

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	55	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.39	0.00	1.00	1.17
time (sec)	N/A	0.104	15.460	2.326	0.000	0.264	0.000	0.382	1.578

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.090	14.359	1.362	0.000	0.271	0.000	0.390	1.560

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.072	13.525	1.423	0.000	0.271	0.000	0.436	1.390

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	276	186	0	0	286	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.196	0.310	0.000	0.000	0.116	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	296	248	0	0	348	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.172	4.127	0.000	0.000	0.127	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	631	631	323	0	0	536	0	0	0
N.S.	1	1.00	0.51	0.00	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	1.033	7.593	0.000	0.000	0.145	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	589	570	400	0	0	0	0	0	0
N.S.	1	0.97	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.632	3.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	374	355	291	0	0	0	0	0	0
N.S.	1	0.95	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.473	0.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	204	169	0	0	0	0	0	0
N.S.	1	1.15	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	0.328	0.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.26
time (sec)	N/A	0.054	2.293	2.823	0.548	0.271	39.070	0.323	0.910

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	1.09	1.26
time (sec)	N/A	0.051	4.060	2.532	0.562	0.285	0.000	0.330	0.903

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	42	0	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.68	0.00	1.00	1.16
time (sec)	N/A	0.079	0.957	2.255	0.519	0.273	0.000	0.354	0.949

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.069	0.123	0.895	0.384	0.277	52.552	0.350	0.877

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.070	0.921	1.103	0.407	0.281	22.423	0.352	0.964

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	45	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.80	0.96	1.00	1.16
time (sec)	N/A	0.075	1.101	2.091	0.412	0.274	140.345	0.353	0.989

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	401	401	194	0	0	238	0	0	0
N.S.	1	1.00	0.48	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	1.836	0.239	0.000	0.000	0.297	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	159	0	0	181	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	1.502	0.279	0.000	0.000	0.279	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	135	118	0	0	125	0	0	0
N.S.	1	1.07	0.94	0.00	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.147	0.222	0.000	0.000	0.275	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	87	37	34	26	30
N.S.	1	1.00	1.08	0.92	3.35	1.42	1.31	1.00	1.15
time (sec)	N/A	0.066	0.637	0.273	0.483	0.264	11.186	0.376	1.811

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	111	39	36	26	30
N.S.	1	1.00	1.08	0.92	4.27	1.50	1.38	1.00	1.15
time (sec)	N/A	0.070	8.065	3.075	0.493	0.259	82.450	0.354	1.446

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [160] had the largest ratio of [.7825999999999999962]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	12	0.417
2	A	4	3	1.00	12	0.250
3	A	6	5	1.00	12	0.417
4	A	3	3	1.00	12	0.250
5	A	5	5	1.00	12	0.417
6	A	2	2	1.00	10	0.200
7	A	5	4	1.00	8	0.500
8	A	6	6	1.00	12	0.500
9	A	2	2	1.00	12	0.167
10	A	4	4	1.00	12	0.333
11	A	4	3	1.00	12	0.250
12	A	5	4	1.00	12	0.333
13	A	4	3	1.00	12	0.250
14	A	6	4	1.00	12	0.333
15	A	5	5	1.00	14	0.357
16	A	8	6	1.00	14	0.429
17	A	4	4	1.00	12	0.333
18	A	7	5	1.00	10	0.500
19	A	6	6	1.00	14	0.429
20	A	4	3	1.00	14	0.214
21	A	4	3	1.00	14	0.214
22	A	5	5	1.00	14	0.357
23	A	5	3	1.00	14	0.214
24	A	10	10	1.00	14	0.714

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	11	8	1.00	14	0.571
26	A	7	7	1.00	12	0.583
27	A	9	6	1.00	10	0.600
28	A	7	7	1.00	14	0.500
29	A	5	3	1.00	14	0.214
30	A	6	6	1.00	14	0.429
31	A	8	6	1.00	14	0.429
32	A	10	6	1.00	14	0.429
33	N/A	0	0	1.00	12	0.000
34	N/A	0	0	1.00	10	0.000
35	N/A	0	0	1.00	14	0.000
36	A	4	4	1.00	14	0.286
37	A	6	6	1.00	14	0.429
38	A	9	5	1.00	14	0.357
39	N/A	0	0	1.00	12	0.000
40	N/A	0	0	1.00	10	0.000
41	N/A	0	0	1.00	14	0.000
42	A	5	5	1.00	14	0.357
43	A	7	7	1.00	14	0.500
44	A	11	6	1.00	14	0.429
45	N/A	0	0	1.00	12	0.000
46	N/A	0	0	1.00	10	0.000
47	N/A	0	0	1.00	14	0.000
48	A	6	5	1.00	14	0.357
49	A	8	7	1.00	14	0.500
50	A	13	6	1.00	14	0.429
51	N/A	0	0	1.00	16	0.000
52	N/A	0	0	1.00	16	0.000
53	A	3	3	1.00	14	0.214
54	N/A	0	0	1.00	16	0.000
55	N/A	0	0	1.00	16	0.000
56	A	11	9	1.00	16	0.562
57	A	10	9	1.00	16	0.562

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
58	A	9	9	1.00	14	0.643
59	A	5	4	1.00	8	0.500
60	A	4	2	1.00	16	0.125
61	A	7	7	1.00	16	0.438
62	A	8	8	1.00	16	0.500
63	A	22	13	1.00	18	0.722
64	A	15	11	1.00	18	0.611
65	A	9	9	1.00	18	0.500
66	A	6	6	1.00	18	0.333
67	A	12	11	1.00	18	0.611
68	A	19	14	1.18	18	0.778
69	A	7	7	1.00	19	0.368
70	A	6	7	1.00	19	0.368
71	A	5	5	1.00	16	0.312
72	A	4	5	1.00	19	0.263
73	A	4	5	1.00	19	0.263
74	A	5	6	1.00	19	0.316
75	A	6	6	1.00	19	0.316
76	A	5	5	1.00	19	0.263
77	A	5	5	1.00	19	0.263
78	A	6	5	1.00	17	0.294
79	A	11	11	1.00	19	0.579
80	A	13	13	1.00	19	0.684
81	A	7	8	1.00	21	0.381
82	A	6	7	1.00	18	0.389
83	A	6	7	1.00	21	0.333
84	A	6	7	1.00	21	0.333
85	A	5	6	1.00	21	0.286
86	A	6	7	1.00	21	0.333
87	A	5	6	1.00	21	0.286
88	A	6	5	1.00	19	0.263
89	A	12	13	1.00	21	0.619
90	A	14	15	1.00	21	0.714
91	A	25	12	1.00	21	0.571
92	A	26	9	1.00	19	0.474

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	19	7	1.00	18	0.389
94	A	19	7	1.00	21	0.333
95	A	24	10	1.00	21	0.476
96	A	31	14	1.00	21	0.667
97	A	29	12	1.00	21	0.571
98	A	7	5	1.00	19	0.263
99	A	24	10	1.00	21	0.476
100	A	51	15	1.00	21	0.714
101	A	27	10	1.00	21	0.476
102	A	47	11	1.00	18	0.611
103	A	50	13	1.00	21	0.619
104	A	33	13	1.00	21	0.619
105	A	6	7	1.00	21	0.333
106	A	8	6	1.00	19	0.316
107	A	28	11	1.00	21	0.524
108	A	35	11	1.00	21	0.524
109	A	63	12	1.00	21	0.571
110	A	81	12	1.00	18	0.667
111	A	12	12	1.00	23	0.522
112	A	11	12	1.00	23	0.522
113	A	9	9	1.00	21	0.429
114	N/A	0	0	1.00	23	0.000
115	N/A	0	0	1.00	23	0.000
116	N/A	0	0	1.00	23	0.000
117	N/A	0	0	1.00	20	0.000
118	N/A	0	0	1.00	23	0.000
119	A	11	11	1.00	23	0.478
120	A	12	12	1.00	23	0.522
121	A	12	12	1.00	23	0.522
122	A	10	10	1.00	21	0.476
123	N/A	0	0	1.00	23	0.000
124	N/A	0	0	1.00	23	0.000
125	N/A	0	0	1.00	23	0.000
126	N/A	0	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
127	N/A	0	0	1.00	23	0.000
128	N/A	0	0	1.00	23	0.000
129	A	12	12	1.00	23	0.522
130	A	13	12	1.00	23	0.522
131	A	11	12	1.00	23	0.522
132	A	10	12	1.00	23	0.522
133	A	9	9	1.00	21	0.429
134	N/A	0	0	1.00	23	0.000
135	N/A	0	0	1.00	23	0.000
136	N/A	0	0	1.00	23	0.000
137	N/A	0	0	1.00	20	0.000
138	A	11	11	1.00	23	0.478
139	A	11	12	1.00	23	0.522
140	A	32	15	1.00	23	0.652
141	A	10	11	1.00	23	0.478
142	A	9	11	1.00	23	0.478
143	A	4	4	1.00	21	0.190
144	N/A	0	0	1.00	23	0.000
145	N/A	0	0	1.00	23	0.000
146	N/A	0	0	1.00	23	0.000
147	N/A	0	0	1.00	23	0.000
148	A	5	5	1.00	20	0.250
149	A	10	11	1.00	23	0.478
150	A	25	14	1.00	23	0.609
151	A	10	11	1.00	23	0.478
152	A	7	8	1.00	23	0.348
153	A	5	5	1.00	21	0.238
154	N/A	0	0	1.00	23	0.000
155	N/A	0	0	1.00	23	0.000
156	N/A	0	0	1.00	23	0.000
157	N/A	0	0	1.00	23	0.000
158	A	10	10	1.00	23	0.435
159	A	10	11	1.00	20	0.550

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	26	18	1.00	23	0.783
161	A	6	7	0.97	23	0.304
162	A	6	7	0.95	23	0.304
163	A	5	6	1.15	21	0.286
164	N/A	0	0	1.00	23	0.000
165	N/A	0	0	1.00	23	0.000
166	N/A	0	0	1.00	25	0.000
167	N/A	0	0	1.00	25	0.000
168	N/A	0	0	1.00	25	0.000
169	N/A	0	0	1.00	25	0.000
170	A	16	11	1.00	26	0.423
171	A	13	11	1.00	26	0.423
172	A	8	9	1.07	26	0.346
173	N/A	0	0	1.00	26	0.000
174	N/A	0	0	1.00	26	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^6(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 114

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5}-\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)-\frac{5b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}$$

```
[Out] 1/7*x^7*(a+b*arcsec(c*x))-5/112*b*arctanh((1-1/c^2/x^2)^(1/2))/c^7-5/112*b*
x^2*(1-1/c^2/x^2)^(1/2)/c^5-5/168*b*x^4*(1-1/c^2/x^2)^(1/2)/c^3-1/42*b*x^6*
(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5328, 272, 44, 65, 214}

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)-\frac{5b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}-\frac{bx^6\sqrt{1-\frac{1}{c^2x^2}}}{42c}-\frac{5bx^2\sqrt{1-\frac{1}{c^2x^2}}}{112c^5}-\frac{5bx^4\sqrt{1-\frac{1}{c^2x^2}}}{168c^3}$$

```
[In] Int[x^6*(a + b*ArcSec[c*x]),x]
```

```
[Out] (-5*b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(112*c^5) - (5*b*Sqrt[1 - 1/(c^2*x^2)]*x^4
)/(168*c^3) - (b*Sqrt[1 - 1/(c^2*x^2)]*x^6)/(42*c) + (x^7*(a + b*ArcSec[c*x
]))/7 - (5*b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(112*c^7)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)-\frac{b\int\frac{x^5}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{7c}$$

$$=\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)+\frac{b\text{Subst}\left(\int\frac{1}{x^4\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{14c}$$

$$=-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right)+\frac{(5b)\text{Subst}\left(\int\frac{1}{x^3\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{84c^3}$$

$$= -\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right) + \frac{(5b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{112c^5}$$

$$= -\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5} - \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right) + \frac{(5b)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{224c^7}$$

$$= -\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5} - \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right) - \frac{(5b)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^5}$$

$$= -\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5} - \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7\left(a+b\sec^{-1}(cx)\right) - \frac{5b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^7}{7} + b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{5x^2}{112c^5} - \frac{5x^4}{168c^3} - \frac{x^6}{42c}\right) + \frac{1}{7}bx^7\sec^{-1}(cx) - \frac{5b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{112c^7}$$

```
[In] Integrate[x^6*(a + b*ArcSec[c*x]),x]
```

```
[Out] (a*x^7)/7 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((-5*x^2)/(112*c^5) - (5*x^4)/
(168*c^3) - x^6/(42*c)) + (b*x^7*ArcSec[c*x])/7 - (5*b*Log[x*(1 + Sqrt[(-1
+ c^2*x^2)/(c^2*x^2)])])/(112*c^7)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.55

method	result
parts	$\frac{a\,x^{7}}{7}+\frac{b\,x^{7}\,\mathrm{arcsec}(cx)}{7}-\frac{b(c^{2}x^{2}-1)x^{4}}{42c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b(c^{2}x^{2}-1)x^{2}}{168c^{5}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b(c^{2}x^{2}-1)}{112c^{7}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{112c^{8}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x}$
derivativedivides	$\frac{\frac{a\,c^{7}x^{7}}{7}+\frac{b\,c^{7}x^{7}\,\mathrm{arcsec}(cx)}{7}-\frac{b\left(c^{2}x^{2}-1\right)c^{4}x^{4}}{42\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\left(c^{2}x^{2}-1\right)c^{2}x^{2}}{168\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\left(c^{2}x^{2}-1\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}}{c^{7}}$
default	$\frac{\frac{a\,c^{7}x^{7}}{7}+\frac{b\,c^{7}x^{7}\,\mathrm{arcsec}(cx)}{7}-\frac{b\left(c^{2}x^{2}-1\right)c^{4}x^{4}}{42\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\left(c^{2}x^{2}-1\right)c^{2}x^{2}}{168\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\left(c^{2}x^{2}-1\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}-\frac{5b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}}{c^{7}}$

```
[In] int(x^6*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a*x^7+1/7*b*x^7*arcsec(c*x)-1/42*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2
)^(1/2)*x^4-5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2-5/112*b
/c^7*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)-5/112*b/c^8*(c^2*x^2-1)^(1/2)/
((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int x^{6}\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{48\,ac^{7}x^{7}+96\,bc^{7}\arctan\left(-cx+\sqrt{c^{2}x^{2}-1}\right)+48\left(bc^{7}x^{7}-bc^{7}\right)\mathrm{arcsec}\left(cx\right)+15\,b\log\left(-cx+\sqrt{c^{2}x^{2}-1}\right)-}{336\,c^{7}}$$

```
[In] integrate(x^6*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/336*(48*a*c^7*x^7 + 96*b*c^7*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 48*(b*c^7
*x^7 - b*c^7)*arcsec(c*x) + 15*b*log(-c*x + sqrt(c^2*x^2 - 1)) - (8*b*c^5*x
^5 + 10*b*c^3*x^3 + 15*b*c*x)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.94

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^7}{7}+\frac{bx^7\operatorname{asec}(cx)}{7}-\frac{b\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

```
[In] integrate(x**6*(a+b*asec(c*x)),x)
```

```
[Out] a*x**7/7 + b*x**7*asec(c*x)/7 - b*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1))
 + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) -
 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2)
 > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 +
 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*
x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.42

$$\int x^6\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{7}\,ax^7+\frac{1}{672}\left(96\,x^7\operatorname{arcsec}(cx)-\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)b$$

```
[In] integrate(x^6*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7 + 1/672*(96*x^7*arcsec(c*x) - (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 4
0*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) -
 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*log
(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6)/
c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8644 vs. 2(96) = 192.

Time = 1.12 (sec) , antiderivative size = 8644, normalized size of antiderivative = 75.82

$$\int x^6 (a + b \sec^{-1}(cx)) \, dx = \text{Too large to display}$$

```
[In] integrate(x^6*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/336*c*(48*b*arccos(1/(c*x))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^
12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) - 15*b*log(abs(sqrt(-1/(c^2*
x^2) + 1) + 1/(c*x) + 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*
x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 +
 c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 15*b*log(abs(sqrt(-1/(c^2*x^2)
 + 1) - 1/(c*x) - 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c
^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x
) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2)
 - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8
*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 48*a/(c^8 + 7*c^8*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^
6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) - 336*b*(1/(
c^2*x^2) - 1)*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/
(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)
^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^2) - 105*b*(1
/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^
8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*
(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1
)^14)*(1/(c*x) + 1)^2) + 105*b*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2)
+ 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c
^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x
) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2)
 - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8
*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^2) - 66*b*sqrt(-1/(c^2
```

```
*x^2) + 1)/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1
/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x
^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)) - 336*a*(1/(c^2*x^2) - 1)/((c^8
 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/
(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*
x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 +
 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c
*x) + 1)^14)*(1/(c*x) + 1)^2) + 1008*b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/
((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/
(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)
^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/
(1/(c*x) + 1)^14)*(1/(c*x) + 1)^4) - 315*b*(1/(c^2*x^2) - 1)^2*log(abs(sqrt
(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^
8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^4) + 315
*b*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8
 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/
(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*
x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 +
 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c
*x) + 1)^14)*(1/(c*x) + 1)^4) + 56*b*(-1/(c^2*x^2) + 1)^(3/2)/((c^8 + 7*c^8
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(
1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)
^14)*(1/(c*x) + 1)^3) + 1008*a*(1/(c^2*x^2) - 1)^2/((c^8 + 7*c^8*(1/(c^2*x^
2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c
^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2)
 - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*
x) + 1)^4) - 1680*b*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2
*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(
1/(c*x) + 1)^6) - 525*b*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1)
+ 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1
/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)
^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(
```

c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^6) + 525*b*(1/(c^2*x^2) - 1
)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^
2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c
^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2)
 - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*
x) + 1)^6) - 170*b*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(
1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)
^14)*(1/(c*x) + 1)^5) - 1680*a*(1/(c^2*x^2) - 1)^3/((c^8 + 7*c^8*(1/(c^2*x^
2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c
^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2)
 - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*
x) + 1)^6) + 1680*b*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2
*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(
1/(c*x) + 1)^8) - 525*b*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1)
+ 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1
/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)
^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(
c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) + 525*b*(1/(c^2*x^2) - 1
)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^
2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c
^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2)
 - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*
x) + 1)^8) + 1680*a*(1/(c^2*x^2) - 1)^4/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*
x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/
(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8)
- 1008*b*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^
6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1
)^10) - 315*b*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x)
+ 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c

```
^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x
) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) -
 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^10) + 315*b*(1/(c^2*x^2) - 1)^5*log(a
bs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(
1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8
+ 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(
1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^1
0) + 170*b*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 3
5*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x
^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/
(c*x) + 1)^9) - 1008*a*(1/(c^2*x^2) - 1)^5/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c
^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8
 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/
(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^
10) + 336*b*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) -
1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x)
+ 1)^12) - 105*b*(1/(c^2*x^2) - 1)^6*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*
x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 3
5*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(
c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2
) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^12) + 105*b*(1/(c^2*x^2) - 1)^6*lo
g(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^
6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1
)^12) + 56*b*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2
*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(
1/(c*x) + 1)^11) + 336*a*(1/(c^2*x^2) - 1)^6/((c^8 + 7*c^8*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^
6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1
)^12) - 48*b*(1/(c^2*x^2) - 1)^7*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2)
```

```
 - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8
*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x)
+ 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) -
 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x)
 + 1)^14) - 15*b*(1/(c^2*x^2) - 1)^7*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*
x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 3
5*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(
c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2
) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^14) + 15*b*(1/(c^2*x^2) - 1)^7*log
(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^
8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6
/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)
^14) + 66*b*(1/(c^2*x^2) - 1)^6*sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/
(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*
x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1
/(c*x) + 1)^13) - 48*a*(1/(c^2*x^2) - 1)^7/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c
^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8
 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/
(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^
14))
```

Mupad [F(-1)]

Timed out.

$$\int x^6(a+b\sec^{-1}(cx))\,dx=\int x^6\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^6*(a + b*acos(1/(c*x))),x)
[Out] int(x^6*(a + b*acos(1/(c*x))), x)
```

3.2 $\int x^5(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{4b\sqrt{1-\frac{1}{c^2x^2}}\,x}{45c^5}-\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)$$

```
[Out] 1/6*x^6*(a+b*arcsec(c*x))-4/45*b*x*(1-1/c^2/x^2)^(1/2)/c^5-2/45*b*x^3*(1-1/
c^2/x^2)^(1/2)/c^3-1/30*b*x^5*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5328, 277, 197}

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)-\frac{bx^5\sqrt{1-\frac{1}{c^2x^2}}}{30c}-\frac{4bx\sqrt{1-\frac{1}{c^2x^2}}}{45c^5}-\frac{2bx^3\sqrt{1-\frac{1}{c^2x^2}}}{45c^3}$$

```
[In] Int[x^5*(a + b*ArcSec[c*x]),x]
```

```
[Out] (-4*b*Sqrt[1 - 1/(c^2*x^2)]*x)/(45*c^5) - (2*b*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(
45*c^3) - (b*Sqrt[1 - 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*ArcSec[c*x]))/
6
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)-\frac{b\int\frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{6c}\\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)-\frac{(2b)\int\frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{15c^3}\\
&= -\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)-\frac{(4b)\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{45c^5}\\
&= -\frac{4b\sqrt{1-\frac{1}{c^2x^2}}\,x}{45c^5}-\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\sec^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^6}{6}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{4x}{45c^5}-\frac{2x^3}{45c^3}-\frac{x^5}{30c}\right)+\frac{1}{6}bx^6\sec^{-1}(cx)$$

```
[In] Integrate[x^5*(a + b*ArcSec[c*x]),x]
[Out] (a*x^6)/6 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((-4*x)/(45*c^5) - (2*x^3)/(45
*c^3) - x^5/(30*c)) + (b*x^6*ArcSec[c*x])/6
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{x^{6}a}{6}+\frac{b\left(\frac{c^{6}x^{6}\,\mathrm{arcsec}(cx)}{6}-\frac{\left(c^{2}x^{2}-1\right)\left(3c^{4}x^{4}+4c^{2}x^{2}+8\right)}{90\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{6}}$	79
derivativedivides	$\frac{\frac{a\,c^{6}x^{6}}{6}+b\left(\frac{c^{6}x^{6}\,\mathrm{arcsec}(cx)}{6}-\frac{\left(c^{2}x^{2}-1\right)\left(3c^{4}x^{4}+4c^{2}x^{2}+8\right)}{90\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{6}}$	83
default	$\frac{\frac{a\,c^{6}x^{6}}{6}+b\left(\frac{c^{6}x^{6}\,\mathrm{arcsec}(cx)}{6}-\frac{\left(c^{2}x^{2}-1\right)\left(3c^{4}x^{4}+4c^{2}x^{2}+8\right)}{90\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{6}}$	83

```
[In] int(x^5*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/6*x^6*a+b/c^6*(1/6*c^6*x^6*arcsec(c*x)-1/90*(c^2*x^2-1)*(3*c^4*x^4+4*c^2*
x^2+8)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{15\,bc^6x^6\,\mathrm{arcsec}\,(cx)+15\,ac^6x^6-(3\,bc^4x^4+4\,bc^2x^2+8\,b)\sqrt{c^2x^2-1}}{90\,c^6}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] 1/90*(15*b*c^6*x^6*arcsec(c*x) + 15*a*c^6*x^6 - (3*b*c^4*x^4 + 4*b*c^2*x^2
+ 8*b)*sqrt(c^2*x^2 - 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.72

$$\int x^5\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^6}{6}+\frac{bx^6\,\mathrm{asec}\,(cx)}{6}-\frac{b\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

```
[In] integrate(x**5*(a+b*asec(c*x)),x)
```

```
[Out] a*x**6/6 + b*x**6*asec(c*x)/6 - b*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c)
 + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5),
Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c*
*2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^5 \left(a + b \sec^{-1}(cx)\right)\, dx = \frac{1}{6}\, a x^6 + \frac{1}{90} \left(15\, x^6 \operatorname{arcsec}(cx) - \frac{3\, c^4 x^5 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{5}{2}} + 10\, c^2 x^3 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{3}{2}} + 15\, x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^5}\right) b$$

```
[In] integrate(x^5*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arcsec(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2)
+ 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3862 vs. 2(75) = 150.

Time = 0.32 (sec) , antiderivative size = 3862, normalized size of antiderivative = 43.39

$$\int x^5 \left(a + b \sec^{-1}(cx)\right)\, dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/90*c*(15*b*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^
2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)
+ 15*a/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c
^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x)
 + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 90*b*(1/(c^2*x^2) -
1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x)
```

```
+ 1)^2) - 30*b*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*
c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12)*(1/(c*x) + 1)) - 90*a*(1/(c^2*x^2) - 1)/((c^7 + 6*c^7*(1/(c^2*x^2
) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^
7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x)
 + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1
)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2) + 225*b*(1/(c^2*x^2) - 1)^2*arccos(1
/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^
2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15
*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*
x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^4) + 7
0*b*(-1/(c^2*x^2) + 1)^(3/2)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c
^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)
*(1/(c*x) + 1)^3) + 225*a*(1/(c^2*x^2) - 1)^2/((c^7 + 6*c^7*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1
/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1
)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/
(1/(c*x) + 1)^12)*(1/(c*x) + 1)^4) - 300*b*(1/(c^2*x^2) - 1)^3*arccos(1/(c*
x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7
*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) +
 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^6) - 156*b
*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^
8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1
/(c*x) + 1)^12)*(1/(c*x) + 1)^5) - 300*a*(1/(c^2*x^2) - 1)^3/((c^7 + 6*c^7*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c
^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^6) + 225*b*(1/(c^2*x^2) - 1)
^4*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x)
+ 1)^8) - 156*b*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^
4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2
*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^7) + 225*a*(1/(c^2*x^2) - 1)^4
/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)
```

```
^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)
^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^8) - 90*b*(1/
(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) +
 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)
^12)*(1/(c*x) + 1)^10) - 70*b*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((
c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/
(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c
^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^9) - 90*a*(1/(c^
2*x^2) - 1)^5/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^
6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/
(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^
10) + 15*b*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6
/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^12) - 30*b*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^
2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
+ 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1
/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^11
) + 15*a*(1/(c^2*x^2) - 1)^6/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c
^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)
*(1/(c*x) + 1)^12))
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a+b\sec^{-1}(cx)\right)\,dx=\int x^5\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^5*(a + b*acos(1/(c*x))),x)
[Out] int(x^5*(a + b*acos(1/(c*x))), x)
```

3.3 $\int x^4(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^4(a+b\sec^{-1}(cx))\,dx=-\frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5(a+b\sec^{-1}(cx))-\frac{3b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}$$

```
[Out] 1/5*x^5*(a+b*arcsec(c*x))-3/40*b*arctanh((1-1/c^2/x^2)^(1/2))/c^5-3/40*b*x^
2*(1-1/c^2/x^2)^(1/2)/c^3-1/20*b*x^4*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5328, 272, 44, 65, 214}

$$\int x^4(a+b\sec^{-1}(cx))\,dx=\frac{1}{5}x^5(a+b\sec^{-1}(cx))-\frac{3b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}-\frac{bx^4\sqrt{1-\frac{1}{c^2x^2}}}{20c}-\frac{3bx^2\sqrt{1-\frac{1}{c^2x^2}}}{40c^3}$$

```
[In] Int[x^4*(a + b*ArcSec[c*x]),x]
```

```
[Out] (-3*b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(40*c^3) - (b*Sqrt[1 - 1/(c^2*x^2)]*x^4)/(
20*c) + (x^5*(a + b*ArcSec[c*x]))/5 - (3*b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/
(40*c^5)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{5}x^5\left(a+b\sec^{-1}(cx)\right)-\frac{b\int\frac{x^3}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{5c}$$

$$=\frac{1}{5}x^5\left(a+b\sec^{-1}(cx)\right)+\frac{b\text{Subst}\left(\int\frac{1}{x^3\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{10c}$$

$$=-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\sec^{-1}(cx)\right)+\frac{(3b)\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{40c^3}$$

$$= -\frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5\left(a+b\sec^{-1}(cx)\right) + \frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{80c^5}$$

$$= -\frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5\left(a+b\sec^{-1}(cx)\right) - \frac{(3b)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^3}$$

$$= -\frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5\left(a+b\sec^{-1}(cx)\right) - \frac{3b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^5}{5} + b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{3x^2}{40c^3} - \frac{x^4}{20c}\right) + \frac{1}{5}bx^5\sec^{-1}(cx) - \frac{3b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{40c^5}$$

```
[In] Integrate[x^4*(a + b*ArcSec[c*x]),x]
```

```
[Out] (a*x^5)/5 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((-3*x^2)/(40*c^3) - x^4/(20*c
)) + (b*x^5*ArcSec[c*x])/5 - (3*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]
)])/(40*c^5)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.58

method	result	size
parts	$\frac{a\,x^5}{5}+\frac{x^5 b\,\operatorname{arcsec}(cx)}{5}-\frac{b(c^2x^2-1)x^2}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b(c^2x^2-1)}{40c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$	141
derivativedivides	$\frac{\frac{a\,c^5x^5}{5}+\frac{b\,c^5x^5\,\operatorname{arcsec}(cx)}{5}-\frac{b\left(c^2x^2-1\right)c^2x^2}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^5}$	148
default	$\frac{\frac{a\,c^5x^5}{5}+\frac{b\,c^5x^5\,\operatorname{arcsec}(cx)}{5}-\frac{b\left(c^2x^2-1\right)c^2x^2}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^5}$	148

```
[In] int(x^4*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arcsec(c*x)-1/20*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2
)^(1/2)*x^2-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)-3/40*b/c^6*(
c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{8\,ac^5x^5+16\,bc^5\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+8\left(bc^5x^5-bc^5\right)\operatorname{arcsec}\left(cx\right)+3\,b\log\left(-cx+\sqrt{c^2x^2-1}\right)-(2\,b}{40\,c^5}$$

```
[In] integrate(x^4*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*x^5 + 16*b*c^5*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 8*(b*c^5*x^
5 - b*c^5)*arcsec(c*x) + 3*b*log(-c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^3*x^3 +
 3*b*c*x)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.97

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^5}{5}+\frac{bx^5\operatorname{asec}\left(cx\right)}{5} - \frac{b\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}\left(cx\right)}{8c^4} & \text{for}\ \left|c^2x^2\right|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}\left(cx\right)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

```
[In] integrate(x**4*(a+b*asec(c*x)),x)
```

```
[Out] a*x**5/5 + b*x**5*asec(c*x)/5 - b*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1))
 + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*ac
osh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
- 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{5}\,ax^5+\frac{1}{80}\left(16\,x^5\,\mathrm{arcsec}\left(cx\right)+\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)b$$

```
[In] integrate(x^4*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/80*(16*x^5*arcsec(c*x) + (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*s
qrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) +
 c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1
) - 1)/c^4)/c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4828 vs. 2(75) = 150.

Time = 0.79 (sec) , antiderivative size = 4828, normalized size of antiderivative = 54.25

$$\int x^4\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x^4*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/40*c*(8*b*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^
2) - 1)^5/(1/(c*x) + 1)^10) - 3*b*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x)
+ 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6
*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1
)^10) + 3*b*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^6 + 5*c^6*(1/
(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(
1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) + 8*a/(c^6 + 5*c
^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) -
```

```
1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 40*b*(1/
(c^2*x^2) - 1)*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^
2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 15*b*(1/(c^2*x^2) - 1)*l
og(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1
/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) + 15*b*(1/(
c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)
^2) - 10*b*sqrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) -
 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/
(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)) - 40*a*(1/(c^2*x^2) - 1)/
((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(
c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)
*(1/(c*x) + 1)^2) + 80*b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^6 + 5*c^6*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)
^4) - 30*b*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1
))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(
1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^
10)*(1/(c*x) + 1)^4) + 30*b*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) +
 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^
6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x)
 + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)
^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) + 4*b*(-1/(c^2*x^2) + 1)^(3/2)/((c^6
+ 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(
c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c
*x) + 1)^3) + 80*a*(1/(c^2*x^2) - 1)^2/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(
c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^
6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) - 80*b*(1/(c^2*x^2
) - 1)^3*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2)
 - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) - 30*b*(1/(c^2*x^2) - 1)^3*log(a
bs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(
```

```
1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) + 30*b*(1/(c^2*
x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^
4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/
(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6
) - 80*a*(1/(c^2*x^2) - 1)^3/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*
x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) + 40*b*(1/(c^2*x^2) - 1)^4*a
rccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/
(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1
/(c*x) + 1)^10)*(1/(c*x) + 1)^8) - 15*b*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1
/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c
^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) + 15*b*(1/(c^2*x^2) - 1)^
4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2)
 - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6
*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) + 4*b*(1
/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^7) + 40*a*(1/(c^2*x
^2) - 1)^4/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*
x) + 1)^10)*(1/(c*x) + 1)^8) - 8*b*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^
6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*
x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/
(c*x) + 1)^10) - 3*b*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1
/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(
c*x) + 1)^10)*(1/(c*x) + 1)^10) + 3*b*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(
c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2
*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) + 10*b*(1/(c^2*x^2) - 1)^4
*sqrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 1
0*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(
```

```
c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2)
- 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^9) - 8*a*(1/(c^2*x^2) - 1)^5/((c^6 +
 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2
) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*
x) + 1)^10))
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\sec^{-1}(cx))\,dx=\int x^4\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^4*(a + b*acos(1/(c*x))),x)
```

```
[Out] int(x^4*(a + b*acos(1/(c*x))), x)
```

3.4 $\int x^3(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int x^3(a+b\sec^{-1}(cx))\,dx=-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4(a+b\sec^{-1}(cx))$$

```
[Out] 1/4*x^4*(a+b*arcsec(c*x))-1/6*b*x*(1-1/c^2/x^2)^(1/2)/c^3-1/12*b*x^3*(1-1/c
^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5328, 277, 197}

$$\int x^3(a+b\sec^{-1}(cx))\,dx=\frac{1}{4}x^4(a+b\sec^{-1}(cx))-\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}}{12c}-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{6c^3}$$

```
[In] Int[x^3*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/6*(b*Sqrt[1 - 1/(c^2*x^2)]*x)/c^3 - (b*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(12*c)
 + (x^4*(a + b*ArcSec[c*x]))/4
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right) - \frac{b\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{4c} \\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c} + \frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right) - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{6c^3} \\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c} + \frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$
\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^4}{4} + b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(-\frac{x}{6c^3}-\frac{x^3}{12c}\right) + \frac{1}{4}bx^4\sec^{-1}(cx)
$$

```
[In] Integrate[x^3*(a + b*ArcSec[c*x]),x]
[Out] (a*x^4)/4 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*(-1/6*x/c^3 - x^3/(12*c)) + (b
*x^4*ArcSec[c*x])/4
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

method	result	size
parts	$\frac{x^4 a}{4}+\frac{b\left(\frac{c^4 x^4 \operatorname{arcsec}(cx)}{4}-\frac{(c^2x^2-1)(c^2x^2+2)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	70
derivativedivides	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4 x^4 \operatorname{arcsec}(cx)}{4}-\frac{(c^2x^2-1)(c^2x^2+2)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	74
default	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4 x^4 \operatorname{arcsec}(cx)}{4}-\frac{(c^2x^2-1)(c^2x^2+2)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	74

```
[In] int(x^3*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*a+b/c^4*(1/4*c^4*x^4*arcsec(c*x)-1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^2
*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{3\,bc^4x^4\operatorname{arcsec}(cx)+3\,ac^4x^4-(bc^2x^2+2\,b)\sqrt{c^2x^2-1}}{12\,c^4}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^4*x^4*arcsec(c*x) + 3*a*c^4*x^4 - (b*c^2*x^2 + 2*b)*sqrt(c^2*x^
2 - 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^4}{4}+\frac{bx^4\operatorname{asec}(cx)}{4}-\frac{b\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

```
[In] integrate(x**3*(a+b*asec(c*x)),x)
```

```
[Out] a*x**4/4 + b*x**4*asec(c*x)/4 - b*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c)
 + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*
x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\,\mathrm{arcsec}\,(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)b$$

```
[In] integrate(x^3*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3
*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. 2(54) = 108.

Time = 0.31 (sec) , antiderivative size = 1926, normalized size of antiderivative = 30.09

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/12*c*(3*b*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 3*a/(c^5 + 4*c^5*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)
^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8) - 12*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8)*(1/(c*x) + 1)^2) - 6*b*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8)*(1/(c*x) + 1)) - 12*a*(1/(c^2*x^2) - 1)/((c^5 + 4*c^5*(1/(c^2*x^2
) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8)*(1/(c*x) + 1)^2) + 18*b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^5 + 4*c
^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 10*b*(-1/(c^2*x^2) + 1)^(3/2)/((c^5 +
```

```
4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 18*a*(1/(c^2*x^2) - 1)^2/((c^5 + 4*
c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 12*b*(1/(c^2*x^2) - 1)^3*arccos(1/(c*
x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(
c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 10*b*(1/(c^2*x^2) - 1)^
2*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) - 12
*a*(1/(c^2*x^2) - 1)^3/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*
c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x
) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 3*b*
(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) -
 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) +
 1)^8) - 6*b*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8)*(1/(c*x) + 1)^7) + 3*a*(1/(c^2*x^2) - 1)^4/((c^5 + 4*c^5*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c
^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
 1)^8)*(1/(c*x) + 1)^8))
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(a + b*acos(1/(c*x))),x)
[Out] int(x^3*(a + b*acos(1/(c*x))), x)
```

3.5 $\int x^2(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

```
[Out] 1/3*x^3*(a+b*arcsec(c*x))-1/6*b*arctanh((1-1/c^2/x^2)^(1/2))/c^3-1/6*b*x^2*
(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {5328, 272, 44, 65, 214}

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}-\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}}{6c}$$

```
[In] Int[x^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/6*(b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/c + (x^3*(a + b*ArcSec[c*x]))/3 - (b*Arc
Tanh[Sqrt[1 - 1/(c^2*x^2)]])/(6*c^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)-\frac{b\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{3c}\\
&= \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)+\frac{b\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{6c}\\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)+\frac{b\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{12c^3}\\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int\frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}\\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)-\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

$$\int x^2\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{ax^3}{3}-\frac{bx^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c}+\frac{1}{3}bx^3\sec^{-1}(cx)-\frac{b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

[In] Integrate[x^2*(a + b*ArcSec[c*x]),x]

[Out] (a*x^3)/3 - (b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcSec[c*x])/3 - (b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

method	result	size
parts	$\frac{x^3a}{3}+\frac{b\left(\frac{c^3x^3\,\mathrm{arcsec}(cx)}{3}-\frac{\sqrt{c^2x^2-1}\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	94
derivativedivides	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\mathrm{arcsec}(cx)}{3}-\frac{\sqrt{c^2x^2-1}\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	98
default	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\mathrm{arcsec}(cx)}{3}-\frac{\sqrt{c^2x^2-1}\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	98

[In] int(x^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/3*x^3*a+b/c^3*(1/3*c^3*x^3*arcsec(c*x)-1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

$$\int x^2(a+b\sec^{-1}(cx))\,dx$$
$$=\frac{2\,ac^3x^3+4\,bc^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}bcx+2\,(bc^3x^3-bc^3)\operatorname{arcsec}(cx)+b\log\left(-cx+\sqrt{\right.}}{6\,c^3}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3 + 4*b*c^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2
- 1)*b*c*x + 2*(b*c^3*x^3 - b*c^3)*arcsec(c*x) + b*log(-c*x + sqrt(c^2*x^2
- 1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int x^2(a+b\sec^{-1}(cx))\,dx=\frac{ax^3}{3}+\frac{bx^3\operatorname{asec}(cx)}{3}-\frac{b\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate(x**2*(a+b*asec(c*x)),x)
```

```
[Out] a*x**3/3 + b*x**3*asec(c*x)/3 - b*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) +
acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1
)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

$$\int x^2(a+b\sec^{-1}(cx))\,dx$$
$$=\frac{1}{3}\,ax^3+\frac{1}{12}\left(4\,x^3\operatorname{arcsec}(cx)-\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)b$$

```
[In] integrate(x^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^
2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2
*x^2) + 1) - 1)/c^2)/c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2101 vs. $2(54) = 108$.

Time = 0.59 (sec) , antiderivative size = 2101, normalized size of antiderivative = 32.83

$$\int x^2 \left(a + b \sec^{-1}(cx)\right) \, dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*c*(2*b*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*
x) + 1)^6) - b*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^4 + 3*c^4*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)
^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + b*log(abs(sqrt(-1/(c^2*x^2)
 + 1) - 1/(c*x) - 1))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^
4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6) + 2*a/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 6*
b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 3*b*(1/(c^2*x^2) - 1)*log(abs(sqrt(
-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 3*b*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1
/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 2*b*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 3*c^
4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)) - 6*a*(1/(c^
2*x^2) - 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/
(c*x) + 1)^2) + 6*b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 3*b*(1/(c^2*x^2
) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 3*b*(1/(c^2*x^2
) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c
```

```
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 6*a*(1/(c^2*x^2
) - 1)^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^
2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*
x) + 1)^4) - 2*b*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4
*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) - b*(1/(c^2*x^2) - 1
)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^
2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + b*(1/(c^2*x^2) - 1)^
3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2)
 - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + 2*b*(1/(c^2*x^2) - 1)^
2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x
) + 1)^6)*(1/(c*x) + 1)^5) - 2*a*(1/(c^2*x^2) - 1)^3/((c^4 + 3*c^4*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4
*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6))
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\sec^{-1}(cx))\,dx=\int x^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(a + b*acos(1/(c*x))),x)
[Out] int(x^2*(a + b*acos(1/(c*x))), x)
```

3.6 $\int x(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)$$

```
[Out] 1/2*x^2*(a+b*arcsec(c*x))-1/2*b*x*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5328, 197}

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2c}$$

```
[In] Int[x*(a + b*ArcSec[c*x]),x]
[Out] -1/2*(b*Sqrt[1 - 1/(c^2*x^2)]*x)/c + (x^2*(a + b*ArcSec[c*x]))/2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
```

```
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right) - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{2c}$$
$$= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int x\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{ax^2}{2} - \frac{bx\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c} + \frac{1}{2}bx^2\sec^{-1}(cx)$$

```
[In] Integrate[x*(a + b*ArcSec[c*x]),x]
[Out] (a*x^2)/2 - (b*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcSec[c*x]
)/2
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.56

method	result	size
parts	$\frac{a\,x^2}{2}+\frac{b\left(\frac{c^2x^2\,\mathrm{arcsec}(cx)}{2}-\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	61
derivativedivides	$\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\,\mathrm{arcsec}(cx)}{2}-\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	65
default	$\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\,\mathrm{arcsec}(cx)}{2}-\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	65

```
[In] int(x*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arcsec(c*x)-1/2/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/
x*(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x(a+b\sec^{-1}(cx))\,dx=\frac{bc^2x^2\,\mathrm{arcsec}\,(cx)+ac^2x^2-\sqrt{c^2x^2-1}b}{2\,c^2}$$

```
[In] integrate(x*(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] 1/2*(b*c^2*x^2*arcsec(c*x) + a*c^2*x^2 - sqrt(c^2*x^2 - 1)*b)/c^2
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int x(a+b\sec^{-1}(cx))\,dx=\frac{ax^2}{2}+\frac{bx^2\,\mathrm{asec}\,(cx)}{2}-\frac{b\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for}\ |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

```
[In] integrate(x*(a+b*asec(c*x)),x)
[Out] a*x**2/2 + b*x**2*asec(c*x)/2 - b*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**
2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x(a+b\sec^{-1}(cx))\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\,\mathrm{arcsec}\,(cx)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)b$$

```
[In] integrate(x*(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*(x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1)/c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(33) = 66$.

Time = 0.29 (sec) , antiderivative size = 634, normalized size of antiderivative = 16.26

$$\int x(a + b\sec^{-1}(cx))\,dx$$
$$= \frac{1}{2}c\left(\frac{b\arccos\left(\frac{1}{cx}\right)}{c^3 + \frac{2c^3\left(\frac{1}{c^2x^2}-1\right)}{\left(\frac{1}{cx}+1\right)^2} + \frac{c^3\left(\frac{1}{c^2x^2}-1\right)^2}{\left(\frac{1}{cx}+1\right)^4}} + \frac{a}{c^3 + \frac{2c^3\left(\frac{1}{c^2x^2}-1\right)}{\left(\frac{1}{cx}+1\right)^2} + \frac{c^3\left(\frac{1}{c^2x^2}-1\right)^2}{\left(\frac{1}{cx}+1\right)^4}} - \frac{2b\left(\frac{1}{c^2x^2}-1\right)\arccos\left(\frac{1}{cx}\right)}{\left(c^3 + \frac{2c^3\left(\frac{1}{c^2x^2}-1\right)}{\left(\frac{1}{cx}+1\right)^2} + \frac{c^3\left(\frac{1}{c^2x^2}-1\right)^2}{\left(\frac{1}{cx}+1\right)^4}\right)\right.$$

```
[In] integrate(x*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*c*(b*arccos(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c
^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + a/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b*(1/(c^2*x^
2) - 1)*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c
^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 2*b*sqrt(-1/(c^2
*x^2) + 1)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^
2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) - 2*a*(1/(c^2*x^2) - 1)/((c^3 + 2
*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
 1)^4)*(1/(c*x) + 1)^2) + b*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^3 + 2*c
^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4)*(1/(c*x) + 1)^4) + 2*b*(-1/(c^2*x^2) + 1)^(3/2)/((c^3 + 2*c^3*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*
x) + 1)^3) + a*(1/(c^2*x^2) - 1)^2/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4))
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x(a + b\sec^{-1}(cx))\,dx = \frac{a\,x^2}{2} + \frac{b\,x^2\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{2} - \frac{b\,x\,\sqrt{1-\frac{1}{c^2\,x^2}}}{2\,c}$$

```
[In] int(x*(a + b*acos(1/(c*x))),x)
```

```
[Out] (a*x^2)/2 + (b*x^2*acos(1/(c*x)))/2 - (b*x*(1 - 1/(c^2*x^2))^(1/2))/(2*c)
```

3.7 $\int (a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int (a+b\sec^{-1}(cx))\,dx = ax+bx\sec^{-1}(cx)-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] a*x+b*x*arcsec(c*x)-b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5322, 272, 65, 214}

$$\int (a+b\sec^{-1}(cx))\,dx = ax-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+bx\sec^{-1}(cx)$$

```
[In] Int[a + b*ArcSec[c*x],x]
```

```
[Out] a*x + b*x*ArcSec[c*x] - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5322

```
Int[ArcSec[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSec[c*x], x] - Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \sec^{-1}(cx)\,dx \\
&= ax + bx\sec^{-1}(cx) - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\,dx}{c} \\
&= ax + bx\sec^{-1}(cx) + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2c} \\
&= ax + bx\sec^{-1}(cx) - (bc)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right) \\
&= ax + bx\sec^{-1}(cx) - \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int \left(a + b\sec^{-1}(cx)\right)\,dx = ax + bx\sec^{-1}(cx) - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

```
[In] Integrate[a + b*ArcSec[c*x],x]
```

```
[Out] a*x + b*x*ArcSec[c*x] - (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

method	result	size
default	$ax + bx \operatorname{arcsec}(cx) - \frac{b \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c}$	38
parts	$ax + bx \operatorname{arcsec}(cx) - \frac{b \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2 x^2}}\right)}{c}$	38
derivativedivides	$\frac{acx + b\left(cx \operatorname{arcsec}(cx) - \ln\left(cx + cx\sqrt{1 - \frac{1}{c^2 x^2}}\right)\right)}{c}$	42

```
[In] int(a+b*arcsec(c*x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arcsec(c*x)-b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int (a + b\sec^{-1}(cx))\, dx = \frac{acx + 2\,bc \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + (bcx - bc) \operatorname{arcsec}(cx) + b \log\left(-cx + \sqrt{c^2x^2 - 1}\right)}{c}$$

```
[In] integrate(a+b*arcsec(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x + 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arcsec(c*x)
 + b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (a + b\sec^{-1}(cx))\, dx = ax + b\left(x \operatorname{asec}(cx) - \frac{\begin{cases} \operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1 \\ -i \operatorname{asin}(cx) & \text{otherwise} \end{cases}}{c}\right)$$

```
[In] integrate(a+b*asec(c*x),x)
```

```
[Out] a*x + b*(x*asec(c*x) - Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = ax + \frac{\left(2\,cx\operatorname{arcsec}\left(cx\right) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)b}{2\,c}$$

```
[In] integrate(a+b*arcsec(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{2}\,bc\left(\frac{2\,x\arccos\left(\frac{1}{cx}\right)}{c} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}\right) + ax$$

```
[In] integrate(a+b*arcsec(c*x),x, algorithm="giac")
```

```
[Out] 1/2*b*c*(2*x*arccos(1/(c*x))/c - (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sq
rt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = a\,x + b\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right) - \frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

```
[In] int(a + b*acos(1/(c*x)),x)
```

```
[Out] a*x + b*x*acos(1/(c*x)) - (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

3.8 $\int \frac{a+b\sec^{-1}(cx)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx = \frac{i(a+b\sec^{-1}(cx))^2}{2b} - (a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right) + \frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)$$

```
[Out] 1/2*I*(a+b*arcsec(c*x))^2/b-(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(
1/2))^2)+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5326, 4722, 3800, 2221, 2317, 2438}

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx = \frac{i(a+b\sec^{-1}(cx))^2}{2b} - \log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx)) + \frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)$$

```
[In] Int[(a + b*ArcSec[c*x])/x,x]
```

```
[Out] ((I/2)*(a + b*ArcSec[c*x])^2)/b - (a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcS
ec[c*x])] + (I/2)*b*PolyLog[2, -E^((2*I)*ArcSec[c*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5326

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)\\
&= \text{Subst}\left(\int (a+bx)\tan(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b} - 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2b} - \left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\
&\quad + b\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \sec^{-1}(cx)\right)
\end{aligned}
$$

$$= \frac{i(a+b\sec^{-1}(cx))^2}{2b} - (a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right) - \frac{1}{2}(ib)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\sec^{-1}(cx)}\right)$$

$$= \frac{i(a+b\sec^{-1}(cx))^2}{2b} - (a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right) + \frac{1}{2}ib\,\text{PolyLog}\left(2, -e^{2i\sec^{-1}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx = \frac{1}{2}ib\sec^{-1}(cx)^2 - b\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) + a\log(x) + \frac{1}{2}ib\,\text{PolyLog}\left(2, -e^{2i\sec^{-1}(cx)}\right)$$

```
[In] Integrate[(a + b*ArcSec[c*x])/x,x]
[Out] (I/2)*b*ArcSec[c*x]^2 - b*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] + a*Lo
g[x] + (I/2)*b*PolyLog[2, -E^((2*I)*ArcSec[c*x])]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.31

method	result
parts	$a\ln(x) + b\left(\frac{i\,\text{arcsec}(cx)^2}{2} - \text{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right) + \frac{i\,\text{polylog}\left(2,-\left(\frac{1}{cx}+i\sqrt{1\right.\right.}}{2}\right.$
derivativedivides	$a\ln(cx) + b\left(\frac{i\,\text{arcsec}(cx)^2}{2} - \text{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right) + \frac{i\,\text{polylog}\left(2,-\left(\frac{1}{cx}+i\sqrt{\right.\right.}}{2}\right.$
default	$a\ln(cx) + b\left(\frac{i\,\text{arcsec}(cx)^2}{2} - \text{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right) + \frac{i\,\text{polylog}\left(2,-\left(\frac{1}{cx}+i\sqrt{\right.\right.}}{2}\right.$

```
[In] int((a+b*arcsec(c*x))/x,x,method=_RETURNVERBOSE)
[Out] a*ln(x)+b*(1/2*I*arcsec(c*x)^2-arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2
))^2)+1/2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

Fricas [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x,x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x}\,dx = \int \frac{a + b\,\mathrm{asec}\,(cx)}{x}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x,x)
[Out] Integral((a + b*asec(c*x))/x, x)
```

Maxima [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x,x, algorithm="maxima")
[Out] -(c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) -
arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b + a*log(x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/x,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expansi
on Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/x,x)
[Out] int((a + b*acos(1/(c*x)))/x, x)
```

3.9 $\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx=bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{x}$$

```
[Out] (-a-b*arcsec(c*x))/x+b*c*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5328, 267}

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx=bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{x}$$

```
[In] Int[(a + b*ArcSec[c*x])/x^2,x]
```

```
[Out] b*c*Sqrt[1 - 1/(c^2*x^2)] - (a + b*ArcSec[c*x])/x
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
```

```
m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\sec^{-1}(cx)}{x}+\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^3}\,dx}{c}\\
&= bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx=-\frac{a}{x}+bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\sec^{-1}(cx)}{x}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/x^2,x]
[Out] -(a/x) + b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*ArcSec[c*x])/x
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

method	result	size
parts	$-\frac{a}{x}+bc\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)$	58
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$	62
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$	62

```
[In] int((a+b*arcsec(c*x))/x^2,x,method=_RETURNVERBOSE)
[Out] -a/x+b*c*(-1/c/x*arcsec(c*x)+1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^2
-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = -\frac{b\,\mathrm{arcsec}\,(cx)-\sqrt{c^2x^2-1}b+a}{x}$$

```
[In] integrate((a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
[Out] -(b*arcsec(c*x) - sqrt(c^2*x^2 - 1)*b + a)/x
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = \begin{cases} -\frac{a}{x}+bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{b\,\mathrm{asec}\,(cx)}{x} & \text{for } c\neq 0 \\ -\frac{a+\tilde{\infty}b}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*asec(c*x))/x**2,x)
[Out] Piecewise((-a/x + b*c*sqrt(1 - 1/(c**2*x**2)) - b*asec(c*x)/x, Ne(c, 0)), (
-(a + zoo*b)/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = \left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\mathrm{arcsec}\,(cx)}{x}\right)b-\frac{a}{x}$$

```
[In] integrate((a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
[Out] (c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b - a/x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = \left(b\sqrt{-\frac{1}{c^2x^2}+1}-\frac{b\arccos\left(\frac{1}{cx}\right)}{cx}-\frac{a}{cx}\right)c$$

```
[In] integrate((a+b*arcsec(c*x))/x^2,x, algorithm="giac")
[Out] (b*sqrt(-1/(c^2*x^2) + 1) - b*arccos(1/(c*x))/(c*x) - a/(c*x))*c
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{a+b\sec^{-1}(cx)}{x^2}\,dx = b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{a}{x}-\frac{b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x}$$

```
[In] int((a + b*acos(1/(c*x)))/x^2,x)
[Out] b*c*(1 - 1/(c^2*x^2))^(1/2) - a/x - (b*acos(1/(c*x)))/x
```

3.10 $\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx=\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}-\frac{1}{4}bc^2\csc^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{2x^2}$$

```
[Out] -1/4*b*c^2*arccsc(c*x)+1/2*(-a-b*arcsec(c*x))/x^2+1/4*b*c*(1-1/c^2/x^2)^(1/
2)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5328, 342, 327, 222}

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx=-\frac{a+b\sec^{-1}(cx)}{2x^2}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}-\frac{1}{4}bc^2\csc^{-1}(cx)$$

```
[In] Int[(a + b*ArcSec[c*x])/x^3,x]
```

```
[Out] (b*c*Sqrt[1 - 1/(c^2*x^2)])/(4*x) - (b*c^2*ArcCsc[c*x])/4 - (a + b*ArcSec[c
*x])/(2*x^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\sec^{-1}(cx)}{2x^2}+\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}\,dx}{2c}\\
&= -\frac{a+b\sec^{-1}(cx)}{2x^2}-\frac{b\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2c}\\
&= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}-\frac{a+b\sec^{-1}(cx)}{2x^2}-\frac{1}{4}(bc)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}-\frac{1}{4}bc^2\csc^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = -\frac{a}{2x^2}+\frac{bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x}-\frac{b\sec^{-1}(cx)}{2x^2}-\frac{1}{4}bc^2\arcsin\left(\frac{1}{cx}\right)$$

```
[In] Integrate[(a + b*ArcSec[c*x])/x^3,x]
[Out] -1/2*a/x^2 + (b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcSec[c*x])/(
2*x^2) - (b*c^2*ArcSin[1/(c*x)])/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(46) = 92.

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.86

method	result	size
parts	$-\frac{a}{2x^2}+b\,c^2\left(-\frac{\operatorname{arcsec}(cx)}{2c^2x^2}+\frac{\sqrt{c^2x^2-1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)$	95
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arcsec}(cx)}{2c^2x^2}+\frac{\sqrt{c^2x^2-1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)\right)$	99
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arcsec}(cx)}{2c^2x^2}+\frac{\sqrt{c^2x^2-1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)\right)$	99

```
[In] int((a+b*arcsec(c*x))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arcsec(c*x)+1/4*(c^2*x^2-1)^(1/2)*(-arctan(1
/(c^2*x^2-1)^(1/2))*c^2*x^2+(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)/
c^3/x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx=\frac{(bc^2x^2-2\,b)\operatorname{arcsec}(cx)+\sqrt{c^2x^2-1}b-2\,a}{4\,x^2}$$

```
[In] integrate((a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
[Out] 1/4*((b*c^2*x^2 - 2*b)*arcsec(c*x) + sqrt(c^2*x^2 - 1)*b - 2*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.33

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{b\,\operatorname{asec}(cx)}{2x^2}+\frac{b\left(\begin{cases}\frac{ic^3\operatorname{acosh}\left(\frac{1}{cx}\right)}{2}-\frac{ic^2}{2x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{2x^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|}>1\\ -\frac{c^3\operatorname{asin}\left(\frac{1}{cx}\right)}{2}+\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{2c}$$

```
[In] integrate((a+b*asec(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) - b*asec(c*x)/(2*x**2) + b*Piecewise((I*c**3*acosh(1/(c*x))/2 -
 I*c**2/(2*x*sqrt(-1 + 1/(c**2*x**2))) + I/(2*x**3*sqrt(-1 + 1/(c**2*x**2))
), 1/Abs(c**2*x**2) > 1), (-c**3*asin(1/(c*x))/2 + c**2*sqrt(1 - 1/(c**2*x*
*2))/(2*x), True))/(2*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = -\frac{1}{4}\,b\left(\frac{\frac{c^4x\sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}-c^3\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c}+\frac{2\,\mathrm{arcsec}\,(cx)}{x^2}\right)-\frac{a}{2\,x^2}$$

```
[In] integrate((a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3
*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) - 1/2*a/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx = \frac{1}{4}\left(bc\arccos\left(\frac{1}{cx}\right)+\frac{b\sqrt{-\frac{1}{c^2x^2}+1}}{x}-\frac{2\,b\arccos\left(\frac{1}{cx}\right)}{cx^2}-\frac{2\,a}{cx^2}\right)c$$

```
[In] integrate((a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*c*arccos(1/(c*x)) + b*sqrt(-1/(c^2*x^2) + 1)/x - 2*b*arccos(1/(c*x))
/(c*x^2) - 2*a/(c*x^2))*c
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{a+b\sec^{-1}(cx)}{x^3}\,dx=\frac{b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}}{4\,x}-\frac{b\,c^2\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2}-1\right)}{4}-\frac{a}{2\,x^2}$$

```
[In] int((a + b*acos(1/(c*x)))/x^3,x)
[Out] (b*c*(1 - 1/(c^2*x^2))^(1/2))/(4*x) - (b*c^2*acos(1/(c*x))*(2/(c^2*x^2) - 1
))/4 - a/(2*x^2)
```

3.11 $\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx=\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}-\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{a+b\sec^{-1}(cx)}{3x^3}$$

```
[Out] -1/9*b*c^3*(1-1/c^2/x^2)^(3/2)+1/3*(-a-b*arcsec(c*x))/x^3+1/3*b*c^3*(1-1/c^
2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5328, 272, 45}

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx=-\frac{a+b\sec^{-1}(cx)}{3x^3}-\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}+\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}$$

```
[In] Int[(a + b*ArcSec[c*x])/x^4,x]
```

```
[Out] (b*c^3*Sqrt[1 - 1/(c^2*x^2)])/3 - (b*c^3*(1 - 1/(c^2*x^2))^(3/2))/9 - (a +
b*ArcSec[c*x])/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\sec^{-1}(cx)}{3x^3}+\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^5}\,dx}{3c}\\
&= -\frac{a+b\sec^{-1}(cx)}{3x^3}-\frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{6c}\\
&= -\frac{a+b\sec^{-1}(cx)}{3x^3}-\frac{b\text{Subst}\left(\int \left(\frac{c^2}{\sqrt{1-\frac{x}{c^2}}}-c^2\sqrt{1-\frac{x}{c^2}}\right)\,dx,x,\frac{1}{x^2}\right)}{6c}\\
&= \frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}-\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{a+b\sec^{-1}(cx)}{3x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = -\frac{a}{3x^3}+b\left(\frac{2c^3}{9}+\frac{c}{9x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\sec^{-1}(cx)}{3x^3}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 + b*((2*c^3)/9 + c/(9*x^2))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*
ArcSec[c*x])/(3*x^3)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

method	result	size
parts	$-\frac{a}{3x^{3}}+b\,c^{3}\left(-\frac{\operatorname{arcsec}(cx)}{3c^{3}x^{3}}+\frac{(c^{2}x^{2}-1)\,(2c^{2}x^{2}+1)}{9\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)$	71
derivativedivides	$c^{3}\left(-\frac{a}{3c^{3}x^{3}}+b\left(-\frac{\operatorname{arcsec}(cx)}{3c^{3}x^{3}}+\frac{(c^{2}x^{2}-1)\,(2c^{2}x^{2}+1)}{9\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)\right)$	75
default	$c^{3}\left(-\frac{a}{3c^{3}x^{3}}+b\left(-\frac{\operatorname{arcsec}(cx)}{3c^{3}x^{3}}+\frac{(c^{2}x^{2}-1)\,(2c^{2}x^{2}+1)}{9\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)\right)$	75

```
[In] int((a+b*arcsec(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arcsec(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx=-\frac{3\,b\operatorname{arcsec}\left(cx\right)-\left(2\,bc^2x^2+b\right)\sqrt{c^2x^2-1}+3\,a}{9\,x^3}$$

```
[In] integrate((a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b*arcsec(c*x) - (2*b*c^2*x^2 + b)*sqrt(c^2*x^2 - 1) + 3*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.83

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx=-\frac{a}{3x^3}-\frac{b\operatorname{asec}\left(cx\right)}{3x^3}+\frac{b\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for}\ \left|c^2x^2\right|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((a+b*asec(c*x))/x**4,x)
```

```
[Out] -a/(3*x**3) - b*asec(c*x)/(3*x**3) + b*Piecewise((2*c**3*sqrt(c**2*x**2 - 1
)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sq
rt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = -\frac{1}{9}\,b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\operatorname{arcsec}\left(cx\right)}{x^3}\right)-\frac{a}{3\,x^3}$$

```
[In] integrate((a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
[Out] -1/9*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3
*arcsec(c*x)/x^3) - 1/3*a/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = \frac{1}{9}\left(2\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{b\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}-\frac{3\,b\arccos\left(\frac{1}{cx}\right)}{cx^3}-\frac{3\,a}{cx^3}\right)c$$

```
[In] integrate((a+b*arcsec(c*x))/x^4,x, algorithm="giac")
[Out] 1/9*(2*b*c^2*sqrt(-1/(c^2*x^2) + 1) + b*sqrt(-1/(c^2*x^2) + 1)/x^2 - 3*b*ar
ccos(1/(c*x))/(c*x^3) - 3*a/(c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/x^4,x)
[Out] int((a + b*acos(1/(c*x)))/x^4, x)
```

3.12 $\int \frac{a+b \sec^{-1}(cx)}{x^5} dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx=\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}+\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}-\frac{3}{32}bc^4\csc^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{4x^4}$$

```
[Out] -3/32*b*c^4*arccsc(c*x)+1/4*(-a-b*arcsec(c*x))/x^4+1/16*b*c*(1-1/c^2/x^2)^(
1/2)/x^3+3/32*b*c^3*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5328, 342, 327, 222}

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx=-\frac{a+b\sec^{-1}(cx)}{4x^4}-\frac{3}{32}bc^4\csc^{-1}(cx)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}+\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}$$

```
[In] Int[(a + b*ArcSec[c*x])/x^5,x]
```

```
[Out] (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*x^3) + (3*b*c^3*Sqrt[1 - 1/(c^2*x^2)])/(32*
x) - (3*b*c^4*ArcCsc[c*x])/32 - (a + b*ArcSec[c*x])/(4*x^4)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\sec^{-1}(cx)}{4x^4}+\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^6}\,dx}{4c}$$

$$=-\frac{a+b\sec^{-1}(cx)}{4x^4}-\frac{b\text{Subst}\left(\int\frac{x^4}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{4c}$$

$$=\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}-\frac{a+b\sec^{-1}(cx)}{4x^4}-\frac{1}{16}(3bc)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}+\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}-\frac{a+b\sec^{-1}(cx)}{4x^4}-\frac{1}{32}\left(3bc^3\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}+\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}-\frac{3}{32}bc^4\csc^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{4x^4}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx = -\frac{a}{4x^4}+b\left(\frac{c}{16x^3}+\frac{3c^3}{32x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\sec^{-1}(cx)}{4x^4}-\frac{3}{32}bc^4\arcsin\left(\frac{1}{cx}\right)$$

```
[In] Integrate[(a + b*ArcSec[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 + b*(c/(16*x^3) + (3*c^3)/(32*x))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]
 - (b*ArcSec[c*x])/(4*x^4) - (3*b*c^4*ArcSin[1/(c*x)])/32
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(67) = 134.

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.82

method	result	siz
parts	$-\frac{a}{4x^4}-\frac{b\,\mathrm{arcsec}(cx)}{4x^4}-\frac{3bc^3\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{3bc(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}+\frac{b(c^2x^2-1)}{16c\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^5}$	13
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arcsec}(cx)}{4c^4x^4}-\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}+\frac{b(c^2x^2-1)}{16\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}\right)$	15
default	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arcsec}(cx)}{4c^4x^4}-\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}+\frac{b(c^2x^2-1)}{16\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}\right)$	15

```
[In] int((a+b*arcsec(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arcsec(c*x)-3/32*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))+3/32*b*c*(c^2*x^2-1)/((c^2*x^2
-1)/c^2/x^2)^(1/2)/x^3+1/16*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx=\frac{(3\,bc^4x^4-8\,b)\,\mathrm{arcsec}\,(cx)+(3\,bc^2x^2+2\,b)\sqrt{c^2x^2-1}-8\,a}{32\,x^4}$$

```
[In] integrate((a+b*arcsec(c*x))/x^5,x, algorithm="fricas")
[Out] 1/32*((3*b*c^4*x^4 - 8*b)*arcsec(c*x) + (3*b*c^2*x^2 + 2*b)*sqrt(c^2*x^2 -
1) - 8*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.53

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx = -\frac{a}{4x^4}-\frac{b\,\mathrm{asec}\,(cx)}{4x^4} + \frac{b\left(\begin{cases}\frac{3ic^5\,\mathrm{acosh}\,\left(\frac{1}{cx}\right)}{8}-\frac{3ic^4}{8x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{ic^2}{8x^3\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{4x^5\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|}>1\\ -\frac{3c^5\,\mathrm{asin}\,\left(\frac{1}{cx}\right)}{8}+\frac{3c^4}{8x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{c^2}{8x^3\sqrt{1-\frac{1}{c^2x^2}}}-\frac{1}{4x^5\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise}\end{cases}\right)}{4c}$$

```
[In] integrate((a+b*asec(c*x))/x**5,x)
[Out] -a/(4*x**4) - b*asec(c*x)/(4*x**4) + b*Piecewise((3*I*c**5*acosh(1/(c*x))/8
 - 3*I*c**4/(8*x*sqrt(-1 + 1/(c**2*x**2))) + I*c**2/(8*x**3*sqrt(-1 + 1/(c*
*2*x**2))) + I/(4*x**5*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-
3*c**5*asin(1/(c*x))/8 + 3*c**4/(8*x*sqrt(1 - 1/(c**2*x**2))) - c**2/(8*x**
3*sqrt(1 - 1/(c**2*x**2))) - 1/(4*x**5*sqrt(1 - 1/(c**2*x**2))), True))/(4*
c)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.64

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx = \frac{1}{32}\,b\left(\frac{3\,c^5\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)+\frac{3\,c^8x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+5\,c^6x\sqrt{-\frac{1}{c^2x^2}+1}}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c}-\frac{8\,\mathrm{arcsec}\left(cx\right)}{x^4}\right) - \frac{a}{4\,x^4}$$

```
[In] integrate((a+b*arcsec(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/32*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*x^2
) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^2
 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 8*arcsec(c*x)/x^4) - 1/4*a/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx = \frac{1}{32}\left(3\,bc^3\arccos\left(\frac{1}{cx}\right)+\frac{3\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x}+\frac{2\,b\sqrt{-\frac{1}{c^2x^2}+1}}{x^3}-\frac{8\,b\arccos\left(\frac{1}{cx}\right)}{cx^4}-\frac{8\,a}{cx^4}\right)c$$

```
[In] integrate((a+b*arcsec(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/32*(3*b*c^3*arccos(1/(c*x)) + 3*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 2*b*sqrt
(-1/(c^2*x^2) + 1)/x^3 - 8*b*arccos(1/(c*x))/(c*x^4) - 8*a/(c*x^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^5}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^5}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/x^5,x)
[Out] int((a + b*acos(1/(c*x)))/x^5, x)
```

3.13 $\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 82

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = \frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}-\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}+\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}-\frac{a+b\sec^{-1}(cx)}{5x^5}$$

```
[Out] -2/15*b*c^5*(1-1/c^2/x^2)^(3/2)+1/25*b*c^5*(1-1/c^2/x^2)^(5/2)+1/5*(-a-b*ar
csec(c*x))/x^5+1/5*b*c^5*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5328, 272, 45}

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = -\frac{a+b\sec^{-1}(cx)}{5x^5}+\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}-\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}+\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}$$

```
[In] Int[(a + b*ArcSec[c*x])/x^6,x]
```

```
[Out] (b*c^5*Sqrt[1 - 1/(c^2*x^2)])/5 - (2*b*c^5*(1 - 1/(c^2*x^2))^(3/2))/15 + (b
*c^5*(1 - 1/(c^2*x^2))^(5/2))/25 - (a + b*ArcSec[c*x])/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\sec^{-1}(cx)}{5x^5}+\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^7}\,dx}{5c}\\
&= -\frac{a+b\sec^{-1}(cx)}{5x^5}-\frac{b\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{10c}\\
&= -\frac{a+b\sec^{-1}(cx)}{5x^5}-\frac{b\text{Subst}\left(\int\left(\frac{c^4}{\sqrt{1-\frac{x}{c^2}}}-2c^4\sqrt{1-\frac{x}{c^2}}+c^4\left(1-\frac{x}{c^2}\right)^{3/2}\right)\,dx,x,\frac{1}{x^2}\right)}{10c}\\
&= \frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}-\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}+\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}-\frac{a+b\sec^{-1}(cx)}{5x^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$
\int\frac{a+b\sec^{-1}(cx)}{x^6}\,dx=-\frac{a}{5x^5}+b\left(\frac{8c^5}{75}+\frac{c}{25x^4}+\frac{4c^3}{75x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\sec^{-1}(cx)}{5x^5}
$$

```
[In] Integrate[(a + b*ArcSec[c*x])/x^6,x]
[Out] -1/5*a/x^5 + b*((8*c^5)/75 + c/(25*x^4) + (4*c^3)/(75*x^2))*Sqrt[(-1 + c^2*
x^2)/(c^2*x^2)] - (b*ArcSec[c*x])/(5*x^5)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\operatorname{arcsec}(cx)}{5c^5x^5}+\frac{(c^2x^2-1)\,(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)$	79
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arcsec}(cx)}{5c^5x^5}+\frac{(c^2x^2-1)\,(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$	83
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arcsec}(cx)}{5c^5x^5}+\frac{(c^2x^2-1)\,(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$	83

```
[In] int((a+b*arcsec(c*x))/x^6,x,method=_RETURNVERBOSE)
[Out] -1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arcsec(c*x)+1/75*(c^2*x^2-1)*(8*c^4*x^4+4*c^
2*x^2+3)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx=-\frac{15\,b\operatorname{arcsec}(cx)-(8\,bc^4x^4+4\,bc^2x^2+3\,b)\sqrt{c^2x^2-1}+15\,a}{75\,x^5}$$

```
[In] integrate((a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
[Out] -1/75*(15*b*arcsec(c*x) - (8*b*c^4*x^4 + 4*b*c^2*x^2 + 3*b)*sqrt(c^2*x^2 -
1) + 15*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.90

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx=-\frac{a}{5x^5}-\frac{b\operatorname{asec}(cx)}{5x^5}+\frac{b\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$

```
[In] integrate((a+b*asec(c*x))/x**6,x)
```

```
[Out] -a/(5*x**5) - b*asec(c*x)/(5*x**5) + b*Piecewise((8*c**5*sqrt(c**2*x**2 - 1
)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*
x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**
3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True)
)/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = \frac{1}{75}\,b\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\mathrm{arcsec}\,(cx)}{x^5}\right) - \frac{a}{5\,x^5}$$

```
[In] integrate((a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/75*b*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2) +
 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcsec(c*x)/x^5) - 1/5*a/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = \frac{1}{75}\left(8\,bc^4\sqrt{-\frac{1}{c^2x^2}+1}+\frac{4\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}+\frac{3\,b\sqrt{-\frac{1}{c^2x^2}+1}}{x^4}-\frac{15\,b\arccos\left(\frac{1}{cx}\right)}{cx^5}-\frac{15\,a}{cx^5}\right)c$$

```
[In] integrate((a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 1/75*(8*b*c^4*sqrt(-1/(c^2*x^2) + 1) + 4*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x^2 +
 3*b*sqrt(-1/(c^2*x^2) + 1)/x^4 - 15*b*arccos(1/(c*x))/(c*x^5) - 15*a/(c*x^
5))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^6}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/x^6,x)
[Out] int((a + b*acos(1/(c*x)))/x^6, x)
```

3.14 $\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx=\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}+\frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}+\frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x}-\frac{5}{96}bc^6\csc^{-1}(cx)-\frac{a+b\sec^{-1}(cx)}{6x^6}$$

```
[Out] -5/96*b*c^6*arccsc(c*x)+1/6*(-a-b*arcsec(c*x))/x^6+1/36*b*c*(1-1/c^2/x^2)^(
1/2)/x^5+5/144*b*c^3*(1-1/c^2/x^2)^(1/2)/x^3+5/96*b*c^5*(1-1/c^2/x^2)^(1/2)
/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5328, 342, 327, 222}

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx=-\frac{a+b\sec^{-1}(cx)}{6x^6}-\frac{5}{96}bc^6\csc^{-1}(cx)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}+\frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x}+\frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}$$

```
[In] Int[(a + b*ArcSec[c*x])/x^7,x]
```

```
[Out] (b*c*Sqrt[1 - 1/(c^2*x^2)])/(36*x^5) + (5*b*c^3*Sqrt[1 - 1/(c^2*x^2)])/(144
*x^3) + (5*b*c^5*Sqrt[1 - 1/(c^2*x^2)])/(96*x) - (5*b*c^6*ArcCsc[c*x])/96 -
 (a + b*ArcSec[c*x])/(6*x^6)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\sec^{-1}(cx)}{6x^6}+\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^8}\,dx}{6c}$$

$$= -\frac{a+b\sec^{-1}(cx)}{6x^6}-\frac{b\text{Subst}\left(\int\frac{x^6}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{6c}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}-\frac{a+b\sec^{-1}(cx)}{6x^6}-\frac{1}{36}(5bc)\text{Subst}\left(\int\frac{x^4}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}+\frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}-\frac{a+b\sec^{-1}(cx)}{6x^6}-\frac{1}{48}\left(5bc^3\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}+\frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}+\frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} -\frac{a+b\sec^{-1}(cx)}{6x^6}-\frac{1}{96}\left(5bc^5\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} + \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} + \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} - \frac{5}{96}bc^6\csc^{-1}(cx) - \frac{a+b\sec^{-1}(cx)}{6x^6}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6} + b\left(\frac{c}{36x^5} + \frac{5c^3}{144x^3} + \frac{5c^5}{96x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\sec^{-1}(cx)}{6x^6} - \frac{5}{96}bc^6\arcsin\left(\frac{1}{cx}\right)$$

```
[In] Integrate[(a + b*ArcSec[c*x])/x^7,x]
```

```
[Out] -1/6*a/x^6 + b*(c/(36*x^5) + (5*c^3)/(144*x^3) + (5*c^5)/(96*x))*Sqrt[(-1 +
 c^2*x^2)/(c^2*x^2)] - (b*ArcSec[c*x])/(6*x^6) - (5*b*c^6*ArcSin[1/(c*x)])/
96
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.72

method	result
parts	$-\frac{a}{6x^6}-\frac{b\,\mathrm{arcsec}(cx)}{6x^6}-\frac{5b\,c^5\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{5b\,c^3\left(c^2x^2-1\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}+\frac{5bc\left(c^2x^2-1\right)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^5}+\frac{b(c^2x^2-}{36c\sqrt{\frac{c^2x^2}{c^2x}}}$
derivativedivides	$c^6\left(-\frac{a}{6c^6x^6}-\frac{b\,\mathrm{arcsec}(cx)}{6c^6x^6}-\frac{5b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{5b\left(c^2x^2-1\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}+\frac{5b\left(c^2x^2-1\right)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}+\right.$
default	$c^6\left(-\frac{a}{6c^6x^6}-\frac{b\,\mathrm{arcsec}(cx)}{6c^6x^6}-\frac{5b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{5b\left(c^2x^2-1\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}+\frac{5b\left(c^2x^2-1\right)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}+\right.$

```
[In] int((a+b*arcsec(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arcsec(c*x)-5/96*b*c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))+5/96*b*c^3*(c^2*x^2-1)/((c^2*x
^2-1)/c^2/x^2)^(1/2)/x^3+5/144*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x^5+1/36*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = \frac{3\,(5\,bc^6x^6-16\,b)\operatorname{arcsec}(cx)+(15\,bc^4x^4+10\,bc^2x^2+8\,b)\sqrt{c^2x^2-1}-48\,a}{288\,x^6}$$

```
[In] integrate((a+b*arcsec(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/288*(3*(5*b*c^6*x^6 - 16*b)*arcsec(c*x) + (15*b*c^4*x^4 + 10*b*c^2*x^2 +
8*b)*sqrt(c^2*x^2 - 1) - 48*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.39

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6}-\frac{b\,\mathrm{asec}\,(cx)}{6x^6} + \frac{b\left(\begin{cases}\frac{5ic^7\,\mathrm{acosh}\left(\frac{1}{cx}\right)}{16}-\frac{5ic^6}{16x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{5ic^4}{48x^3\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{ic^2}{24x^5\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{6x^7\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|}>1\\ -\frac{5c^7\,\mathrm{asin}\left(\frac{1}{cx}\right)}{16}+\frac{5c^6}{16x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{5c^4}{48x^3\sqrt{1-\frac{1}{c^2x^2}}}-\frac{c^2}{24x^5\sqrt{1-\frac{1}{c^2x^2}}}-\frac{1}{6x^7\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise}\end{cases}\right)}{6c}$$

```
[In] integrate((a+b*asec(c*x))/x**7,x)
```

```
[Out] -a/(6*x**6) - b*asec(c*x)/(6*x**6) + b*Piecewise((5*I*c**7*acosh(1/(c*x))/1
6 - 5*I*c**6/(16*x*sqrt(-1 + 1/(c**2*x**2))) + 5*I*c**4/(48*x**3*sqrt(-1 +
1/(c**2*x**2))) + I*c**2/(24*x**5*sqrt(-1 + 1/(c**2*x**2))) + I/(6*x**7*sqr
t(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-5*c**7*asin(1/(c*x))/16 +
5*c**6/(16*x*sqrt(1 - 1/(c**2*x**2))) - 5*c**4/(48*x**3*sqrt(1 - 1/(c**2*x*
*2))) - c**2/(24*x**5*sqrt(1 - 1/(c**2*x**2))) - 1/(6*x**7*sqrt(1 - 1/(c**2
*x**2))), True))/(6*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.63

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = \frac{1}{288}\,b\left(\frac{15\,c^7\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)-\frac{15\,c^{12}x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+40\,c^{10}x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\,c^8x\sqrt{-\frac{1}{c^2x^2}+1}}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3-3\,c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}}{c}-\frac{48\,\mathrm{arcsec}\,(c\right. \\ -\frac{a}{6\,x^6}$$

```
[In] integrate((a+b*arcsec(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] 1/288*b*((15*c^7*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) - (15*c^12*x^5*(-1/(c^2
*x^2) + 1)^(5/2) + 40*c^10*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(-1/
(c^2*x^2) + 1))/(c^6*x^6*(1/(c^2*x^2) - 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) - 1)^
2 + 3*c^2*x^2*(1/(c^2*x^2) - 1) - 1))/c - 48*arcsec(c*x)/x^6) - 1/6*a/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sec^{-1}(cx)}{x^7}\,dx = \frac{1}{288}\left(15\,bc^5\arccos\left(\frac{1}{cx}\right)+\frac{15\,bc^4\sqrt{-\frac{1}{c^2x^2}+1}}{x}+\frac{10\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x^3}+\frac{8\,b\sqrt{-\frac{1}{c^2x^2}+1}}{x^5}-\frac{48\,b\arccos\left(\frac{1}{cx}\right)}{cx^6}\right.$$

```
[In] integrate((a+b*arcsec(c*x))/x^7,x, algorithm="giac")
```

```
[Out] 1/288*(15*b*c^5*arccos(1/(c*x)) + 15*b*c^4*sqrt(-1/(c^2*x^2) + 1)/x + 10*b*
c^2*sqrt(-1/(c^2*x^2) + 1)/x^3 + 8*b*sqrt(-1/(c^2*x^2) + 1)/x^5 - 48*b*arcc
os(1/(c*x))/(c*x^6) - 48*a/(c*x^6))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x^7} \, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{cx}\right)}{x^7} \, dx$$

```
[In] int((a + b*acos(1/(c*x)))/x^7,x)
[Out] int((a + b*acos(1/(c*x)))/x^7, x)
```

3.15 $\int x^3(a+b\sec^{-1}(cx))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 107

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx=\frac{b^2x^2}{12c^2}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\sec^{-1}(cx))}{3c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\sec^{-1}(cx))}{6c}+\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^2+\frac{b^2\log(x)}{3c^4}$$

```
[Out] 1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arcsec(c*x))^2+1/3*b^2*ln(x)/c^4-1/3*b*x*(a+b
*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/c^3-1/6*b*x^3*(a+b*arcsec(c*x))*(1-1/c^2/
x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5330, 4494, 4270, 4269, 3556}

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx=-\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{6c}-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{3c^3}+\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^2+\frac{b^2\log(x)}{3c^4}+\frac{b^2x^2}{12c^2}$$

```
[In] Int[x^3*(a + b*ArcSec[c*x])^2,x]
```

```
[Out] (b^2*x^2)/(12*c^2) - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*ArcSec[c*x]))/(3*c^3
) - (b*Sqrt[1 - 1/(c^2*x^2)]*x^3*(a + b*ArcSec[c*x]))/(6*c) + (x^4*(a + b*A
rcSec[c*x])^2)/4 + (b^2*Log[x])/(3*c^4)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)^2 \sec^4(x)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{c^4} \\
&= \frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^2 - \frac{b\text{Subst}\left(\int (a+bx)\sec^4(x)\,dx, x, \sec^{-1}(cx)\right)}{2c^4} \\
&= \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3\left(a+b\sec^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^2 \\
&\quad - \frac{b\text{Subst}\left(\int (a+bx)\sec^2(x)\,dx, x, \sec^{-1}(cx)\right)}{3c^4}
\end{aligned}
$$

$$= \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\sec^{-1}(cx))}{3c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\sec^{-1}(cx))}{6c} + \frac{1}{4}x^4(a+b\sec^{-1}(cx))^2 + \frac{b^2\text{Subst}\left(\int\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{3c^4}$$

$$= \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\sec^{-1}(cx))}{3c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\sec^{-1}(cx))}{6c} + \frac{1}{4}x^4(a+b\sec^{-1}(cx))^2 + \frac{b^2\log(x)}{3c^4}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16

$$\int x^3(a+b\sec^{-1}(cx))^2\,dx = \frac{cx\left(b^2cx+3a^2c^3x^3-2ab\sqrt{1-\frac{1}{c^2x^2}}(2+c^2x^2)\right)-2bcx\left(-3ac^3x^3+b\sqrt{1-\frac{1}{c^2x^2}}(2+c^2x^2)\right)\sec^{-1}(cx)+3b}{12c^4}$$

```
[In] Integrate[x^3*(a + b*ArcSec[c*x])^2,x]
[Out] (c*x*(b^2*c*x + 3*a^2*c^3*x^3 - 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))
- 2*b*c*x*(-3*a*c^3*x^3 + b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))*ArcSec[c*x
] + 3*b^2*c^4*x^4*ArcSec[c*x]^2 + 4*b^2*Log[x])/(12*c^4)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.59

method	result
parts	$\frac{a^2x^4}{4} + \frac{b^2\left(\frac{\text{arcsec}(cx)^2c^4x^4}{4} - \frac{\text{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6} + \frac{c^2x^2}{12} - \frac{\text{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3} - \frac{\ln\left(\frac{1}{cx}\right)}{3}\right)}{c^4} + \frac{2ab\left(\frac{c^4x^4\,\text{arcsec}(c\right.}{4}}{}$
derivativedivides	$\frac{\frac{a^2c^4x^4}{4} + b^2\left(\frac{\text{arcsec}(cx)^2c^4x^4}{4} - \frac{\text{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6} + \frac{c^2x^2}{12} - \frac{\text{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3} - \frac{\ln\left(\frac{1}{cx}\right)}{3}\right) + 2ab\left(\frac{c^4x^4\,\text{arcsec}(cx)}{4} -\right.}{c^4}$
default	$\frac{\frac{a^2c^4x^4}{4} + b^2\left(\frac{\text{arcsec}(cx)^2c^4x^4}{4} - \frac{\text{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6} + \frac{c^2x^2}{12} - \frac{\text{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3} - \frac{\ln\left(\frac{1}{cx}\right)}{3}\right) + 2ab\left(\frac{c^4x^4\,\text{arcsec}(cx)}{4} -\right.}{c^4}$

```
[In] int(x^3*(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+b^2/c^4*(1/4*arcsec(c*x)^2*c^4*x^4-1/6*arcsec(c*x)*((c^2*x^2-1)
/c^2/x^2)^(1/2)*c^3*x^3+1/12*c^2*x^2-1/3*arcsec(c*x)*c*x*((c^2*x^2-1)/c^2/x
^2)^(1/2)-1/3*ln(1/c/x))+2*a*b/c^4*(1/4*c^4*x^4*arcsec(c*x)-1/12*(c^2*x^2-1
)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx$$
$$=\frac{3\,b^2c^4x^4\operatorname{arcsec}\left(cx\right)^2+3\,a^2c^4x^4+12\,abc^4\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+b^2c^2x^2+4\,b^2\log\left(x\right)+6\left(abc^4x^4-a\right.}{12\,c^4}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*arcsec(c*x)^2 + 3*a^2*c^4*x^4 + 12*a*b*c^4*arctan(-c*x
+ sqrt(c^2*x^2 - 1)) + b^2*c^2*x^2 + 4*b^2*log(x) + 6*(a*b*c^4*x^4 - a*b*c^
4)*arcsec(c*x) - 2*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^2)*arcsec(c*x)
)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [F]

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx=\int x^3\left(a+b\operatorname{asec}\left(cx\right)\right)^2\,dx$$

```
[In] integrate(x**3*(a+b*asec(c*x))**2,x)
```

```
[Out] Integral(x**3*(a + b*asec(c*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx=\frac{1}{4}\,b^2x^4\operatorname{arcsec}\left(cx\right)^2+\frac{1}{4}\,a^2x^4$$
$$+\frac{1}{6}\left(3\,x^4\operatorname{arcsec}\left(cx\right)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)ab$$
$$+\frac{\left(\left(c^2x^2+2\,\log\left(x^2\right)\right)\sqrt{cx+1}\sqrt{cx-1}-2\left(c^4x^4+c^2x^2-2\right)\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)\right)b^2}{12\,\sqrt{cx+1}\sqrt{cx-1}c^4}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arcsec(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arcsec(c*x) - (c^2*x^3
*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a*b + 1/12*((c
^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) - 2*(c^4*x^4 + c^2*x^2 - 2
)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c^4
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6625 vs. $2(93) = 186$.

Time = 0.61 (sec) , antiderivative size = 6625, normalized size of antiderivative = 61.92

$$\int x^3 \left(a + b\sec^{-1}(cx)\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*arccos(1/(c*x))^2/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 6*a*b*arccos(1/
(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x
))^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/
(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 4*b^2*log(2)/(c^5 + 4*
c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8) + 4*b^2*log(2/(c*x) + 2)/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 4
*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^5 + 4*c^5*(1/(c^2*x^
2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5
*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1
)^8) - 4*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^5 + 4*c^5*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8) - 12*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/((c^5 + 4*c^5*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)
^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8)*(1/(c*x) + 1)) + 3*a^2/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + b^2/(c^
5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/
```

(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8) - 24*a*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^5 +
 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 18*b^2*(1/(c^2*x^2) - 1)^2*arccos(
1/(c*x))^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c
^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 16*b^2*(1/(c^2*x
^2) - 1)*log(2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/
(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^
6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 16*b^2*(1/(
c^2*x^2) - 1)*log(2/(c*x) + 2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2
) - 16*b^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))
/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2
*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 16*b^2*(1/(c^2*x^2) - 1)*l
og(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*
(1/(c*x) + 1)^2) - 12*a*b*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2)
 - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^
8)*(1/(c*x) + 1)) + 20*b^2*(-1/(c^2*x^2) + 1)^(3/2)*arccos(1/(c*x))/((c^5 +
 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) - 12*a^2*(1/(c^2*x^2) - 1)/((c^5 + 4
c^5(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 36*a*b*(1/(c^2*x^2) - 1)^2*arccos(1/
(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(
1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 12*b^2*(1/(c^2*x^2)
 - 1)^3*arccos(1/(c*x))^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c
x) + 1)^6 + c^5(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 24
b^2(1/(c^2*x^2) - 1)^2*log(2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^
4) + 24*b^2*(1/(c^2*x^2) - 1)^2*log(2/(c*x) + 2)/((c^5 + 4*c^5*(1/(c^2*x^2)
 - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^
8)*(1/(c*x) + 1)^4) - 24*b^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2)
+ 1) + 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^

5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 24*b^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 20*a*b*(-1/(c^2*x^2) + 1)^(3/2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) - 20*b^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) + 18*a^2*(1/(c^2*x^2) - 1)^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 2*b^2*(1/(c^2*x^2) - 1)^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 24*a*b*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 3*b^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 16*b^2*(1/(c^2*x^2) - 1)^3*log(2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 16*b^2*(1/(c^2*x^2) - 1)^3*log(2/(c*x) + 2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 16*b^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 16*b^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 20*a*b*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) - 12*b^2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) - 12*

```
a^2*(1/(c^2*x^2) - 1)^3/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6
*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*
x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 6*a
*b*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(
c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x
) + 1)^8) - 4*b^2*(1/(c^2*x^2) - 1)^4*log(2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c
^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(
1/(c*x) + 1)^8) + 4*b^2*(1/(c^2*x^2) - 1)^4*log(2/(c*x) + 2)/((c^5 + 4*c^5*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)
^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 4*b^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1
/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^
8) - 4*b^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1
))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c
^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 12*a*b*(1/(c^2*x^2) - 1)
^3*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) + 3
*a^2*(1/(c^2*x^2) - 1)^4/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + b^
2*(1/(c^2*x^2) - 1)^4/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c
^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x)
 + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^2\,dx=\int x^3\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

```
[In] int(x^3*(a + b*acos(1/(c*x)))^2,x)
```

```
[Out] int(x^3*(a + b*acos(1/(c*x)))^2, x)
```

3.16 $\int x^2(a+b\sec^{-1}(cx))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 147

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^2\,dx=\frac{b^2x}{3c^2}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\sec^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^2+\frac{2ib(a+b\sec^{-1}(cx))\arctan\left(e^{i\sec^{-1}(cx)}\right)}{3c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\operatorname{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{3c^3}$$

```
[Out] 1/3*b^2*x/c^2+1/3*x^3*(a+b*arcsec(c*x))^2+2/3*I*b*(a+b*arcsec(c*x))*arctan(
1/c/x+I*(1-1/c^2/x^2)^(1/2))/c^3-1/3*I*b^2*polylog(2,-I*(1/c/x+I*(1-1/c^2/x
^2)^(1/2)))/c^3+1/3*I*b^2*polylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3-1/
3*b*x^2*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5330, 4494, 4270, 4266, 2317, 2438}

$$\int x^2(a+b\sec^{-1}(cx))^2\,dx=\frac{2ib\arctan\left(e^{i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))}{3c^3}-\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{3c}+\frac{1}{3}x^3(a+b\sec^{-1}(cx))^2-\frac{ib^2\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\operatorname{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{3c^3}+\frac{b^2x}{3c^2}$$

```
[In] Int[x^2*(a + b*ArcSec[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) - (b*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcSec[c*x]))/(3*c) +
 (x^3*(a + b*ArcSec[c*x])^2)/3 + (((2*I)/3)*b*(a + b*ArcSec[c*x])*ArcTan[E^
(I*ArcSec[c*x])])/c^3 - ((I/3)*b^2*PolyLog[2, (-I)*E^(I*ArcSec[c*x])])/c^3
+ ((I/3)*b^2*PolyLog[2, I*E^(I*ArcSec[c*x])])/c^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
```

```
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)^2\sec^3(x)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{c^3}\\
&= \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^2 - \frac{(2b)\text{Subst}\left(\int (a+bx)\sec^3(x)\,dx, x, \sec^{-1}(cx)\right)}{3c^3}\\
&= \frac{b^2x}{3c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\sec^{-1}(cx))}{3c} + \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^2\\
&\quad - \frac{b\text{Subst}\left(\int (a+bx)\sec(x)\,dx, x, \sec^{-1}(cx)\right)}{3c^3}\\
&= \frac{b^2x}{3c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\sec^{-1}(cx))}{3c} + \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^2\\
&\quad + \frac{2ib(a+b\sec^{-1}(cx))\arctan\left(e^{i\sec^{-1}(cx)}\right)}{3c^3}\\
&\quad + \frac{b^2\text{Subst}\left(\int \log\left(1-ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{3c^3}\\
&\quad - \frac{b^2\text{Subst}\left(\int \log\left(1+ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{3c^3}
\end{aligned}
$$

$$= \frac{b^2x}{3c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\sec^{-1}(cx))}{3c} + \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^2 + \frac{2ib(a+b\sec^{-1}(cx))\arctan\left(e^{i\sec^{-1}(cx)}\right)}{3c^3} - \frac{(ib^2)\,\mathrm{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{i\sec^{-1}(cx)}\right)}{3c^3} + \frac{(ib^2)\,\mathrm{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{i\sec^{-1}(cx)}\right)}{3c^3}$$

$$= \frac{b^2x}{3c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\sec^{-1}(cx))}{3c} + \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^2 + \frac{2ib(a+b\sec^{-1}(cx))\arctan\left(e^{i\sec^{-1}(cx)}\right)}{3c^3} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{3c^3} + \frac{ib^2\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{3c^3}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.54

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^2\,dx = \frac{1}{3}\left(a^2x^3 + \frac{ab\left(2x^4\sec^{-1}(cx) + \frac{cx-c^3x^3+\sqrt{-1+c^2x^2}\log\left(-cx+\sqrt{-1+c^2x^2}\right)}{c^4\sqrt{1-\frac{1}{c^2x^2}}}\right)}{x} + \frac{b^2\left(cx - c^2\sqrt{1-\frac{1}{c^2x^2}}x^2\sec^{-1}(cx) + c^3x^3\sec^{-1}(cx)^2 - \sec^{-1}(cx)\log\left(1-ie^{i\sec^{-1}(cx)}\right) + \sec^{-1}(cx)\log\left(1\right.\right.}{c^3}\right.$$

```
[In] Integrate[x^2*(a + b*ArcSec[c*x])^2,x]
```

```
[Out] (a^2*x^3 + (a*b*(2*x^4*ArcSec[c*x] + (c*x - c^3*x^3 + Sqrt[-1 + c^2*x^2]*Lo
g[-(c*x) + Sqrt[-1 + c^2*x^2]])/(c^4*Sqrt[1 - 1/(c^2*x^2)])))/x + (b^2*(c*x
 - c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*ArcSec[c*x] + c^3*x^3*ArcSec[c*x]^2 - ArcS
ec[c*x]*Log[1 - I*E^(I*ArcSec[c*x])] + ArcSec[c*x]*Log[1 + I*E^(I*ArcSec[c*
x])] - I*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] + I*PolyLog[2, I*E^(I*ArcSec[c*
x])]))/c^3)/3
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.94

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{\left(c^2x^2\operatorname{arcsec}(cx)^2-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}+\frac{\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{3}-\frac{\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}\right.\right.}{3}\right.}{c^3}$
derivativedivides	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arcsec}(cx)^2-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}+\frac{\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{3}-\frac{\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}\right.\right.}{3}\right.}{}$
default	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arcsec}(cx)^2-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}+\frac{\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{3}-\frac{\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}\right.\right.}{3}\right.}{}$

```
[In] int(x^2*(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*x^3+b^2/c^3*(1/3*(c^2*x^2*arcsec(c*x)^2-arcsec(c*x)*c*x*((c^2*x^2-1
)/c^2/x^2)^(1/2)+1)*c*x+1/3*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)
))-1/3*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/3*I*dilog(1+I*(1
/c/x+I*(1-1/c^2/x^2)^(1/2)))+1/3*I*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))
)+2*a*b/c^3*(1/3*c^3*x^3*arcsec(c*x)-1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x^2-1)
^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [F]

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^2\,dx=\int(b\operatorname{arcsec}(cx)+a)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arcsec(c*x)^2 + 2*a*b*x^2*arcsec(c*x) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^2\,dx=\int x^2(a+b\operatorname{asec}(cx))^2\,dx$$

```
[In] integrate(x**2*(a+b*asec(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*asec(c*x))**2, x)
```

Maxima [F]

$$\int x^2 \left(a+b\sec^{-1}(cx)\right)^2 \,dx = \int (b\,\mathrm{arcsec}\,(cx)+a)^2 x^2 \,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^
2*x^2) + 1) - 1)/c^2)/c)*a*b + 1/12*(4*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x -
1))^2 - x^3*log(c^2*x^2)^2 - 2*c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/
c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 + 36*c^2*integrate(1/3*x^4*log(c^2*x^2)/
(c^2*x^2 - 1), x)*log(c) - 72*c^2*integrate(1/3*x^4*log(x)/(c^2*x^2 - 1), x
)*log(c) + 36*c^2*integrate(1/3*x^4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) -
 36*c^2*integrate(1/3*x^4*log(x)^2/(c^2*x^2 - 1), x) + 12*c^2*integrate(1/3
*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x) + 6*(2*x/c^2 - log(c*x + 1)/c^3 + log(c
*x - 1)/c^3)*log(c)^2 - 36*integrate(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)
*log(c) + 72*integrate(1/3*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) - 24*integra
te(1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/
(c^2*x^2 - 1), x) - 36*integrate(1/3*x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1),
 x) + 36*integrate(1/3*x^2*log(x)^2/(c^2*x^2 - 1), x) - 12*integrate(1/3*x^
2*log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2
```

Giac [F]

$$\int x^2 \left(a+b\sec^{-1}(cx)\right)^2 \,dx = \int (b\,\mathrm{arcsec}\,(cx)+a)^2 x^2 \,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\sec^{-1}(cx)\right)^2 \,dx = \int x^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2 \,dx$$

```
[In] int(x^2*(a + b*acos(1/(c*x)))^2,x)
```

```
[Out] int(x^2*(a + b*acos(1/(c*x)))^2, x)
```

3.17 $\int x(a+b\sec^{-1}(cx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

```
[Out] 1/2*x^2*(a+b*arcsec(c*x))^2+b^2*ln(x)/c^2-b*x*(a+b*arcsec(c*x))*(1-1/c^2/x^
2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5330, 4494, 4269, 3556}

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

```
[In] Int[x*(a + b*ArcSec[c*x])^2,x]
```

```
[Out] -((b*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*ArcSec[c*x]))/c) + (x^2*(a + b*ArcSec[c
*x])^2)/2 + (b^2*Log[x])/c^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)^2\sec^2(x)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{c^2}\\
&= \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^2 - \frac{b\text{Subst}\left(\int (a+bx)\sec^2(x)\,dx, x, \sec^{-1}(cx)\right)}{c^2}\\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\sec^{-1}(cx)\right)}{c} + \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^2 + \frac{b^2\text{Subst}\left(\int \tan(x)\,dx, x, \sec^{-1}(cx)\right)}{c^2}\\
&= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\sec^{-1}(cx)\right)}{c} + \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^2 + \frac{b^2\log(x)}{c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx = \frac{acx\left(-2b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)+2bcx\left(-b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)\sec^{-1}(cx)+b^2c^2x^2\sec^{-1}(cx)^2+2b^2\log(cx)}{2c^2}$$

```
[In] Integrate[x*(a + b*ArcSec[c*x])^2,x]
```

```
[Out] (a*c*x*(-2*b*Sqrt[1 - 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(-(b*Sqrt[1 - 1/(c^2*
x^2)]) + a*c*x)*ArcSec[c*x] + b^2*c^2*x^2*ArcSec[c*x]^2 + 2*b^2*Log[c*x])/(
2*c^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(52) = 104$.

Time = 0.67 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.20

method	result	size
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\frac{c^2x^2\operatorname{arcsec}(cx)^2}{2}-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-\ln\left(\frac{1}{cx}\right)\right)}{c^2}+\frac{2ab\left(\frac{c^2x^2\operatorname{arcsec}(cx)}{2}-\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	123
derivativedivides	$\frac{\frac{a^2c^2x^2}{2}+b^2\left(\frac{c^2x^2\operatorname{arcsec}(cx)^2}{2}-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-\ln\left(\frac{1}{cx}\right)\right)+2ab\left(\frac{c^2x^2\operatorname{arcsec}(cx)}{2}-\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	124
default	$\frac{\frac{a^2c^2x^2}{2}+b^2\left(\frac{c^2x^2\operatorname{arcsec}(cx)^2}{2}-\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-\ln\left(\frac{1}{cx}\right)\right)+2ab\left(\frac{c^2x^2\operatorname{arcsec}(cx)}{2}-\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	124

```
[In] int(x*(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^2+b^2/c^2*(1/2*c^2*x^2*arcsec(c*x)^2-arcsec(c*x)*c*x*((c^2*x^2-1)
/c^2/x^2)^(1/2)-ln(1/c/x))+2*a*b/c^2*(1/2*c^2*x^2*arcsec(c*x)-1/2/((c^2*x^2
-1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.98

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx$$
$$=\frac{b^2c^2x^2\operatorname{arcsec}(cx)^2+a^2c^2x^2+4\,abc^2\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\,b^2\log(x)+2\left(abc^2x^2-abc^2\right)\operatorname{arcsec}(}{2\,c^2}$$

```
[In] integrate(x*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*arcsec(c*x)^2 + a^2*c^2*x^2 + 4*a*b*c^2*arctan(-c*x + sqrt
(c^2*x^2 - 1)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 - a*b*c^2)*arcsec(c*x) - 2*s
qrt(c^2*x^2 - 1)*(b^2*arcsec(c*x) + a*b))/c^2
```

Sympy [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^{2}\,dx=\int x(a+b\operatorname{asec}(cx))^{2}\,dx$$

```
[In] integrate(x*(a+b*asec(c*x))**2,x)
[Out] Integral(x*(a + b*asec(c*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int x\left(a+b\sec^{-1}(cx)\right)^{2}\,dx=\frac{1}{2}\,b^{2}x^{2}\operatorname{arcsec}(cx)^{2}+\frac{1}{2}\,a^{2}x^{2}+\left(x^{2}\operatorname{arcsec}(cx)-\frac{x\sqrt{-\frac{1}{c^{2}x^{2}}+1}}{c}\right)ab-\left(\frac{x\sqrt{-\frac{1}{c^{2}x^{2}}+1}\operatorname{arcsec}(cx)}{c}-\frac{\log(x)}{c^{2}}\right)b^{2}$$

```
[In] integrate(x*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
[Out] 1/2*b^2*x^2*arcsec(c*x)^2 + 1/2*a^2*x^2 + (x^2*arcsec(c*x) - x*sqrt(-1/(c^2
*x^2) + 1)/c)*a*b - (x*sqrt(-1/(c^2*x^2) + 1)*arcsec(c*x)/c - log(x)/c^2)*b
^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2181 vs. 2(52) = 104.

Time = 0.45 (sec) , antiderivative size = 2181, normalized size of antiderivative = 38.95

$$\int x\left(a+b\sec^{-1}(cx)\right)^{2}\,dx=\text{Too large to display}$$

```
[In] integrate(x*(a+b*arcsec(c*x))^2,x, algorithm="giac")
[Out] 1/2*(b^2*arccos(1/(c*x))^2/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
 c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*a*b*arccos(1/(c*x))/(c^3 + 2*
c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4) - 2*b^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))^2/((c^3 + 2*c^3*(1/(c^2*x^2
```

```
) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x)
+ 1)^2) - 2*b^2*log(2)/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b^2*log(2/(c*x) + 2)/(c^3 + 2*c^3
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^
4) - 2*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^3 + 2*c^3*(1/(
c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) -
2*b^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^3 + 2*c^3*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 4*b^2
*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) + a^
2/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/
(1/(c*x) + 1)^4) - 4*a*b*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^3 + 2*c^3*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*
(1/(c*x) + 1)^2) + b^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))^2/((c^3 + 2*c^3*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
)*(1/(c*x) + 1)^4) - 4*b^2*(1/(c^2*x^2) - 1)*log(2)/((c^3 + 2*c^3*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x
) + 1)^2) + 4*b^2*(1/(c^2*x^2) - 1)*log(2/(c*x) + 2)/((c^3 + 2*c^3*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*
x) + 1)^2) - 4*b^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*
x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 4*b^2*(1/(c^2*x^2) - 1)*log(abs
(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) -
4*a*b*sqrt(-1/(c^2*x^2) + 1)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) + 4*b^2*(-1/(c^
2*x^2) + 1)^(3/2)*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^3) - 2*a^2*
(1/(c^2*x^2) - 1)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/
(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + 2*a*b*(1/(c^2*x^2) - 1
)^2*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(
1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 2*b^2*(1/(c^2*x^2) -
 1)^2*log(2)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + 2*b^2*(1/(c^2*x^2) - 1)^2*l
og(2/(c*x) + 2)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 2*b^2*(1/(c^2*x^2) - 1)^
2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2)
 - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) +
 1)^4) - 2*b^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x)
 - 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + 4*a*b*(-1/(c^2*x^2) + 1)^(3/2)/((
c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/
(c*x) + 1)^4)*(1/(c*x) + 1)^3) + a^2*(1/(c^2*x^2) - 1)^2/((c^3 + 2*c^3*(1/(
c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1
/(c*x) + 1)^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sec^{-1}(cx)\right)^2\,dx=\int x\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

```
[In] int(x*(a + b*acos(1/(c*x)))^2,x)
[Out] int(x*(a + b*acos(1/(c*x)))^2, x)
```

3.18 $\int (a+b\sec^{-1}(cx))^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 92

$$\int (a+b\sec^{-1}(cx))^2 \, dx = x(a+b\sec^{-1}(cx))^2 + \frac{4ib(a+b\sec^{-1}(cx))\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c} - \frac{2ib^2 \operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{2ib^2 \operatorname{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arcsec(c*x))^2+4*I*b*(a+b*arcsec(c*x))*arctan(1/c/x+I*(1-1/c^2/x^2)^
(1/2))/c-2*I*b^2*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c+2*I*b^2*poly
log(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5324, 4494, 4266, 2317, 2438}

$$\int (a+b\sec^{-1}(cx))^2 \, dx = \frac{4ib\arctan\left(e^{i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))}{c} + x(a+b\sec^{-1}(cx))^2 - \frac{2ib^2 \operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{2ib^2 \operatorname{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c}$$

```
[In] Int[(a + b*ArcSec[c*x])^2,x]
```

```
[Out] x*(a + b*ArcSec[c*x])^2 + ((4*I)*b*(a + b*ArcSec[c*x])*ArcTan[E^(I*ArcSec[c
*x])])/c - ((2*I)*b^2*PolyLog[2, (-I)*E^(I*ArcSec[c*x])])/c + ((2*I)*b^2*Po
lyLog[2, I*E^(I*ArcSec[c*x])])/c
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5324

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/c, Subst[I
nt[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n}
, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)^2\sec(x)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{c}\\
&= x\left(a+b\sec^{-1}(cx)\right)^2 - \frac{(2b)\text{Subst}\left(\int (a+bx)\sec(x)\,dx, x, \sec^{-1}(cx)\right)}{c}\\
&= x\left(a+b\sec^{-1}(cx)\right)^2 + \frac{4ib(a+b\sec^{-1}(cx))\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c}\\
&\quad+ \frac{(2b^2)\,\text{Subst}\left(\int \log\left(1-ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c}\\
&\quad- \frac{(2b^2)\,\text{Subst}\left(\int \log\left(1+ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c}
\end{aligned}
$$

$$= x\left(a + b\sec^{-1}(cx)\right)^2 + \frac{4ib\left(a + b\sec^{-1}(cx)\right)\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c} - \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{c} + \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{c}$$

$$= x\left(a + b\sec^{-1}(cx)\right)^2 + \frac{4ib\left(a + b\sec^{-1}(cx)\right)\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c} - \frac{2ib^2\,\text{PolyLog}\left(2, -ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{2ib^2\,\text{PolyLog}\left(2, ie^{i\sec^{-1}(cx)}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.77

$$\int \left(a + b\sec^{-1}(cx)\right)^2\,dx$$

$$= \frac{a^2cx + 2ab\left(cx\sec^{-1}(cx) + \log\left(\cos\left(\frac{1}{2}\sec^{-1}(cx)\right) - \sin\left(\frac{1}{2}\sec^{-1}(cx)\right)\right) - \log\left(\cos\left(\frac{1}{2}\sec^{-1}(cx)\right) + \sin\left(\frac{1}{2}\text{se}\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSec[c*x])^2,x]
```

```
[Out] (a^2*c*x + 2*a*b*(c*x*ArcSec[c*x] + Log[Cos[ArcSec[c*x]/2] - Sin[ArcSec[c*x
]/2]] - Log[Cos[ArcSec[c*x]/2] + Sin[ArcSec[c*x]/2]]) + b^2*(ArcSec[c*x]*(c
*x*ArcSec[c*x] - 2*Log[1 - I*E^(I*ArcSec[c*x])] + 2*Log[1 + I*E^(I*ArcSec[c
*x])]) - (2*I)*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] + (2*I)*PolyLog[2, I*E^(I
*ArcSec[c*x])]))/c
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.11

method	result
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arcsec}(cx)^2cx+2\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2i\operatorname{dilog}\left(1\right.\right.}{c}$
default	$\frac{a^2cx+b^2\left(\operatorname{arcsec}(cx)^2cx+2\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2i\operatorname{dilog}\left(1\right.\right.}{c}$
parts	$a^2x+\frac{b^2\left(\operatorname{arcsec}(cx)^2cx+2\operatorname{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2\operatorname{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-2i\operatorname{dilog}\right.}{c}$

```
[In] int((a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2*c*x+b^2*(arcsec(c*x)^2*c*x+2*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/
x^2)^(1/2)))-2*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-2*I*dilog(
1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+2*I*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))))+2*a*b*(c*x*arcsec(c*x)-ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))))
```

Fricas [F]

$$\int \left(a+b\sec^{-1}(cx)\right)^2\,dx=\int \left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*arcsec(c*x))^2,x, algorithm="fricas")
[Out] integral(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\sec^{-1}(cx)\right)^2\,dx=\int \left(a+b\,\mathrm{asec}\left(cx\right)\right)^2\,dx$$

```
[In] integrate((a+b*asec(c*x))**2,x)
[Out] Integral((a + b*asec(c*x))**2, x)
```

Maxima [F]

$$\int \left(a+b\sec^{-1}(cx)\right)^2\,dx=\int \left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*arcsec(c*x))^2,x, algorithm="maxima")
[Out] -1/4*(2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 4*c^
2*integrate(x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 8*c^2*integrate(x^2
*log(x)/(c^2*x^2 - 1), x)*log(c) - 4*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^
2 - 4*c^2*integrate(x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 4*c^2*integ
rate(x^2*log(x)^2/(c^2*x^2 - 1), x) - 4*c^2*integrate(x^2*log(c^2*x^2)/(c^2
*x^2 - 1), x) + x*log(c^2*x^2)^2 + 2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(
c)^2 + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 8*integrate(log(
x)/(c^2*x^2 - 1), x)*log(c) + 8*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*arcta
n(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x) + 4*integrate(log(c^2*x^2)
*log(x)/(c^2*x^2 - 1), x) - 4*integrate(log(x)^2/(c^2*x^2 - 1), x) + 4*inte
grate(log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2 + a^2*x + (2*c*x*arcsec(c*x) - lo
g(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*a*b/c
```

Giac [F]

$$\int \left(a + b\sec^{-1}(cx)\right)^2 \, dx = \int \left(b\,\mathrm{arcsec}\,(cx) + a\right)^2 dx$$

```
[In] integrate((a+b*arcsec(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + b\sec^{-1}(cx)\right)^2 \, dx = \int \left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

```
[In] int((a + b*acos(1/(c*x)))^2,x)
[Out] int((a + b*acos(1/(c*x)))^2, x)
```

3.19 $\int \frac{(a+b\sec^{-1}(cx))^2}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 93

$$\begin{aligned}\int \frac{(a+b\sec^{-1}(cx))^2}{x} dx = & \frac{i(a+b\sec^{-1}(cx))^3}{3b} - (a+b\sec^{-1}(cx))^2 \log\left(1+e^{2i\sec^{-1}(cx)}\right) \\ & + ib(a+b\sec^{-1}(cx)) \operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) \\ & - \frac{1}{2}b^2 \operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)\end{aligned}$$

```
[Out] 1/3*I*(a+b*arcsec(c*x))^3/b-(a+b*arcsec(c*x))^2*ln(1+(1/c/x+I*(1-1/c^2/x^2)
^(1/2))^2)+I*b*(a+b*arcsec(c*x))*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2
)-1/2*b^2*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5330, 3800, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\sec^{-1}(cx))^2}{x} dx = & ib \operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx)) \\ & + \frac{i(a+b\sec^{-1}(cx))^3}{3b} - \log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))^2 \\ & - \frac{1}{2}b^2 \operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)\end{aligned}$$

```
[In] Int[(a + b*ArcSec[c*x])^2/x,x]
```

```
[Out] ((I/3)*(a + b*ArcSec[c*x])^3)/b - (a + b*ArcSec[c*x])^2*Log[1 + E^((2*I)*Ar
cSec[c*x])] + I*b*(a + b*ArcSec[c*x])*PolyLog[2, -E^((2*I)*ArcSec[c*x])] -
(b^2*PolyLog[3, -E^((2*I)*ArcSec[c*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (a+bx)^2\tan(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b} - 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^2}{1+e^{2ix}}\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b} - \left(a+b\sec^{-1}(cx)\right)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\
&\quad + (2b)\text{Subst}\left(\int (a+bx)\log\left(1+e^{2ix}\right)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b} - \left(a+b\sec^{-1}(cx)\right)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\
&\quad + ib\left(a+b\sec^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\\
&\quad - \left(ib^2\right)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{2ix}\right)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b} - \left(a+b\sec^{-1}(cx)\right)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\
&\quad + ib\left(a+b\sec^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\\
&\quad - \frac{1}{2}b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{2i\sec^{-1}(cx)}\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^3}{3b} - \left(a+b\sec^{-1}(cx)\right)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\
&\quad + ib\left(a+b\sec^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) - \frac{1}{2}b^2\,\text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.39

$$
\begin{aligned}
\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\,dx &= iab\sec^{-1}(cx)^2 + \frac{1}{3}ib^2\sec^{-1}(cx)^3\\
&\quad - 2ab\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\
&\quad - b^2\sec^{-1}(cx)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right) + a^2\log(cx)\\
&\quad + ib\left(a+b\sec^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\\
&\quad - \frac{1}{2}b^2\,\text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSec[c*x])^2/x,x]
```

[Out] I*a*b*ArcSec[c*x]^2 + (I/3)*b^2*ArcSec[c*x]^3 - 2*a*b*ArcSec[c*x]*Log[1 + E
^((2*I)*ArcSec[c*x])] - b^2*ArcSec[c*x]^2*Log[1 + E^((2*I)*ArcSec[c*x])] +
a^2*Log[c*x] + I*b*(a + b*ArcSec[c*x])*PolyLog[2, -E^((2*I)*ArcSec[c*x])] -
(b^2*PolyLog[3, -E^((2*I)*ArcSec[c*x])])/2

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.22

method	result
parts	$a^2\ln(x)+b^2\left(\frac{i\,\mathrm{arcsec}(cx)^3}{3}-\mathrm{arcsec}(cx)^2\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+i\,\mathrm{arcsec}(cx)\,\mathrm{p}\right.$
derivativedivides	$a^2\ln(cx)+b^2\left(\frac{i\,\mathrm{arcsec}(cx)^3}{3}-\mathrm{arcsec}(cx)^2\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+i\,\mathrm{arcsec}(cx)\right.$
default	$a^2\ln(cx)+b^2\left(\frac{i\,\mathrm{arcsec}(cx)^3}{3}-\mathrm{arcsec}(cx)^2\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+i\,\mathrm{arcsec}(cx)\right.$

[In] int((a+b*arcsec(c*x))^2/x,x,method=_RETURNVERBOSE)

[Out] a^2*ln(x)+b^2*(1/3*I*arcsec(c*x)^3-arcsec(c*x)^2*ln(1+(1/c/x+I*(1-1/c^2/x^2
)^(1/2))^2)+I*arcsec(c*x)*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-1/2*p
olylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))+I*a*b*arcsec(c*x)^2-2*a*b*arcse
c(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+I*a*b*polylog(2,-(1/c/x+I*(1-1
/c^2/x^2)^(1/2))^2)

Fricas [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx=\int \frac{(b\,\mathrm{arcsec}(cx)+a)^2}{x}\,dx$$

[In] integrate((a+b*arcsec(c*x))^2/x,x, algorithm="fricas")

[Out] integral((b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2)/x, x)

Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^2}{x}\,dx$$

```
[In] integrate((a+b*asec(c*x))**2/x,x)
[Out] Integral((a + b*asec(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))^2/x,x, algorithm="maxima")
[Out] -1/2*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 + b^2*c^2*integ
rate(x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 2*b^2*c^2*integrate(x^2*lo
g(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*c^2*integrate(x^2*log(c^2*x^2)*log(x)
/(c^2*x^3 - x), x) - b^2*c^2*integrate(x^2*log(x)^2/(c^2*x^3 - x), x) + 2*a
*b*c^2*integrate(x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x)
+ 1/2*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + b^2*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1))^2*log(x) - 1/4*b^2*log(c^2*x^2)^2*log(x) - b^2*i
ntegrate(log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 2*b^2*integrate(log(x)/(c^
2*x^3 - x), x)*log(c) - 2*b^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^3 - x), x) - 2*b^2*integrate(log
(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + b^2*integrate(log(x)^2/(c^2*x^3 - x),
x) - 2*a*b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x)
+ a^2*log(x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))^2/x,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:ln of unsigned or minus infinity Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))^2/x,x)
[Out] int((a + b*acos(1/(c*x)))^2/x, x)
```

3.20 $\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = \frac{2b^2}{x} + 2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx)) - \frac{(a+b\sec^{-1}(cx))^2}{x}$$

```
[Out] 2*b^2/x-(a+b*arcsec(c*x))^2/x+2*b*c*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5330, 3377, 2718}

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = 2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx)) - \frac{(a+b\sec^{-1}(cx))^2}{x} + \frac{2b^2}{x}$$

```
[In] Int[(a + b*ArcSec[c*x])^2/x^2,x]
```

```
[Out] (2*b^2)/x + 2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x]) - (a + b*ArcSec
[c*x])^2/x
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c\text{Subst}\left(\int (a+bx)^2 \sin(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= -\frac{\left(a+b\sec^{-1}(cx)\right)^2}{x} + (2bc)\text{Subst}\left(\int (a+bx)\cos(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= 2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}\\
&\quad - \left(2b^2c\right)\text{Subst}\left(\int \sin(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{2b^2}{x} + 2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right) - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^2}\,dx = \frac{-a^2+2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x+2b\left(-a+bc\sqrt{1-\frac{1}{c^2x^2}}x\right)\sec^{-1}(cx)-b^2\sec^{-1}(cx)^2}{x}$$

```
[In] Integrate[(a + b*ArcSec[c*x])^2/x^2,x]
```

```
[Out] (-a^2 + 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 2*b*(-a + b*c*Sqrt[1 - 1/
(c^2*x^2)]*x)*ArcSec[c*x] - b^2*ArcSec[c*x]^2)/x
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(48) = 96$.

Time = 0.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.28

method	result
parts	$-\frac{a^2}{x}+b^2c\left(-\frac{\operatorname{arcsec}(cx)^2}{cx}+\frac{2}{cx}+2\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2abc\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)$
derivativedivides	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{cx}+\frac{2}{cx}+2\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2ab\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right.\right.$
default	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{cx}+\frac{2}{cx}+2\operatorname{arcsec}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2ab\left(-\frac{\operatorname{arcsec}(cx)}{cx}+\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right.\right.$

```
[In] int((a+b*arcsec(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/x+b^2*c*(-1/c/x*arcsec(c*x)^2+2/c/x+2*arcsec(c*x)*((c^2*x^2-1)/c^2/x^2
)^(1/2))+2*a*b*c*(-1/c/x*arcsec(c*x)+1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*
(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{b^2\operatorname{arcsec}(cx)^2+2\,ab\operatorname{arcsec}(cx)+a^2-2\,b^2-2\,\sqrt{c^2x^2-1}\left(b^2\operatorname{arcsec}(cx)+ab\right)}{x}$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] -(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2 - 2*b^2 - 2*sqrt(c^2*x^2 - 1)
*(b^2*arcsec(c*x) + a*b))/x
```

Sympy [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{\left(a+b\operatorname{asec}(cx)\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*asec(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*asec(c*x))**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = 2\left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)ab + 2\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arcsec}(cx)+\frac{1}{x}\right)b^2 - \frac{b^2\operatorname{arcsec}(cx)^2}{x} - \frac{a^2}{x}$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*a*b + 2*(c*sqrt(-1/(c^2*x^2) +
 1)*arcsec(c*x) + 1/x)*b^2 - b^2*arcsec(c*x)^2/x - a^2/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(48) = 96$.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.10

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = \left(2b^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)+2ab\sqrt{-\frac{1}{c^2x^2}+1}-\frac{b^2\arccos\left(\frac{1}{cx}\right)^2}{cx}-\frac{2ab\arccos\left(\frac{1}{cx}\right)}{cx}-\frac{a^2}{cx}+\frac{2b^2}{cx}\right)c$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] (2*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x)) + 2*a*b*sqrt(-1/(c^2*x^2) + 1
) - b^2*arccos(1/(c*x))^2/(c*x) - 2*a*b*arccos(1/(c*x))/(c*x) - a^2/(c*x) +
 2*b^2/(c*x))*c
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.78

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^2}\,dx = 2\,b^2\,c\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{b^2\,\left(\mathrm{acos}\left(\frac{1}{c\,x}\right)^2-2\right)}{x}-\frac{a^2}{x}+2\,a\,b\,c\,\left(\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{\mathrm{acos}\left(\frac{1}{c\,x}\right)}{c\,x}\right)$$

```
[In] int((a + b*acos(1/(c*x)))^2/x^2,x)
```

```
[Out] 2*b^2*c*acos(1/(c*x))*(1 - 1/(c^2*x^2))^(1/2) - (b^2*(acos(1/(c*x))^2 - 2))
/x - a^2/x + 2*a*b*c*((1 - 1/(c^2*x^2))^(1/2) - acos(1/(c*x))/(c*x))
```

3.21 $\int \frac{(a+b\sec^{-1}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 94

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^3}\,dx = \frac{b^2}{4x^2}-\frac{1}{2}abc^2\sec^{-1}(cx)-\frac{1}{4}b^2c^2\sec^{-1}(cx)^2+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{2x}+\frac{1}{2}\left(c^2-\frac{1}{x^2}\right)(a+b\sec^{-1}(cx))^2$$

```
[Out] 1/4*b^2/x^2-1/2*a*b*c^2*arcsec(c*x)-1/4*b^2*c^2*arcsec(c*x)^2+1/2*(c^2-1/x^
2)*(a+b*arcsec(c*x))^2+1/2*b*c*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5330, 4489, 3391}

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^3}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{2x}+\frac{1}{2}\left(c^2-\frac{1}{x^2}\right)(a+b\sec^{-1}(cx))^2-\frac{1}{2}abc^2\sec^{-1}(cx)-\frac{1}{4}b^2c^2\sec^{-1}(cx)^2+\frac{b^2}{4x^2}$$

```
[In] Int[(a + b*ArcSec[c*x])^2/x^3,x]
```

```
[Out] b^2/(4*x^2) - (a*b*c^2*ArcSec[c*x])/2 - (b^2*c^2*ArcSec[c*x]^2)/4 + (b*c*Sq
rt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(2*x) + ((c^2 - x^(-2))*(a + b*Arc
Sec[c*x])^2)/2
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^2\text{Subst}\left(\int (a+bx)^2\cos(x)\sin(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2-\left(bc^2\right)\text{Subst}\left(\int (a+bx)\sin^2(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{b^2}{4x^2}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2x}+\frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2\\
&\quad-\frac{1}{2}\left(bc^2\right)\text{Subst}\left(\int (a+bx)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{b^2}{4x^2}-\frac{1}{2}abc^2\sec^{-1}(cx)-\frac{1}{4}b^2c^2\sec^{-1}(cx)^2\\
&\quad+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{2x}+\frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^2
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^3}\,dx$$
$$= \frac{-2a^2+b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x+2b\left(-2a+bc\sqrt{1-\frac{1}{c^2x^2}}x\right)\sec^{-1}(cx)+b^2(-2+c^2x^2)\sec^{-1}(cx)^2-2abc^2x^2}{4x^2}$$

```
[In] Integrate[(a + b*ArcSec[c*x])^2/x^3,x]
```

```
[Out] (-2*a^2 + b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 2*b*(-2*a + b*c*Sqrt[1 -
1/(c^2*x^2)]*x)*ArcSec[c*x] + b^2*(-2 + c^2*x^2)*ArcSec[c*x]^2 - 2*a*b*c^2*
x^2*ArcSin[1/(c*x)])/(4*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(82) = 164.

Time = 0.49 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.88

method	result
derivativedivides	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\mathrm{arcsec}(cx)^2}{2c^2x^2}+\frac{\mathrm{arcsec}(cx)\left(cx\,\mathrm{arcsec}(cx)+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}-\frac{\mathrm{arcsec}(cx)^2}{4}-\frac{1}{4}+\frac{1}{4c^2x^2}\right)+$
default	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\mathrm{arcsec}(cx)^2}{2c^2x^2}+\frac{\mathrm{arcsec}(cx)\left(cx\,\mathrm{arcsec}(cx)+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}-\frac{\mathrm{arcsec}(cx)^2}{4}-\frac{1}{4}+\frac{1}{4c^2x^2}\right)+$
parts	$-\frac{a^2}{2x^2}+b^2c^2\left(-\frac{\mathrm{arcsec}(cx)^2}{2c^2x^2}+\frac{\mathrm{arcsec}(cx)\left(cx\,\mathrm{arcsec}(cx)+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}-\frac{\mathrm{arcsec}(cx)^2}{4}-\frac{1}{4}+\frac{1}{4c^2x^2}\right)+2ab$

```
[In] int((a+b*arcsec(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^2/c^2/x^2+b^2*(-1/2/c^2/x^2*arcsec(c*x)^2+1/2*arcsec(c*x)*(c*x*
arcsec(c*x)+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x-1/4*arcsec(c*x)^2-1/4+1/4/c^2/
x^2)+2*a*b*(-1/2/c^2/x^2*arcsec(c*x)+1/4*(c^2*x^2-1)^(1/2)*(-arctan(1/(c^2*
x^2-1)^(1/2))*c^2*x^2+(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^3/x^
3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^3}\,dx = \frac{(b^2c^2x^2 - 2\,b^2)\operatorname{arcsec}(cx)^2 - 2\,a^2 + b^2 + 2\,(abc^2x^2 - 2\,ab)\operatorname{arcsec}(cx) + 2\,\sqrt{c^2x^2 - 1}(b^2\operatorname{arcsec}(cx) + ab)}{4\,x^2}$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^3,x, algorithm="fricas")
[Out] 1/4*((b^2*c^2*x^2 - 2*b^2)*arcsec(c*x)^2 - 2*a^2 + b^2 + 2*(a*b*c^2*x^2 - 2
*a*b)*arcsec(c*x) + 2*sqrt(c^2*x^2 - 1)*(b^2*arcsec(c*x) + a*b))/x^2
```

Sympy [F]

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^3}\,dx = \int \frac{(a + b\operatorname{asec}(cx))^2}{x^3}\,dx$$

```
[In] integrate((a+b*asec(c*x))**2/x**3,x)
[Out] Integral((a + b*asec(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^3}\,dx = \int \frac{(b\operatorname{arcsec}(cx) + a)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^3,x, algorithm="maxima")
[Out] -1/2*a*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c
^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) - 1/8*(4*(c^2
*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 - 4*c^2*integrate(1/2*x^
2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 8*c^2*integrate(1/2*x^2*log(x)/
(c^2*x^5 - x^3), x)*log(c) - 4*c^2*integrate(1/2*x^2*log(c^2*x^2)*log(x)/(c
^2*x^5 - x^3), x) + 4*c^2*integrate(1/2*x^2*log(x)^2/(c^2*x^5 - x^3), x) +
2*c^2*integrate(1/2*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x) - (c^2*log(c*x + 1
) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*log(c)^2 + 4*integrate(1/2*log
(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 8*integrate(1/2*log(x)/(c^2*x^5 - x^
3), x)*log(c) - 4*integrate(1/2*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(sqrt(c*x
 + 1)*sqrt(c*x - 1))/(c^2*x^5 - x^3), x) + 4*integrate(1/2*log(c^2*x^2)*log
(x)/(c^2*x^5 - x^3), x) - 4*integrate(1/2*log(x)^2/(c^2*x^5 - x^3), x) - 2*
integrate(1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x))*x^2 + 4*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1))^2 - log(c^2*x^2)^2)*b^2/x^2 - 1/2*a^2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.56

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^3}\,dx$$
$$= \frac{1}{8}\left(2\,b^2c\arccos\left(\frac{1}{cx}\right)^2 + 4\,abc\arccos\left(\frac{1}{cx}\right) - b^2c + \frac{4\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)}{x} + \frac{4\,ab\sqrt{-\frac{1}{c^2x^2}+1}}{x} - \frac{4\,b^2}{}\right.$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*c*arccos(1/(c*x))^2 + 4*a*b*c*arccos(1/(c*x)) - b^2*c + 4*b^2*sq
rt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x + 4*a*b*sqrt(-1/(c^2*x^2) + 1)/x - 4
*b^2*arccos(1/(c*x))^2/(c*x^2) - 8*a*b*arccos(1/(c*x))/(c*x^2) - 4*a^2/(c*x
^2) + 2*b^2/(c*x^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\sec^{-1}(cx))^2}{x^3}\,dx = \int \frac{\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}{x^3}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))^2/x^3,x)
```

```
[Out] int((a + b*acos(1/(c*x)))^2/x^3, x)
```

3.22 $\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx = \frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}+\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx)) + \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{9x^2}-\frac{(a+b\sec^{-1}(cx))^2}{3x^3}$$

```
[Out] 2/27*b^2/x^3+4/9*b^2*c^2/x-1/3*(a+b*arcsec(c*x))^2/x^3+4/9*b*c^3*(a+b*arcse
c(c*x))*(1-1/c^2/x^2)^(1/2)+2/9*b*c*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/x
^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5330, 4490, 3391, 3377, 2718}

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx = \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{9x^2}+\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx)) - \frac{(a+b\sec^{-1}(cx))^2}{3x^3}+\frac{4b^2c^2}{9x}+\frac{2b^2}{27x^3}$$

```
[In] Int[(a + b*ArcSec[c*x])^2/x^4,x]
```

```
[Out] (2*b^2)/(27*x^3) + (4*b^2*c^2)/(9*x) + (4*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a +
b*ArcSec[c*x]))/9 + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(9*x^
2) - (a + b*ArcSec[c*x])^2/(3*x^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^3\text{Subst}\left(\int (a+bx)^2\cos^2(x)\sin(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= -\frac{\left(a+b\sec^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(2bc^3\right)\text{Subst}\left(\int (a+bx)\cos^3(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{2b^2}{27x^3}+\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\sec^{-1}(cx)\right)^2}{3x^3}\\
&\quad+\frac{1}{9}\left(4bc^3\right)\text{Subst}\left(\int (a+bx)\cos(x)\,dx, x, \sec^{-1}(cx)\right)
\end{aligned}$$

$$= \frac{2b^2}{27x^3} + \frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx)) + \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{9x^2} - \frac{(a+b\sec^{-1}(cx))^2}{3x^3} - \frac{1}{9}(4b^2c^3)\,\text{Subst}\left(\int \sin(x)\,dx, x, \sec^{-1}(cx)\right)$$

$$= \frac{2b^2}{27x^3} + \frac{4b^2c^2}{9x} + \frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx)) + \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{9x^2} - \frac{(a+b\sec^{-1}(cx))^2}{3x^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx = \frac{-9a^2+6abc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)+2b^2(1+6c^2x^2)+6b\left(-3a+bc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)\right)\sec^{-1}(cx) -}{27x^3}$$

```
[In] Integrate[(a + b*ArcSec[c*x])^2/x^4,x]
```

```
[Out] (-9*a^2 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c^
2*x^2) + 6*b*(-3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*ArcSec[c*
x] - 9*b^2*ArcSec[c*x]^2)/(27*x^3)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

method	result
parts	$-\frac{a^2}{3x^3}+b^2c^3\left(-\frac{\operatorname{arcsec}(cx)^2}{3c^3x^3}+\frac{2\operatorname{arcsec}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\,c^3\left(-\frac{\operatorname{arcsec}(c}{3c^3x^3}\right.$
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{3c^3x^3}+\frac{2\operatorname{arcsec}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\operatorname{arcsec}(}{3c^3x}\right.\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arcsec}(cx)^2}{3c^3x^3}+\frac{2\operatorname{arcsec}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\operatorname{arcsec}(}{3c^3x}\right.\right.$

```
[In] int((a+b*arcsec(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arcsec(c*x)^2+2/9*arcsec(c*x)*(2*c^2*x^2
+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/x)+2*a*b*c^3*(-1
```

```
/3/c^3/x^3*arcsec(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/c^4/x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx$$
$$= \frac{12\,b^2c^2x^2 - 9\,b^2\operatorname{arcsec}(cx)^2 - 18\,ab\operatorname{arcsec}(cx) - 9\,a^2 + 2\,b^2 + 6\left(2\,abc^2x^2 + ab + \left(2\,b^2c^2x^2 + b^2\right)\operatorname{arcsec}(cx\right.}{27\,x^3}$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(12*b^2*c^2*x^2 - 9*b^2*arcsec(c*x)^2 - 18*a*b*arcsec(c*x) - 9*a^2 + 2
*b^2 + 6*(2*a*b*c^2*x^2 + a*b + (2*b^2*c^2*x^2 + b^2)*arcsec(c*x))*sqrt(c^2
*x^2 - 1))/x^3
```

Sympy [F]

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{\left(a+b\operatorname{asec}(cx)\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*asec(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*asec(c*x))**2/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.61

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^2}{x^4}\,dx$$
$$= -\frac{2}{9}\,ab\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{3\operatorname{arcsec}(cx)}{x^3}\right) - \frac{b^2\operatorname{arcsec}(cx)^2}{3\,x^3} - \frac{a^2}{3\,x^3}$$
$$+ \frac{2\left(\left(6\,c^3x^2+c\right)\sqrt{cx+1}\sqrt{cx-1} + 3\left(2\,c^5x^4 - c^3x^2 - c\right)\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)\right)b^2}{27\,\sqrt{cx+1}\sqrt{cx-1}cx^3}$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -2/9*a*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c +
 3*arcsec(c*x)/x^3) - 1/3*b^2*arcsec(c*x)^2/x^3 - 1/3*a^2/x^3 + 2/27*((6*c^
3*x^2 + c)*sqrt(c*x + 1)*sqrt(c*x - 1) + 3*(2*c^5*x^4 - c^3*x^2 - c)*arctan
(sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.65

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx$$
$$= \frac{1}{27}\left(12\,b^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)+12\,abc^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{12\,b^2c}{x}+\frac{6\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)}{x^2}+\cdots\right.$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] 1/27*(12*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x)) + 12*a*b*c^2*sqrt(-
1/(c^2*x^2) + 1) + 12*b^2*c/x + 6*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x)
)/x^2 + 6*a*b*sqrt(-1/(c^2*x^2) + 1)/x^2 - 9*b^2*arccos(1/(c*x))^2/(c*x^3)
- 18*a*b*arccos(1/(c*x))/(c*x^3) - 9*a^2/(c*x^3) + 2*b^2/(c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^4}\,dx = \int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}{x^4}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))^2/x^4,x)
```

```
[Out] int((a + b*acos(1/(c*x)))^2/x^4, x)
```

3.23 $\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 134

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx = \frac{b^2}{32x^4}+\frac{3b^2c^2}{32x^2}+\frac{3}{16}abc^4\sec^{-1}(cx)+\frac{3}{32}b^2c^4\sec^{-1}(cx)^2+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{8x^3}+\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{16x}-\frac{(a+b\sec^{-1}(cx))^2}{4x^4}$$

```
[Out] 1/32*b^2/x^4+3/32*b^2*c^2/x^2+3/16*a*b*c^4*arcsec(c*x)+3/32*b^2*c^4*arcsec(
c*x)^2-1/4*(a+b*arcsec(c*x))^2/x^4+1/8*b*c*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^
(1/2)/x^3+3/16*b*c^3*(a+b*arcsec(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5330, 4490, 3391}

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx = \frac{3}{16}abc^4\sec^{-1}(cx)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{8x^3}+\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{16x}-\frac{(a+b\sec^{-1}(cx))^2}{4x^4}+\frac{3}{32}b^2c^4\sec^{-1}(cx)^2+\frac{3b^2c^2}{32x^2}+\frac{b^2}{32x^4}$$

```
[In] Int[(a + b*ArcSec[c*x])^2/x^5,x]
```

```
[Out] b^2/(32*x^4) + (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcSec[c*x])/16 + (3*b^2*c
^4*ArcSec[c*x]^2)/32 + (b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(8*x
^3) + (3*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x]))/(16*x) - (a + b*A
rcSec[c*x])^2/(4*x^4)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c^4\text{Subst}\left(\int (a+bx)^2\cos^3(x)\sin(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= -\frac{\left(a+b\sec^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bc^4\right)\text{Subst}\left(\int (a+bx)\cos^4(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{b^2}{32x^4} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{8x^3} - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{4x^4}\\
&\quad + \frac{1}{8}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)\cos^2(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{8x^3} + \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)}{16x}\\
&\quad - \frac{\left(a+b\sec^{-1}(cx)\right)^2}{4x^4} + \frac{1}{16}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)\,dx, x, \sec^{-1}(cx)\right)
\end{aligned}
$$

$$= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4\sec^{-1}(cx) + \frac{3}{32}b^2c^4\sec^{-1}(cx)^2 + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{8x^3} + \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))}{16x} - \frac{(a+b\sec^{-1}(cx))^2}{4x^4}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx = \frac{-8a^2+b^2+4abc\sqrt{1-\frac{1}{c^2x^2}}x+3b^2c^2x^2+6abc^3\sqrt{1-\frac{1}{c^2x^2}}x^3+2b\left(-8a+bc\sqrt{1-\frac{1}{c^2x^2}}x(2+3c^2x^2)\right)\sec^{-1}}{32x^4}$$

```
[In] Integrate[(a + b*ArcSec[c*x])^2/x^5,x]
```

```
[Out] (-8*a^2 + b^2 + 4*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 + 6*a*b*c^3
*Sqrt[1 - 1/(c^2*x^2)]*x^3 + 2*b*(-8*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3
*c^2*x^2))*ArcSec[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcSec[c*x]^2 - 6*a*b*c^4*x^4
*ArcSin[1/(c*x)])/(32*x^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(116) = 232$.

Time = 0.82 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.98

method	result
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\mathrm{arcsec}(cx)^2}{4c^4x^4}+\frac{\mathrm{arcsec}(cx)\left(3c^3x^3\,\mathrm{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\,\mathrm{arcsec}(cx)^2}{32}+\right.$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\mathrm{arcsec}(cx)^2}{4c^4x^4}+\frac{\mathrm{arcsec}(cx)\left(3c^3x^3\,\mathrm{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\,\mathrm{arcsec}(cx)^2}{32}\right.\right.$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\mathrm{arcsec}(cx)^2}{4c^4x^4}+\frac{\mathrm{arcsec}(cx)\left(3c^3x^3\,\mathrm{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\,\mathrm{arcsec}(cx)^2}{32}\right.\right.$

```
[In] int((a+b*arcsec(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^2/x^4+b^2*c^4*(-1/4/c^4/x^4*arcsec(c*x)^2+1/16*arcsec(c*x)*(3*c^3*x^
3*arcsec(c*x)+3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2*((c^2*x^2-1)/c^2/x^2)
^(1/2))/c^3/x^3-3/32*arcsec(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-1/2*a*b/x
^4*arcsec(c*x)-3/16*a*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

```
*arctan(1/(c^2*x^2-1)^(1/2))+3/16*a*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(
1/2)/x^3+1/8*a*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx = \frac{3\,b^2c^2x^2 + (3\,b^2c^4x^4 - 8\,b^2)\,\mathrm{arcsec}\,(cx)^2 - 8\,a^2 + b^2 + 2\,(3\,abc^4x^4 - 8\,ab)\,\mathrm{arcsec}\,(cx) + 2\,(3\,abc^2x^2 + 2\,ab +}{32\,x^4}$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*arcsec(c*x)^2 - 8*a^2 + b^2 +
 2*(3*a*b*c^4*x^4 - 8*a*b)*arcsec(c*x) + 2*(3*a*b*c^2*x^2 + 2*a*b + (3*b^2*
c^2*x^2 + 2*b^2)*arcsec(c*x))*sqrt(c^2*x^2 - 1))/x^4
```

Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))^2}{x^5}\,dx$$

```
[In] integrate((a+b*asec(c*x))**2/x**5,x)
```

```
[Out] Integral((a + b*asec(c*x))**2/x**5, x)
```

Maxima [F]

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)^2}{x^5}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*a*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*x
^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1)
^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 8*arcsec(c*x)/x^4) - 1/16*(4*(2*
(c^2*log(c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*c^2*log(c)^2 -
 16*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 32*c^2*
integrate(1/4*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) - 16*c^2*integrate(1/4*
x^2*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 16*c^2*integrate(1/4*x^2*log(
```

```
x)^2/(c^2*x^7 - x^5), x) + 4*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 -
x^5), x) - (2*c^4*log(c*x + 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x) + (2*c^2
*x^2 + 1)/x^4)*log(c)^2 + 16*integrate(1/4*log(c^2*x^2)/(c^2*x^7 - x^5), x)
*log(c) - 32*integrate(1/4*log(x)/(c^2*x^7 - x^5), x)*log(c) - 8*integrate(
1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^
7 - x^5), x) + 16*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) - 1
6*integrate(1/4*log(x)^2/(c^2*x^7 - x^5), x) - 4*integrate(1/4*log(c^2*x^2)
/(c^2*x^7 - x^5), x))*x^4 + 4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - log(c
^2*x^2)^2)*b^2/x^4 - 1/4*a^2/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx$$
$$= \frac{1}{256}\left(24\,b^2c^3\arccos\left(\frac{1}{cx}\right)^2 + 48\,abc^3\arccos\left(\frac{1}{cx}\right) - 15\,b^2c^3 + \frac{48\,b^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)}{x} + \frac{48\,abc^2\sqrt{\cdots}}{\cdots}\right.$$

```
[In] integrate((a+b*arcsec(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] 1/256*(24*b^2*c^3*arccos(1/(c*x))^2 + 48*a*b*c^3*arccos(1/(c*x)) - 15*b^2*c
^3 + 48*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x + 48*a*b*c^2*sqrt(
-1/(c^2*x^2) + 1)/x + 24*b^2*c/x^2 + 32*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1
/(c*x))/x^3 + 32*a*b*sqrt(-1/(c^2*x^2) + 1)/x^3 - 64*b^2*arccos(1/(c*x))^2/
(c*x^4) - 128*a*b*arccos(1/(c*x))/(c*x^4) - 64*a^2/(c*x^4) + 8*b^2/(c*x^4))
*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec^{-1}(cx))^2}{x^5}\,dx = \int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))^2/x^5,x)
```

```
[Out] int((a + b*acos(1/(c*x)))^2/x^5, x)
```

3.24 $\int x^3 (a+b\sec^{-1}(cx))^3 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 207

$$\begin{aligned}\int x^3 (a+b\sec^{-1}(cx))^3 \, dx = & -\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3} + \frac{b^2x^2(a+b\sec^{-1}(cx))}{4c^2} \\ & + \frac{ib(a+b\sec^{-1}(cx))^2}{2c^4} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\sec^{-1}(cx))^2}{2c^3} \\ & - \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\sec^{-1}(cx))^2}{4c} + \frac{1}{4}x^4(a+b\sec^{-1}(cx))^3 \\ & - \frac{b^2(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^4} \\ & + \frac{ib^3\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2c^4}\end{aligned}$$

```
[Out] 1/4*b^2*x^2*(a+b*arcsec(c*x))/c^2+1/2*I*b*(a+b*arcsec(c*x))^2/c^4+1/4*x^4*(
a+b*arcsec(c*x))^3-b^2*(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))
^2)/c^4+1/2*I*b^3*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/c^4-1/4*b^3*x
*(1-1/c^2/x^2)^(1/2)/c^3-1/2*b*x*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)/c^
3-1/4*b*x^3*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5330, 4494, 4271, 3852, 8, 4269, 3800, 2221, 2317, 2438}

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx=-\frac{b^2\log\left(1+e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)}{c^4}+\frac{b^2x^2(a+b\sec^{-1}(cx))}{4c^2}+\frac{ib(a+b\sec^{-1}(cx))^2}{2c^4}-\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{4c}-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{2c^3}+\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^3+\frac{ib^3\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2c^4}-\frac{b^3x\sqrt{1-\frac{1}{c^2x^2}}}{4c^3}$$

```
[In] Int[x^3*(a + b*ArcSec[c*x])^3,x]
```

```
[Out] -1/4*(b^3*Sqrt[1 - 1/(c^2*x^2)]*x)/c^3 + (b^2*x^2*(a + b*ArcSec[c*x]))/(4*c
^2) + ((I/2)*b*(a + b*ArcSec[c*x])^2)/c^4 - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a +
 b*ArcSec[c*x])^2)/(2*c^3) - (b*Sqrt[1 - 1/(c^2*x^2)]*x^3*(a + b*ArcSec[c*x
])^2)/(4*c) + (x^4*(a + b*ArcSec[c*x])^3)/4 - (b^2*(a + b*ArcSec[c*x])*Log[
1 + E^((2*I)*ArcSec[c*x])])/c^4 + ((I/2)*b^3*PolyLog[2, -E^((2*I)*ArcSec[c*
x])])/c^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
```

```
LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)^3\sec^4(x)\tan(x)\,dx,x,\sec^{-1}(cx)\right)}{c^4}\\
&= \frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{(3b)\text{Subst}\left(\int (a+bx)^2\sec^4(x)\,dx,x,\sec^{-1}(cx)\right)}{4c^4}\\
&= \frac{b^2x^2\left(a+b\sec^{-1}(cx)\right)}{4c^2}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3\left(a+b\sec^{-1}(cx)\right)^2}{4c}+\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^3\\
&\quad-\frac{b\text{Subst}\left(\int (a+bx)^2\sec^2(x)\,dx,x,\sec^{-1}(cx)\right)}{2c^4}-\frac{b^3\text{Subst}\left(\int \sec^2(x)\,dx,x,\sec^{-1}(cx)\right)}{4c^4}\\
&= \frac{b^2x^2\left(a+b\sec^{-1}(cx)\right)}{4c^2}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)^2}{2c^3}\\
&\quad-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3\left(a+b\sec^{-1}(cx)\right)^2}{4c}+\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^3\\
&\quad+\frac{b^2\text{Subst}\left(\int (a+bx)\tan(x)\,dx,x,\sec^{-1}(cx)\right)}{c^4}+\frac{b^3\text{Subst}\left(\int 1\,dx,x,-c\sqrt{1-\frac{1}{c^2x^2}}x\right)}{4c^4}\\
&= -\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}x}{4c^3}+\frac{b^2x^2\left(a+b\sec^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\sec^{-1}(cx)\right)^2}{2c^4}\\
&\quad-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)^2}{2c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3\left(a+b\sec^{-1}(cx)\right)^2}{4c}\\
&\quad+\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{(2ib^2)\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}\,dx,x,\sec^{-1}(cx)\right)}{c^4}\\
&= -\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}x}{4c^3}+\frac{b^2x^2\left(a+b\sec^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\sec^{-1}(cx)\right)^2}{2c^4}\\
&\quad-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)^2}{2c^3}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3\left(a+b\sec^{-1}(cx)\right)^2}{4c}\\
&\quad+\frac{1}{4}x^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{b^2\left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^4}\\
&\quad+\frac{b^3\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx,x,\sec^{-1}(cx)\right)}{c^4}
\end{aligned}$$

$$= -\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2(a+b\sec^{-1}(cx))}{4c^2} + \frac{ib(a+b\sec^{-1}(cx))^2}{2c^4} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\sec^{-1}(cx))^2}{2c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\sec^{-1}(cx))^2}{4c} + \frac{1}{4}x^4(a+b\sec^{-1}(cx))^3 - \frac{b^2(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^4} - \frac{(ib^3)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2i\sec^{-1}(cx)}\right)}{2c^4}$$

$$= -\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2(a+b\sec^{-1}(cx))}{4c^2} + \frac{ib(a+b\sec^{-1}(cx))^2}{2c^4} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\sec^{-1}(cx))^2}{2c^3} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\sec^{-1}(cx))^2}{4c} + \frac{1}{4}x^4(a+b\sec^{-1}(cx))^3 - \frac{b^2(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^4} + \frac{ib^3\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2c^4}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.39

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx = \frac{-2a^2bc\sqrt{1-\frac{1}{c^2x^2}}x - b^3c\sqrt{1-\frac{1}{c^2x^2}}x + ab^2c^2x^2 - a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}x^3 + a^3c^4x^4 - b^2\left(-3ac^4x^4 + b\left(-2i + 2\right.\right.}{}$$

```
[In] Integrate[x^3*(a + b*ArcSec[c*x])^3,x]
```

```
[Out] (-2*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*b^2
*c^2*x^2 - a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 - b^2*(-3*a*c^
4*x^4 + b*(-2*I + 2*c*Sqrt[1 - 1/(c^2*x^2)]*x + c^3*Sqrt[1 - 1/(c^2*x^2)]*x
^3))*ArcSec[c*x]^2 + b^3*c^4*x^4*ArcSec[c*x]^3 + b*ArcSec[c*x]*(c*x*(b^2*c*
x + 3*a^2*c^3*x^3 - 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2)) - 4*b^2*Log[
1 + E^((2*I)*ArcSec[c*x])]) - 4*a*b^2*Log[1/(c*x)] + (2*I)*b^3*PolyLog[2, -
E^((2*I)*ArcSec[c*x])])/(4*c^4)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.76

method	result
derivativedivides	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arcsec}(cx)^3c^4x^4}{4}-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\operatorname{arcsec}(cx)^2c^3x^3}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arcsec}(cx)^2}{2}+\frac{c^2x^2\operatorname{arcsec}(cx)}{4}-\frac{xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4}\cdots\right.}{\cdots}$
default	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arcsec}(cx)^3c^4x^4}{4}-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\operatorname{arcsec}(cx)^2c^3x^3}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arcsec}(cx)^2}{2}+\frac{c^2x^2\operatorname{arcsec}(cx)}{4}-\frac{xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4}\cdots\right.}{\cdots}$
parts	$\frac{a^3x^4}{4}+\frac{b^3\left(\frac{\operatorname{arcsec}(cx)^3c^4x^4}{4}-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\operatorname{arcsec}(cx)^2c^3x^3}{4}-\frac{\operatorname{arcsec}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arcsec}(cx)^2}{2}+\frac{c^2x^2\operatorname{arcsec}(cx)}{4}-\frac{xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4}\cdots\right.}{c^4}$

```
[In] int(x^3*(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*a^3*c^4*x^4+b^3*(1/4*arcsec(c*x)^3*c^4*x^4-1/4*((c^2*x^2-1)/c^2/
x^2)^(1/2)*arcsec(c*x)^2*c^3*x^3-1/2*arcsec(c*x)^2*((c^2*x^2-1)/c^2/x^2)^(1
/2)*c*x+1/2*I*arcsec(c*x)^2+1/4*c^2*x^2*arcsec(c*x)-1/4*x*c*((c^2*x^2-1)/c^
2/x^2)^(1/2)-1/4*I-arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+1/2*I*
polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))+3*a*b^2*(1/4*arcsec(c*x)^2*c^4
*x^4-1/6*arcsec(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*x^3+1/12*c^2*x^2-1/3*a
rcsec(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/3*ln(1/c/x))+3*a^2*b*(1/4*c^4*
x^4*arcsec(c*x)-1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/
x))
```

Fricas [F]

$$\int x^3\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int (b\operatorname{arcsec}(cx)+a)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arcsec(c*x)^3 + 3*a*b^2*x^3*arcsec(c*x)^2 + 3*a^2*b*x^3*ar
csec(c*x) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a + b\sec^{-1}(cx)\right)^3 \, dx = \int x^3 (a + b\,\mathrm{asec}\,(cx))^3 \, dx$$

```
[In] integrate(x**3*(a+b*asec(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*asec(c*x))**3, x)
```

Maxima [F]

$$\int x^3 \left(a + b\sec^{-1}(cx)\right)^3 \, dx = \int (b\,\mathrm{arcsec}\,(cx) + a)^3 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arcsec(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arcsec(c*x) - (c^2*x
^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a^2*b + 1/16
*(4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x^4*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))*log(c^2*x^2)^2 - 16*integrate(3/16*((4*x^3*arctan(sqrt(c*x +
 1)*sqrt(c*x - 1))^2 - x^3*log(c^2*x^2)^2)*sqrt(c*x + 1)*sqrt(c*x - 1) + 4*
(4*c^2*x^5*log(c)^2 - 4*x^3*log(c)^2 + 4*(c^2*x^5 - x^3)*log(x)^2 - ((4*c^2
*log(c) + c^2)*x^5 - x^3*(4*log(c) + 1) + 4*(c^2*x^5 - x^3)*log(x))*log(c^2
*x^2) + 8*(c^2*x^5*log(c) - x^3*log(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c
*x - 1)))/(c^2*x^2 - 1), x))*b^3 + 1/4*((c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1
)*sqrt(c*x - 1) - 2*(c^4*x^4 + c^2*x^2 - 2)*arctan(sqrt(c*x + 1)*sqrt(c*x -
 1)))*a*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c^4)
```

Giac [F]

$$\int x^3 \left(a + b\sec^{-1}(cx)\right)^3 \, dx = \int (b\,\mathrm{arcsec}\,(cx) + a)^3 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int x^3 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3 dx$$

```
[In] int(x^3*(a + b*acos(1/(c*x)))^3,x)
[Out] int(x^3*(a + b*acos(1/(c*x)))^3, x)
```

3.25 $\int x^2(a+b\sec^{-1}(cx))^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 236

$$\begin{aligned}\int x^2(a+b\sec^{-1}(cx))^3\,dx=&\frac{b^2x(a+b\sec^{-1}(cx))}{c^2}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\sec^{-1}(cx))^2}{2c}\\&+\frac{1}{3}x^3(a+b\sec^{-1}(cx))^3\\&+\frac{ib(a+b\sec^{-1}(cx))^2\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c^3}\\&-\frac{b^3\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3}\\&-\frac{ib^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c^3}\\&+\frac{ib^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c^3}\\&+\frac{b^3\operatorname{PolyLog}\left(3,-ie^{i\sec^{-1}(cx)}\right)}{c^3}-\frac{b^3\operatorname{PolyLog}\left(3,ie^{i\sec^{-1}(cx)}\right)}{c^3}\end{aligned}$$

```
[Out] b^2*x*(a+b*arcsec(c*x))/c^2+1/3*x^3*(a+b*arcsec(c*x))^3+I*b*(a+b*arcsec(c*x
))^2*arctan(1/c/x+I*(1-1/c^2/x^2)^(1/2))/c^3-b^3*arctanh((1-1/c^2/x^2)^(1/2
))/c^3-I*b^2*(a+b*arcsec(c*x))*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/
c^3+I*b^2*(a+b*arcsec(c*x))*polylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3+
b^3*polylog(3,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c^3-b^3*polylog(3,I*(1/c/x+
I*(1-1/c^2/x^2)^(1/2)))/c^3-1/2*b*x^2*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/
2)/c
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5330, 4494, 4271, 3855, 4266, 2611, 2320, 6724}

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^3\,dx=\frac{ib\arctan\left(e^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2}{c^3}-\frac{ib^2\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)}{c^3}+\frac{ib^2\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)}{c^3}+\frac{b^2x\left(a+b\sec^{-1}(cx)\right)}{c^2}-\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^3-\frac{b^3\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3}+\frac{b^3\,\text{PolyLog}\left(3,-ie^{i\sec^{-1}(cx)}\right)}{c^3}-\frac{b^3\,\text{PolyLog}\left(3,ie^{i\sec^{-1}(cx)}\right)}{c^3}$$

```
[In] Int[x^2*(a + b*ArcSec[c*x])^3,x]
```

```
[Out] (b^2*x*(a + b*ArcSec[c*x]))/c^2 - (b*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcSe
c[c*x])^2)/(2*c) + (x^3*(a + b*ArcSec[c*x])^3)/3 + (I*b*(a + b*ArcSec[c*x])
^2*ArcTan[E^(I*ArcSec[c*x])])/c^3 - (b^3*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c^
3 - (I*b^2*(a + b*ArcSec[c*x])*PolyLog[2, (-I)*E^(I*ArcSec[c*x])])/c^3 + (I
*b^2*(a + b*ArcSec[c*x])*PolyLog[2, I*E^(I*ArcSec[c*x])])/c^3 + (b^3*PolyLo
g[3, (-I)*E^(I*ArcSec[c*x])])/c^3 - (b^3*PolyLog[3, I*E^(I*ArcSec[c*x])])/c
^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+bx)^3 \sec^3(x)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{c^3}$$

$$= \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^3 - \frac{b\text{Subst}\left(\int (a+bx)^2 \sec^3(x)\,dx, x, \sec^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x(a+b\sec^{-1}(cx))}{c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2\left(a+b\sec^{-1}(cx)\right)^2}{2c} + \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^3 - \frac{b\text{Subst}\left(\int (a+bx)^2\sec(x)\,dx, x, \sec^{-1}(cx)\right)}{2c^3} - \frac{b^3\text{Subst}\left(\int \sec(x)\,dx, x, \sec^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x(a+b\sec^{-1}(cx))}{c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2\left(a+b\sec^{-1}(cx)\right)^2}{2c} + \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^3 + \frac{ib\left(a+b\sec^{-1}(cx)\right)^2\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c^3} - \frac{b^3\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3} + \frac{b^2\text{Subst}\left(\int (a+bx)\log\left(1-ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c^3} - \frac{b^2\text{Subst}\left(\int (a+bx)\log\left(1+ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x(a+b\sec^{-1}(cx))}{c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2\left(a+b\sec^{-1}(cx)\right)^2}{2c} + \frac{1}{3}x^3\left(a+b\sec^{-1}(cx)\right)^3 + \frac{ib\left(a+b\sec^{-1}(cx)\right)^2\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c^3} - \frac{b^3\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3} - \frac{ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c^3} + \frac{ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c^3} + \frac{(ib^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c^3} - \frac{(ib^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2,ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c^3}$$

$$
\begin{aligned}
&= \frac{b^2x(a+b\sec^{-1}(cx))}{c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\sec^{-1}(cx))^2}{2c} \\
&\quad + \frac{1}{3}x^3(a+b\sec^{-1}(cx))^3 + \frac{ib(a+b\sec^{-1}(cx))^2\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c^3} \\
&\quad - \frac{b^3\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3} - \frac{ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c^3} \\
&\quad + \frac{ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c^3} \\
&\quad + \frac{b^3\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{i\sec^{-1}(cx)}\right)}{c^3} \\
&\quad - \frac{b^3\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{i\sec^{-1}(cx)}\right)}{c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^2x(a+b\sec^{-1}(cx))}{c^2} - \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\sec^{-1}(cx))^2}{2c} \\
&\quad + \frac{1}{3}x^3(a+b\sec^{-1}(cx))^3 + \frac{ib(a+b\sec^{-1}(cx))^2\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c^3} \\
&\quad - \frac{b^3\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3} - \frac{ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c^3} \\
&\quad + \frac{ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c^3} \\
&\quad + \frac{b^3\,\text{PolyLog}\left(3,-ie^{i\sec^{-1}(cx)}\right)}{c^3} - \frac{b^3\,\text{PolyLog}\left(3,ie^{i\sec^{-1}(cx)}\right)}{c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.71

$$
\int x^2(a+b\sec^{-1}(cx))^3\,dx
$$

$$
= \frac{6ab^2cx - 3a^2bc^2\sqrt{1-\frac{1}{c^2x^2}}x^2 + 2a^3c^3x^3 + 6b^3cx\sec^{-1}(cx) - 6ab^2c^2\sqrt{1-\frac{1}{c^2x^2}}x^2\sec^{-1}(cx) + 6a^2bc^3x^3\sec^{-}}{}
$$

```
[In] Integrate[x^2*(a + b*ArcSec[c*x])^3,x]
```

```
[Out] (6*a*b^2*c*x - 3*a^2*b*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2 + 2*a^3*c^3*x^3 + 6*b^
3*c*x*ArcSec[c*x] - 6*a*b^2*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*ArcSec[c*x] + 6*a
^2*b*c^3*x^3*ArcSec[c*x] - 3*b^3*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*ArcSec[c*x]^
2 + 6*a*b^2*c^3*x^3*ArcSec[c*x]^2 + 2*b^3*c^3*x^3*ArcSec[c*x]^3 + (6*I)*b^3
```

```
*ArcSec[c*x]^2*ArcTan[E^(I*ArcSec[c*x])] - 6*b^3*ArcTanh[Sqrt[1 - 1/(c^2*x^
2)]] - 6*a*b^2*ArcSec[c*x]*Log[1 - I*E^(I*ArcSec[c*x])] + 6*a*b^2*ArcSec[c*
x]*Log[1 + I*E^(I*ArcSec[c*x])] - 3*a^2*b*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x
] - (6*I)*b^2*(a + b*ArcSec[c*x])*PolyLog[2, (-I)*E^(I*ArcSec[c*x])] + (6*I
)*b^2*(a + b*ArcSec[c*x])*PolyLog[2, I*E^(I*ArcSec[c*x])] + 6*b^3*PolyLog[3
, (-I)*E^(I*ArcSec[c*x])] - 6*b^3*PolyLog[3, I*E^(I*ArcSec[c*x])])/(6*c^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(286) = 572$.

Time = 1.44 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.43

method	result
derivativedivides	$\frac{\frac{a^3c^3x^3}{3}+b^3\left(\frac{\operatorname{arcsec}(cx)\left(2c^2x^2\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}+\frac{\operatorname{arcsec}(cx)^2\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{2}-i\operatorname{arcsec}(cx\right.}{}$
default	$\frac{\frac{a^3c^3x^3}{3}+b^3\left(\frac{\operatorname{arcsec}(cx)\left(2c^2x^2\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}+\frac{\operatorname{arcsec}(cx)^2\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{2}-i\operatorname{arcsec}(cx\right.}{}$
parts	$\frac{a^3x^3}{3}+\frac{b^3\left(\frac{\operatorname{arcsec}(cx)\left(2c^2x^2\operatorname{arcsec}(cx)^2-3\operatorname{arcsec}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}+\frac{\operatorname{arcsec}(cx)^2\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{2}-i\operatorname{arcsec}(c\right.}{}$

```
[In] int(x^2*(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*a^3*c^3*x^3+b^3*(1/6*arcsec(c*x)*(2*c^2*x^2*arcsec(c*x)^2-3*arcs
ec(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)+6)*c*x+1/2*arcsec(c*x)^2*ln(1+I*(1/
c/x+I*(1-1/c^2/x^2)^(1/2)))-I*arcsec(c*x)*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^
2)^(1/2)))+polylog(3,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/2*arcsec(c*x)^2*ln
(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I*arcsec(c*x)*polylog(2,I*(1/c/x+I*(1-1
/c^2/x^2)^(1/2)))-polylog(3,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+2*I*arctan(1/c
/x+I*(1-1/c^2/x^2)^(1/2)))+3*a*b^2*(1/3*(c^2*x^2*arcsec(c*x)^2-arcsec(c*x)*
c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1)*c*x+1/3*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1
/c^2/x^2)^(1/2)))-1/3*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/3
*I*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+1/3*I*dilog(1-I*(1/c/x+I*(1-1/c
^2/x^2)^(1/2))))+3*a^2*b*(1/3*c^3*x^3*arcsec(c*x)-1/6*(c^2*x^2-1)^(1/2)*(c*
x*(c^2*x^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/
c/x))
```

Fricas [F]

$$\int x^2 (a + b \sec^{-1}(cx))^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
[Out] integral(b^3*x^2*arcsec(c*x)^3 + 3*a*b^2*x^2*arcsec(c*x)^2 + 3*a^2*b*x^2*ar
csec(c*x) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2 (a + b \sec^{-1}(cx))^3 \, dx = \int x^2 (a + b \operatorname{asec}(cx))^3 \, dx$$

```
[In] integrate(x**2*(a+b*asec(c*x))**3,x)
[Out] Integral(x**2*(a + b*asec(c*x))**3, x)
```

Maxima [F]

$$\int x^2 (a + b \sec^{-1}(cx))^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
[Out] 1/3*b^3*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 1/4*b^3*x^3*arctan(sqrt
(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 1/2*a*b^2*c^2*(2*(c^2*x^3 + 3*x)/
c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 - 12*b^3*c^2*integr
ate(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x)*log(c)^2
+ 12*b^3*c^2*integrate(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*
x^2)/(c^2*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^4*arctan(sqrt(c*
x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2*integr
ate(1/4*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*integrate(
1/4*x^4*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/3*a^3*x^3 + 12*b^3*c^2*integrat
e(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^2
- 1), x) - 12*b^3*c^2*integrate(1/4*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
*log(x)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^4*arctan(sqrt(c*
x + 1)*sqrt(c*x - 1))^2/(c^2*x^2 - 1), x) + 4*b^3*c^2*integrate(1/4*x^4*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c
^2*integrate(1/4*x^4*log(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integr
ate(1/4*x^4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^2*c^2*integrate(
```

```
1/4*x^4*log(x)^2/(c^2*x^2 - 1), x) + 3/2*a*b^2*(2*x/c^2 - log(c*x + 1)/c^3
+ log(c*x - 1)/c^3)*log(c)^2 + 12*b^3*integrate(1/4*x^2*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1))/(c^2*x^2 - 1), x)*log(c)^2 - 12*b^3*integrate(1/4*x^2*arct
an(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*
b^3*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^2 -
 1), x)*log(c) - 12*a*b^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*
log(c) + 24*a*b^2*integrate(1/4*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/4*(
4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2)
 + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^
2)/c)*a^2*b - 4*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^2 - 1), x) + b^3*integrate(1/4*sqrt(c*x
 + 1)*sqrt(c*x - 1)*x^2*log(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*b^3*integrate
(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^2 -
 1), x) + 12*b^3*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(
x)^2/(c^2*x^2 - 1), x) - 12*a*b^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1))^2/(c^2*x^2 - 1), x) - 4*b^3*integrate(1/4*x^2*arctan(sqrt(c*x
+ 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^2 - 1), x) + 3*a*b^2*integrate(1/4*
x^2*log(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*a*b^2*integrate(1/4*x^2*log(c^2*x
^2)*log(x)/(c^2*x^2 - 1), x) + 12*a*b^2*integrate(1/4*x^2*log(x)^2/(c^2*x^2
 - 1), x)
```

Giac [F]

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int(b\,\mathrm{arcsec}\,(cx)+a)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)^3*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int x^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

```
[In] int(x^2*(a + b*acos(1/(c*x)))^3,x)
[Out] int(x^2*(a + b*acos(1/(c*x)))^3, x)
```

3.26 $\int x(a+b\sec^{-1}(cx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 126

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\frac{3ib(a+b\sec^{-1}(cx))^2}{2c^2}-\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\sec^{-1}(cx))^2}{2c}+\frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^3-\frac{3b^2(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^2}+\frac{3ib^3\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2c^2}$$

```
[Out] 3/2*I*b*(a+b*arcsec(c*x))^2/c^2+1/2*x^2*(a+b*arcsec(c*x))^3-3*b^2*(a+b*arcs
ec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*I*b^3*polylog(2,-(1/
c/x+I*(1-1/c^2/x^2)^(1/2))^2)/c^2-3/2*b*x*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)
^(1/2)/c
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used

$= \{5330, 4494, 4269, 3800, 2221, 2317, 2438\}$

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx = -\frac{3b^2\log\left(1+e^{2i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)}{c^2} - \frac{3bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{2c} + \frac{3ib\left(a+b\sec^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^3 + \frac{3ib^3\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2c^2}$$

```
[In] Int[x*(a + b*ArcSec[c*x])^3,x]
```

```
[Out] (((3*I)/2)*b*(a + b*ArcSec[c*x])^2)/c^2 - (3*b*Sqrt[1 - 1/(c^2*x^2)]*x*(a +
 b*ArcSec[c*x])^2)/(2*c) + (x^2*(a + b*ArcSec[c*x])^3)/2 - (3*b^2*(a + b*Ar
cSec[c*x])*Log[1 + E^((2*I)*ArcSec[c*x])])/c^2 + (((3*I)/2)*b^3*PolyLog[2,
-E^((2*I)*ArcSec[c*x])])/c^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)^3\sec^2(x)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{c^2}\\
&= \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^3 - \frac{(3b)\text{Subst}\left(\int (a+bx)^2\sec^2(x)\,dx, x, \sec^{-1}(cx)\right)}{2c^2}\\
&= -\frac{3b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)^2}{2c} + \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^3\\
&\quad + \frac{(3b^2)\,\text{Subst}\left(\int (a+bx)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{c^2}\\
&= \frac{3ib\left(a+b\sec^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)^2}{2c}\\
&\quad + \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^3 - \frac{(6ib^2)\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}\,dx, x, \sec^{-1}(cx)\right)}{c^2}\\
&= \frac{3ib\left(a+b\sec^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)^2}{2c}\\
&\quad + \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^3 - \frac{3b^2\left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^2}\\
&\quad + \frac{(3b^3)\,\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c^2}\\
&= \frac{3ib\left(a+b\sec^{-1}(cx)\right)^2}{2c^2} - \frac{3b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\sec^{-1}(cx)\right)^2}{2c} + \frac{1}{2}x^2\left(a+b\sec^{-1}(cx)\right)^3\\
&\quad - \frac{3b^2\left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^2} - \frac{(3ib^3)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\sec^{-1}(cx)}\right)}{2c^2}
\end{aligned}$$

$$= \frac{3ib(a+b\sec^{-1}(cx))^2}{2c^2} - \frac{3b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\sec^{-1}(cx))^2}{2c} + \frac{1}{2}x^2(a+b\sec^{-1}(cx))^3 - \frac{3b^2(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{c^2} + \frac{3ib^3\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.46

$$\int x(a+b\sec^{-1}(cx))^3\,dx$$

$$= \frac{-3b^2\left(-ac^2x^2+b\left(-i+c\sqrt{1-\frac{1}{c^2x^2}}x\right)\right)\sec^{-1}(cx)^2+b^3c^2x^2\sec^{-1}(cx)^3-3b\sec^{-1}(cx)\left(acx\left(2b\sqrt{1-\frac{1}{c^2x^2}}\right.\right.}{}$$

```
[In] Integrate[x*(a + b*ArcSec[c*x])^3,x]
```

```
[Out] (-3*b^2*(-(a*c^2*x^2) + b*(-I + c*Sqrt[1 - 1/(c^2*x^2)]*x))*ArcSec[c*x]^2 +
 b^3*c^2*x^2*ArcSec[c*x]^3 - 3*b*ArcSec[c*x]*(a*c*x*(2*b*Sqrt[1 - 1/(c^2*x^
2)] - a*c*x) + 2*b^2*Log[1 + E^((2*I)*ArcSec[c*x])]) + a*(a*c*x*(-3*b*Sqrt[
1 - 1/(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + (3*I)*b^3*PolyLog[2, -E^(
(2*I)*ArcSec[c*x])])/(2*c^2)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{\frac{a^3c^2x^2}{2}+b^3\left(\frac{\mathrm{arcsec}(cx)^2\left(c^2x^2\,\mathrm{arcsec}(cx)-3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}+3i\,\mathrm{arcsec}(cx)^2-3\,\mathrm{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{3i}{}\right.}{}$
default	$\frac{\frac{a^3c^2x^2}{2}+b^3\left(\frac{\mathrm{arcsec}(cx)^2\left(c^2x^2\,\mathrm{arcsec}(cx)-3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}+3i\,\mathrm{arcsec}(cx)^2-3\,\mathrm{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{3i}{}\right.}{}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{\mathrm{arcsec}(cx)^2\left(c^2x^2\,\mathrm{arcsec}(cx)-3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}+3i\,\mathrm{arcsec}(cx)^2-3\,\mathrm{arcsec}(cx)\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)+\frac{3}{}\right.}{c^2}$

```
[In] int(x*(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*a^3*c^2*x^2+b^3*(1/2*arcsec(c*x)^2*(c^2*x^2*arcsec(c*x)-3*x*c*((
c^2*x^2-1)/c^2/x^2)^(1/2)-3*I)+3*I*arcsec(c*x)^2-3*arcsec(c*x)*ln(1+(1/c/x+
I*(1-1/c^2/x^2)^(1/2))^2)+3/2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

```
)+3*a*b^2*(1/2*c^2*x^2*arcsec(c*x)^2-arcsec(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^
(1/2)-ln(1/c/x))+3*a^2*b*(1/2*c^2*x^2*arcsec(c*x)-1/2/((c^2*x^2-1)/c^2/x^2)
^(1/2)/c/x*(c^2*x^2-1)))
```

Fricas [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int\left(b\,\mathrm{arcsec}\,(cx)+a\right)^3x\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
[Out] integral(b^3*x*arcsec(c*x)^3 + 3*a*b^2*x*arcsec(c*x)^2 + 3*a^2*b*x*arcsec(c
*x) + a^3*x, x)
```

Sympy [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int x(a+b\,\mathrm{asec}\,(cx))^3\,dx$$

```
[In] integrate(x*(a+b*asec(c*x))**3,x)
[Out] Integral(x*(a + b*asec(c*x))**3, x)
```

Maxima [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int\left(b\,\mathrm{arcsec}\,(cx)+a\right)^3x\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
[Out] 3/2*a*b^2*x^2*arcsec(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arcsec(c*x) - x*sqrt(-
1/(c^2*x^2) + 1)/c)*a^2*b - 3*(x*sqrt(-1/(c^2*x^2) + 1)*arcsec(c*x)/c - log
(x)/c^2)*a*b^2 + 1/8*(4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x^2*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 8*integrate(3/8*((4*x*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x*log(c^2*x^2)^2)*sqrt(c*x + 1)*sqrt
(c*x - 1) + 4*(2*c^2*x^3*log(c)^2 - 2*x*log(c)^2 + 2*(c^2*x^3 - x)*log(x)^2
 - ((2*c^2*log(c) + c^2)*x^3 - x*(2*log(c) + 1) + 2*(c^2*x^3 - x)*log(x))*l
og(c^2*x^2) + 4*(c^2*x^3*log(c) - x*log(c))*log(x))*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1)))/(c^2*x^2 - 1), x))*b^3
```

Giac [F]

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arcsec}\,(cx)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sec^{-1}(cx)\right)^3\,dx=\int x\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

```
[In] int(x*(a + b*acos(1/(c*x)))^3,x)
[Out] int(x*(a + b*acos(1/(c*x)))^3, x)
```

3.27 $\int (a+b\sec^{-1}(cx))^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 158

$$\int (a+b\sec^{-1}(cx))^3\,dx = x(a+b\sec^{-1}(cx))^3 + \frac{6ib(a+b\sec^{-1}(cx))^2\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c} - \frac{6ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{6ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{6b^3\,\text{PolyLog}\left(3,-ie^{i\sec^{-1}(cx)}\right)}{c} - \frac{6b^3\,\text{PolyLog}\left(3,ie^{i\sec^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arcsec(c*x))^3+6*I*b*(a+b*arcsec(c*x))^2*arctan(1/c/x+I*(1-1/c^2/x^2
)^(1/2))/c-6*I*b^2*(a+b*arcsec(c*x))*polylog(2,-I*(1/c/x+I*(1-1/c^2/x^2)^(1
/2)))/c+6*I*b^2*(a+b*arcsec(c*x))*polylog(2,I*(1/c/x+I*(1-1/c^2/x^2)^(1/2))
)/c+6*b^3*polylog(3,-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c-6*b^3*polylog(3,I*(
1/c/x+I*(1-1/c^2/x^2)^(1/2)))/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used

$= \{5324, 4494, 4266, 2611, 2320, 6724\}$

$$\int \left(a+b\sec^{-1}(cx)\right)^3\,dx = \frac{6ib\arctan\left(e^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)^2}{c} - \frac{6ib^2\,\mathrm{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)}{c} + \frac{6ib^2\,\mathrm{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)\left(a+b\sec^{-1}(cx)\right)}{c} + x\left(a+b\sec^{-1}(cx)\right)^3 + \frac{6b^3\,\mathrm{PolyLog}\left(3,-ie^{i\sec^{-1}(cx)}\right)}{c} - \frac{6b^3\,\mathrm{PolyLog}\left(3,ie^{i\sec^{-1}(cx)}\right)}{c}$$

```
[In] Int[(a + b*ArcSec[c*x])^3,x]
```

```
[Out] x*(a + b*ArcSec[c*x])^3 + ((6*I)*b*(a + b*ArcSec[c*x])^2*ArcTan[E^(I*ArcSec
[c*x])])/c - ((6*I)*b^2*(a + b*ArcSec[c*x])*PolyLog[2, (-I)*E^(I*ArcSec[c*x
])])/c + ((6*I)*b^2*(a + b*ArcSec[c*x])*PolyLog[2, I*E^(I*ArcSec[c*x])])/c
+ (6*b^3*PolyLog[3, (-I)*E^(I*ArcSec[c*x])])/c - (6*b^3*PolyLog[3, I*E^(I*A
rcSec[c*x])])/c
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4494

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5324

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/c, Subst[I
nt[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n}
, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+bx)^3 \sec(x)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{c}\\
&= x\left(a+b\sec^{-1}(cx)\right)^3 - \frac{(3b)\text{Subst}\left(\int (a+bx)^2 \sec(x)\,dx, x, \sec^{-1}(cx)\right)}{c}\\
&= x\left(a+b\sec^{-1}(cx)\right)^3 + \frac{6ib(a+b\sec^{-1}(cx))^2 \arctan\left(e^{i\sec^{-1}(cx)}\right)}{c}\\
&\quad + \frac{(6b^2)\,\text{Subst}\left(\int (a+bx)\log\left(1-ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c}\\
&\quad - \frac{(6b^2)\,\text{Subst}\left(\int (a+bx)\log\left(1+ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c}\\
&= x\left(a+b\sec^{-1}(cx)\right)^3 + \frac{6ib(a+b\sec^{-1}(cx))^2 \arctan\left(e^{i\sec^{-1}(cx)}\right)}{c}\\
&\quad - \frac{6ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c}\\
&\quad + \frac{6ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c}\\
&\quad + \frac{(6ib^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c}\\
&\quad - \frac{(6ib^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2,ie^{ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{c}
\end{aligned}$$

$$= x(a+b\sec^{-1}(cx))^3 + \frac{6ib(a+b\sec^{-1}(cx))^2\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c} - \frac{6ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{6ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{(6b^3)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{c} - \frac{(6b^3)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{c}$$

$$= x(a+b\sec^{-1}(cx))^3 + \frac{6ib(a+b\sec^{-1}(cx))^2\arctan\left(e^{i\sec^{-1}(cx)}\right)}{c} - \frac{6ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,-ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{6ib^2(a+b\sec^{-1}(cx))\,\text{PolyLog}\left(2,ie^{i\sec^{-1}(cx)}\right)}{c} + \frac{6b^3\,\text{PolyLog}\left(3,-ie^{i\sec^{-1}(cx)}\right)}{c} - \frac{6b^3\,\text{PolyLog}\left(3,ie^{i\sec^{-1}(cx)}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.83

$$\int (a+b\sec^{-1}(cx))^3\,dx$$

$$= \frac{a^3cx + 3a^2bcx\sec^{-1}(cx) + 3ab^2cx\sec^{-1}(cx)^2 + b^3cx\sec^{-1}(cx)^3 - 6ab^2\sec^{-1}(cx)\log\left(1-ie^{i\sec^{-1}(cx)}\right) - 3b^3}{}$$

```
[In] Integrate[(a + b*ArcSec[c*x])^3,x]
```

```
[Out] (a^3*c*x + 3*a^2*b*c*x*ArcSec[c*x] + 3*a*b^2*c*x*ArcSec[c*x]^2 + b^3*c*x*Ar
cSec[c*x]^3 - 6*a*b^2*ArcSec[c*x]*Log[1 - I*E^(I*ArcSec[c*x])] - 3*b^3*ArcS
ec[c*x]^2*Log[1 - I*E^(I*ArcSec[c*x])] + 6*a*b^2*ArcSec[c*x]*Log[1 + I*E^(I
*ArcSec[c*x])] + 3*b^3*ArcSec[c*x]^2*Log[1 + I*E^(I*ArcSec[c*x])] - 3*a^2*b
*Log[c*(1 + Sqrt[1 - 1/(c^2*x^2)])*x] - (6*I)*b^2*(a + b*ArcSec[c*x])*PolyL
og[2, (-I)*E^(I*ArcSec[c*x])] + (6*I)*b^2*(a + b*ArcSec[c*x])*PolyLog[2, I*
E^(I*ArcSec[c*x])] + 6*b^3*PolyLog[3, (-I)*E^(I*ArcSec[c*x])] - 6*b^3*PolyL
og[3, I*E^(I*ArcSec[c*x])])/c
```

Maple [F]

$$\int (a + b \operatorname{arcsec}(cx))^3 \, dx$$

[In] int((a+b*arcsec(c*x))^3,x)

[Out] int((a+b*arcsec(c*x))^3,x)

Fricas [F]

$$\int \left(a + b\sec^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 \, dx$$

[In] integrate((a+b*arcsec(c*x))^3,x, algorithm="fricas")

[Out] integral(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x) + a^3, x)

Sympy [F]

$$\int \left(a + b\sec^{-1}(cx)\right)^3 \, dx = \int (a + b \operatorname{asec}(cx))^3 \, dx$$

[In] integrate((a+b*asec(c*x))**3,x)

[Out] Integral((a + b*asec(c*x))**3, x)

Maxima [F]

$$\int \left(a + b\sec^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsec}(cx) + a)^3 \, dx$$

[In] integrate((a+b*arcsec(c*x))^3,x, algorithm="maxima")

[Out] -3/2*a*b^2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x)*log(c)^2 + b^3*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3/4*b^3*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x^2*log(x)/(c^2*x^2 -

```
1), x)*log(c) + 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b^3*c^2*integrate(1/4*x^2
*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^2 - 1), x) + 12*a*b^2*
c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^2 - 1),
x) + 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c
^2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)^2/(c
^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x
^2 - 1), x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)^2/(c^2*x^2 - 1), x) - 3
/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(c)^2 + 12*b^3*integrate(1/4*
arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^2 - 1), x)*log(c)^2 - 12*b^3*int
egrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^2 - 1),
x)*log(c) + 24*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)
/(c^2*x^2 - 1), x)*log(c) - 12*a*b^2*integrate(1/4*log(c^2*x^2)/(c^2*x^2 -
1), x)*log(c) + 24*a*b^2*integrate(1/4*log(x)/(c^2*x^2 - 1), x)*log(c) + a^
3*x - 12*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1))^2/(c^2*x^2 - 1), x) + 3*b^3*integrate(1/4*sqrt(c*x + 1)*sqr
t(c*x - 1)*log(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*b^3*integrate(1/4*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*b^3*
integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^2 - 1), x
) - 12*a*b^2*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^2 -
 1), x) - 12*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*
x^2)/(c^2*x^2 - 1), x) + 3*a*b^2*integrate(1/4*log(c^2*x^2)^2/(c^2*x^2 - 1)
, x) - 12*a*b^2*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*a*
b^2*integrate(1/4*log(x)^2/(c^2*x^2 - 1), x) + 3/2*(2*c*x*arcsec(c*x) - log
(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*a^2*b/c
```

Giac [F]

$$\int \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int (b\,\mathrm{arcsec}\,(cx)+a)^3 \, dx$$

```
[In] integrate((a+b*arcsec(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int((a + b*acos(1/(c*x)))^3,x)
[Out] int((a + b*acos(1/(c*x)))^3, x)
```

3.28 $\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 128

$$\begin{aligned}\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = {} & \frac{i(a+b\sec^{-1}(cx))^4}{4b} - (a+b\sec^{-1}(cx))^3\log\left(1+e^{2i\sec^{-1}(cx)}\right) \\ & + \frac{3}{2}ib(a+b\sec^{-1}(cx))^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) \\ & - \frac{3}{2}b^2(a+b\sec^{-1}(cx))\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right) \\ & - \frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-e^{2i\sec^{-1}(cx)}\right)\end{aligned}$$

```
[Out] 1/4*I*(a+b*arcsec(c*x))^4/b-(a+b*arcsec(c*x))^3*ln(1+(1/c/x+I*(1-1/c^2/x^2)
^(1/2))^2)+3/2*I*b*(a+b*arcsec(c*x))^2*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1
/2))^2)-3/2*b^2*(a+b*arcsec(c*x))*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^
2)-3/4*I*b^3*polylog(4,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5330, 3800, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\sec^{-1}(cx))^3}{x}\,dx = {} & -\frac{3}{2}b^2\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx)) \\ & + \frac{3}{2}ib\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))^2 \\ & + \frac{i(a+b\sec^{-1}(cx))^4}{4b} - \log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))^3 \\ & - \frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-e^{2i\sec^{-1}(cx)}\right)\end{aligned}$$

```
[In] Int[(a + b*ArcSec[c*x])^3/x,x]
```

```
[Out] ((I/4)*(a + b*ArcSec[c*x])^4)/b - (a + b*ArcSec[c*x])^3*Log[1 + E^((2*I)*Ar
cSec[c*x])] + ((3*I)/2)*b*(a + b*ArcSec[c*x])^2*PolyLog[2, -E^((2*I)*ArcSec
[c*x])] - (3*b^2*(a + b*ArcSec[c*x])*PolyLog[3, -E^((2*I)*ArcSec[c*x])])/2
- ((3*I)/4)*b^3*PolyLog[4, -E^((2*I)*ArcSec[c*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int (a+bx)^3\tan(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^4}{4b} - 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^3}{1+e^{2ix}}\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^4}{4b} - \left(a+b\sec^{-1}(cx)\right)^3\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\
&\quad + (3b)\text{Subst}\left(\int (a+bx)^2\log\left(1+e^{2ix}\right)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^4}{4b} - \left(a+b\sec^{-1}(cx)\right)^3\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\
&\quad + \frac{3}{2}ib\left(a+b\sec^{-1}(cx)\right)^2\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\\
&\quad - \left(3ib^2\right)\text{Subst}\left(\int (a+bx)\,\text{PolyLog}\left(2,-e^{2ix}\right)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{i\left(a+b\sec^{-1}(cx)\right)^4}{4b} - \left(a+b\sec^{-1}(cx)\right)^3\log\left(1+e^{2i\sec^{-1}(cx)}\right)\\
&\quad + \frac{3}{2}ib\left(a+b\sec^{-1}(cx)\right)^2\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\\
&\quad - \frac{3}{2}b^2\left(a+b\sec^{-1}(cx)\right)\text{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right)\\
&\quad + \frac{1}{2}\left(3b^3\right)\text{Subst}\left(\int \text{PolyLog}\left(3,-e^{2ix}\right)\,dx, x, \sec^{-1}(cx)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{i(a+b\sec^{-1}(cx))^4}{4b} - \left(a+b\sec^{-1}(cx)\right)^3\log\left(1+e^{2i\sec^{-1}(cx)}\right) \\
&\quad + \frac{3}{2}ib\left(a+b\sec^{-1}(cx)\right)^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b^2\left(a+b\sec^{-1}(cx)\right)\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right) \\
&\quad - \frac{1}{4}\left(3ib^3\right)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,-x)}{x}\,dx,x,e^{2i\sec^{-1}(cx)}\right) \\
&= \frac{i(a+b\sec^{-1}(cx))^4}{4b} - \left(a+b\sec^{-1}(cx)\right)^3\log\left(1+e^{2i\sec^{-1}(cx)}\right) \\
&\quad + \frac{3}{2}ib\left(a+b\sec^{-1}(cx)\right)^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b^2\left(a+b\sec^{-1}(cx)\right)\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right) - \frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-e^{2i\sec^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.59

$$
\begin{aligned}
\int\frac{\left(a+b\sec^{-1}(cx)\right)^3}{x}\,dx &= \frac{1}{4}\Big(6ia^2b\sec^{-1}(cx)^2 + 4iab^2\sec^{-1}(cx)^3 + ib^3\sec^{-1}(cx)^4 \\
&\quad - 12a^2b\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) \\
&\quad - 12ab^2\sec^{-1}(cx)^2\log\left(1+e^{2i\sec^{-1}(cx)}\right) \\
&\quad - 4b^3\sec^{-1}(cx)^3\log\left(1+e^{2i\sec^{-1}(cx)}\right) + 4a^3\log(cx) \\
&\quad + 6ib\left(a+b\sec^{-1}(cx)\right)^2\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right) \\
&\quad - 6b^2\left(a+b\sec^{-1}(cx)\right)\operatorname{PolyLog}\left(3,-e^{2i\sec^{-1}(cx)}\right) \\
&\quad - 3ib^3\operatorname{PolyLog}\left(4,-e^{2i\sec^{-1}(cx)}\right)\Big)
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSec[c*x])^3/x,x]
```

```
[Out] ((6*I)*a^2*b*ArcSec[c*x]^2 + (4*I)*a*b^2*ArcSec[c*x]^3 + I*b^3*ArcSec[c*x]^
4 - 12*a^2*b*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 12*a*b^2*ArcSec[c
*x]^2*Log[1 + E^((2*I)*ArcSec[c*x])] - 4*b^3*ArcSec[c*x]^3*Log[1 + E^((2*I)
*ArcSec[c*x])] + 4*a^3*Log[c*x] + (6*I)*b*(a + b*ArcSec[c*x])^2*PolyLog[2,
-E^((2*I)*ArcSec[c*x])] - 6*b^2*(a + b*ArcSec[c*x])*PolyLog[3, -E^((2*I)*Ar
cSec[c*x])] - (3*I)*b^3*PolyLog[4, -E^((2*I)*ArcSec[c*x])])/4
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(177) = 354.

Time = 0.86 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.84

method	result
parts	$a^3 \ln(x) + b^3 \left(\frac{i \operatorname{arcsec}(cx)^4}{4} - \operatorname{arcsec}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + \frac{3i \operatorname{arcsec}(cx)^2 \operatorname{poly}}{}\right.$
derivativedivides	$a^3 \ln(cx) + b^3 \left(\frac{i \operatorname{arcsec}(cx)^4}{4} - \operatorname{arcsec}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + \frac{3i \operatorname{arcsec}(cx)^2 \operatorname{pol}}{}\right.$
default	$a^3 \ln(cx) + b^3 \left(\frac{i \operatorname{arcsec}(cx)^4}{4} - \operatorname{arcsec}(cx)^3 \ln\left(1 + \left(\frac{1}{cx} + i\sqrt{1 - \frac{1}{c^2x^2}}\right)^2\right) + \frac{3i \operatorname{arcsec}(cx)^2 \operatorname{pol}}{}\right.$

```
[In] int((a+b*arcsec(c*x))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*ln(x)+b^3*(1/4*I*arcsec(c*x)^4-arcsec(c*x)^3*ln(1+(1/c/x+I*(1-1/c^2/x^2
)^(1/2))^2)+3/2*I*arcsec(c*x)^2*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
-3/2*arcsec(c*x)*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)-3/4*I*polylog(
4,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))+3*a*b^2*(1/3*I*arcsec(c*x)^3-arcsec(c*
x)^2*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+I*arcsec(c*x)*polylog(2,-(1/c/x+
I*(1-1/c^2/x^2)^(1/2))^2)-1/2*polylog(3,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))+
3*a^2*b*(1/2*I*arcsec(c*x)^2-arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))
^2)+1/2*I*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x}\, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)^3}{x}\, dx$$

```
[In] integrate((a+b*arcsec(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x) +
 a^3)/x, x)
```

Sympy [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{(a + b\,\mathrm{asec}\,(cx))^3}{x}\,dx$$

[In] integrate((a+b*asec(c*x))**3/x,x)

[Out] Integral((a + b*asec(c*x))**3/x, x)

Maxima [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)^3}{x}\,dx$$

[In] integrate((a+b*arcsec(c*x))^3/x,x, algorithm="maxima")

[Out] -3/2*a*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 - 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x)*log(c)^2 + 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^3 - x), x)*log(c) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x^2*log(x)/(c^2*x^3 - x), x)*log(c) + b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3*log(x) - 3/4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2*log(x) + 24*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) - 12*b^3*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^3 - x), x) + 12*a*b^2*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^3 - x), x) - 3*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)^2/(c^2*x^3 - x), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)^2/(c^2*x^3 - x), x) + 12*a^2*b*c^2*integrate(1/4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x) + 3/2*a*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + 12*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^3 - x), x)*log(c)^2 - 12*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 24*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^3 - x), x)*log(c) - 12*a*b^2*integrate(1/4*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 24*a*b^2*integrate(1/4*log(x)/(c^2*x^3 - x), x)*log(c) - 12*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2*log(x)/(c^2*x^3 - x), x) + 3*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2*log(x)/(c^2*x^3 - x), x) - 24*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + 12*b^3*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^3 - x),

```
x) - 12*a*b^2*integrate(1/4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^3
 - x), x) + 3*a*b^2*integrate(1/4*log(c^2*x^2)^2/(c^2*x^3 - x), x) - 12*a*b
^2*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + 12*a*b^2*integrate
(1/4*log(x)^2/(c^2*x^3 - x), x) - 12*a^2*b*integrate(1/4*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))/(c^2*x^3 - x), x) + a^3*log(x)
```

Giac [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)^3}{x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))^3/x,x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x}\,dx = \int \frac{\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))^3/x,x)
[Out] int((a + b*acos(1/(c*x)))^3/x, x)
```

3.29 $\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx = -6b^3c\sqrt{1-\frac{1}{c^2x^2}}+\frac{6b^2(a+b\sec^{-1}(cx))}{x} + 3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-\frac{(a+b\sec^{-1}(cx))^3}{x}$$

```
[Out] 6*b^2*(a+b*arcsec(c*x))/x-(a+b*arcsec(c*x))^3/x-6*b^3*c*(1-1/c^2/x^2)^(1/2)
+3*b*c*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5330, 3377, 2717}

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx = \frac{6b^2(a+b\sec^{-1}(cx))}{x}+3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2 -\frac{(a+b\sec^{-1}(cx))^3}{x}-6b^3c\sqrt{1-\frac{1}{c^2x^2}}$$

```
[In] Int[(a + b*ArcSec[c*x])^3/x^2,x]
```

```
[Out] -6*b^3*c*Sqrt[1 - 1/(c^2*x^2)] + (6*b^2*(a + b*ArcSec[c*x]))/x + 3*b*c*Sqrt
[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x])^2 - (a + b*ArcSec[c*x])^3/x
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c\text{Subst}\left(\int (a+bx)^3\sin(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= -\frac{\left(a+b\sec^{-1}(cx)\right)^3}{x} + (3bc)\text{Subst}\left(\int (a+bx)^2\cos(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= 3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2 - \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x}\\
&\quad -\left(6b^2c\right)\text{Subst}\left(\int (a+bx)\sin(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{6b^2\left(a+b\sec^{-1}(cx)\right)}{x} + 3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2\\
&\quad -\frac{\left(a+b\sec^{-1}(cx)\right)^3}{x} - \left(6b^3c\right)\text{Subst}\left(\int \cos(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= -6b^3c\sqrt{1-\frac{1}{c^2x^2}} + \frac{6b^2\left(a+b\sec^{-1}(cx)\right)}{x}\\
&\quad + 3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2 - \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.76

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx$$
$$= \frac{-a^3+6ab^2+3a^2bc\sqrt{1-\frac{1}{c^2x^2}}x-6b^3c\sqrt{1-\frac{1}{c^2x^2}}x+3b\left(-a^2+2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x\right)\sec^{-1}(cx)+3b^2\Big(-}{x}$$

[In] Integrate[(a + b*ArcSec[c*x])^3/x^2,x]

[Out] (-a^3 + 6*a*b^2 + 3*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 - 1/(c ^2*x^2)]*x + 3*b*(-a^2 + 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcSec[c* x] + 3*b^2*(-a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcSec[c*x]^2 - b^3*ArcSec[c* x]^3)/x

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(76) = 152.

Time = 0.68 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.45

method	result
parts	$-\frac{a^3}{x}+b^3c\left(-\frac{\operatorname{arcsec}(cx)^3}{cx}+3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\operatorname{arcsec}(cx)^2-6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arcsec}(cx)}{cx}\right)+3a\,b^2c\Big(-$
derivativedivides	$c\Big(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{cx}+3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\operatorname{arcsec}(cx)^2-6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arcsec}(cx)}{cx}\right)+3a\,b^2\Big(-$
default	$c\Big(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{cx}+3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\operatorname{arcsec}(cx)^2-6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arcsec}(cx)}{cx}\right)+3a\,b^2\Big(-$

[In] int((a+b*arcsec(c*x))^3/x^2,x,method=_RETURNVERBOSE)

[Out] -a^3/x+b^3*c*(-1/c/x*arcsec(c*x)^3+3*((c^2*x^2-1)/c^2/x^2)^(1/2)*arcsec(c*x)^2-6*((c^2*x^2-1)/c^2/x^2)^(1/2)+6/c/x*arcsec(c*x))+3*a*b^2*c*(-1/c/x*arcs ec(c*x)^2+2/c/x+2*arcsec(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a^2*b*c*(-1/c/ x*arcsec(c*x)+1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^2-1))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx = -\frac{b^3\operatorname{arcsec}(cx)^3+3\,ab^2\operatorname{arcsec}(cx)^2+a^3-6\,ab^2+3\,(a^2b-2\,b^3)\operatorname{arcsec}(cx)-3\,\left(b^3\operatorname{arcsec}(cx)^2+2\,ab^2\operatorname{a}\right.}{x}$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + a^3 - 6*a*b^2 + 3*(a^2*b - 2*
b^3)*arcsec(c*x) - 3*(b^3*arcsec(c*x)^2 + 2*a*b^2*arcsec(c*x) + a^2*b - 2*b
^3)*sqrt(c^2*x^2 - 1))/x
```

Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^3}{x^2}\,dx$$

```
[In] integrate((a+b*asec(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*asec(c*x))**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.82

$$\begin{aligned}\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx &= -\frac{b^3\operatorname{arcsec}(cx)^3}{x}+3\left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)a^2b \\ &\quad +6\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arcsec}(cx)+\frac{1}{x}\right)ab^2 \\ &\quad +3\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arcsec}(cx)^2-2\,c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{2\operatorname{arcsec}(cx)}{x}\right)b^3 \\ &\quad -\frac{3\,ab^2\operatorname{arcsec}(cx)^2}{x}-\frac{a^3}{x}\end{aligned}$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*arcsec(c*x)^3/x + 3*(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*a^2*b +
 6*(c*sqrt(-1/(c^2*x^2) + 1)*arcsec(c*x) + 1/x)*a*b^2 + 3*(c*sqrt(-1/(c^2*x
^2) + 1)*arcsec(c*x)^2 - 2*c*sqrt(-1/(c^2*x^2) + 1) + 2*arcsec(c*x)/x)*b^3
- 3*a*b^2*arcsec(c*x)^2/x - a^3/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(76) = 152$.

Time = 0.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.45

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx$$
$$= \left(3\,b^3\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)^2 + 6\,ab^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right) - \frac{b^3\arccos\left(\frac{1}{cx}\right)^3}{cx} + 3\,a^2b\sqrt{-\frac{1}{c^2x^2}+1}\right.$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] (3*b^3*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2 + 6*a*b^2*sqrt(-1/(c^2*x^2)
 + 1)*arccos(1/(c*x)) - b^3*arccos(1/(c*x))^3/(c*x) + 3*a^2*b*sqrt(-1/(c^2*
x^2) + 1) - 6*b^3*sqrt(-1/(c^2*x^2) + 1) - 3*a*b^2*arccos(1/(c*x))^2/(c*x)
- 3*a^2*b*arccos(1/(c*x))/(c*x) + 6*b^3*arccos(1/(c*x))/(c*x) - a^3/(c*x) +
 6*a*b^2/(c*x))*c
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.95

$$\begin{aligned}\int \frac{(a+b\sec^{-1}(cx))^3}{x^2}\,dx &= \frac{b^3\left(6\,\mathrm{acos}\left(\frac{1}{c\,x}\right)-\mathrm{acos}\left(\frac{1}{c\,x}\right)^3\right)}{x}-\frac{a^3}{x}\\&\quad+3\,a^2\,b\,c\left(\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{\mathrm{acos}\left(\frac{1}{c\,x}\right)}{c\,x}\right)\\&\quad+b^3\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}\left(3\,\mathrm{acos}\left(\frac{1}{c\,x}\right)^2-6\right)\\&\quad+3\,a\,b^2\,c\left(2\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{\mathrm{acos}\left(\frac{1}{c\,x}\right)^2-2}{c\,x}\right)\end{aligned}$$

```
[In] int((a + b*acos(1/(c*x)))^3/x^2,x)
```

```
[Out] (b^3*(6*acos(1/(c*x)) - acos(1/(c*x))^3))/x - a^3/x + 3*a^2*b*c*((1 - 1/(c^
2*x^2))^(1/2) - acos(1/(c*x))/(c*x)) + b^3*c*(1 - 1/(c^2*x^2))^(1/2)*(3*aco
s(1/(c*x))^2 - 6) + 3*a*b^2*c*(2*acos(1/(c*x))*(1 - 1/(c^2*x^2))^(1/2) - (a
cos(1/(c*x))^2 - 2)/(c*x))
```

3.30 $\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 137

$$\begin{aligned}\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx = &-\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}+\frac{3}{8}b^3c^2\sec^{-1}(cx)\\&-\frac{3}{4}b^2\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)\\&+\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{4x}\\&-\frac{1}{4}c^2\left(a+b\sec^{-1}(cx)\right)^3+\frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3\end{aligned}$$

```
[Out] 3/8*b^3*c^2*arcsec(c*x)-3/4*b^2*(c^2-1/x^2)*(a+b*arcsec(c*x))-1/4*c^2*(a+b*
arcsec(c*x))^3+1/2*(c^2-1/x^2)*(a+b*arcsec(c*x))^3-3/8*b^3*c*(1-1/c^2/x^2)^
(1/2)/x+3/4*b*c*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{5330, 4489, 3392, 32, 2715, 8\}$

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^3}\,dx = -\frac{3}{4}b^2\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right) + \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{4x} + \frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3 - \frac{1}{4}c^2\left(a+b\sec^{-1}(cx)\right)^3 - \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x} + \frac{3}{8}b^3c^2\sec^{-1}(cx)$$

```
[In] Int[(a + b*ArcSec[c*x])^3/x^3,x]
```

```
[Out] (-3*b^3*c*Sqrt[1 - 1/(c^2*x^2)])/(8*x) + (3*b^3*c^2*ArcSec[c*x])/8 - (3*b^2
*(c^2 - x^(-2))*(a + b*ArcSec[c*x]))/4 + (3*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a +
b*ArcSec[c*x])^2)/(4*x) - (c^2*(a + b*ArcSec[c*x])^3)/4 + ((c^2 - x^(-2))*(
a + b*ArcSec[c*x])^3)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\text{integral} = c^2\text{Subst}\left(\int (a+bx)^3\cos(x)\sin(x)\,dx, x, \sec^{-1}(cx)\right)$$

$$= \frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3 - \frac{1}{2}(3bc^2)\,\text{Subst}\left(\int (a+bx)^2\sin^2(x)\,dx, x, \sec^{-1}(cx)\right)$$

$$= -\frac{3}{4}b^2\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right) + \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{4x} + \frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3 - \frac{1}{4}(3bc^2)\,\text{Subst}\left(\int (a+bx)^2\,dx, x, \sec^{-1}(cx)\right) + \frac{1}{4}(3b^3c^2)\,\text{Subst}\left(\int \sin^2(x)\,dx, x, \sec^{-1}(cx)\right)$$

$$= -\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x} - \frac{3}{4}b^2\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right) + \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{4x} - \frac{1}{4}c^2\left(a+b\sec^{-1}(cx)\right)^3 + \frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3 + \frac{1}{8}(3b^3c^2)\,\text{Subst}\left(\int 1\,dx, x, \sec^{-1}(cx)\right)$$

$$= -\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x} + \frac{3}{8}b^3c^2\sec^{-1}(cx) - \frac{3}{4}b^2\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right) + \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{4x} - \frac{1}{4}c^2\left(a+b\sec^{-1}(cx)\right)^3 + \frac{1}{2}\left(c^2-\frac{1}{x^2}\right)\left(a+b\sec^{-1}(cx)\right)^3$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^3}\,dx$$
$$= \frac{-4a^3 + 6ab^2 + 6a^2bc\sqrt{1 - \frac{1}{c^2x^2}}x - 3b^3c\sqrt{1 - \frac{1}{c^2x^2}}x + 6b\left(-2a^2 + b^2 + 2abc\sqrt{1 - \frac{1}{c^2x^2}}x\right)\sec^{-1}(cx) + 6b^2\Big(b}{8x^2}$$

```
[In] Integrate[(a + b*ArcSec[c*x])^3/x^3,x]
```

```
[Out] (-4*a^3 + 6*a*b^2 + 6*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 3*b^3*c*Sqrt[1 - 1/
(c^2*x^2)]*x + 6*b*(-2*a^2 + b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcSec[
c*x] + 6*b^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*(-2 + c^2*x^2))*ArcSec[c*x]^2
 + 2*b^3*(-2 + c^2*x^2)*ArcSec[c*x]^3 + 3*b*(-2*a^2 + b^2)*c^2*x^2*ArcSin[1
/(c*x)])/(8*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(121) = 242.

Time = 0.81 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.14

method	result
derivativedivides	$c^2\left(-\frac{a^3}{2c^2x^2} + b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{2c^2x^2} + \frac{3\operatorname{arcsec}(cx)^2\left(cx\operatorname{arcsec}(cx) + \sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx} + \frac{3\operatorname{arcsec}(cx)}{4c^2x^2} - \frac{3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{8cx} - \right.\right.$
default	$c^2\left(-\frac{a^3}{2c^2x^2} + b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{2c^2x^2} + \frac{3\operatorname{arcsec}(cx)^2\left(cx\operatorname{arcsec}(cx) + \sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx} + \frac{3\operatorname{arcsec}(cx)}{4c^2x^2} - \frac{3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{8cx} - \right.\right.$
parts	$-\frac{a^3}{2x^2} + b^3c^2\left(-\frac{\operatorname{arcsec}(cx)^3}{2c^2x^2} + \frac{3\operatorname{arcsec}(cx)^2\left(cx\operatorname{arcsec}(cx) + \sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx} + \frac{3\operatorname{arcsec}(cx)}{4c^2x^2} - \frac{3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{8cx} - \frac{3\operatorname{ar}}{}\right.$

```
[In] int((a+b*arcsec(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^3/c^2/x^2+b^3*(-1/2/c^2/x^2*arcsec(c*x)^3+3/4*arcsec(c*x)^2*(c*
x*arcsec(c*x)+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x+3/4/c^2/x^2*arcsec(c*x)-3/8*
((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x-3/8*arcsec(c*x)-1/2*arcsec(c*x)^3)+3*a*b^2*
(-1/2/c^2/x^2*arcsec(c*x)^2+1/2*arcsec(c*x)*(c*x*arcsec(c*x)+((c^2*x^2-1)/c
^2/x^2)^(1/2))/c/x-1/4*arcsec(c*x)^2-1/4+1/4/c^2/x^2)+3*a^2*b*(-1/2/c^2/x^2
*arcsec(c*x)-1/4*(c^2*x^2-1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*c^2*x^2-(c^
2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^3/c^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^3}\,dx = \frac{2\,(b^3c^2x^2 - 2\,b^3)\,\mathrm{arcsec}\,(cx)^3 - 4\,a^3 + 6\,ab^2 + 6\,(ab^2c^2x^2 - 2\,ab^2)\,\mathrm{arcsec}\,(cx)^2 + 3\,((2\,a^2b - b^3)c^2x^2 - 4\,a^2b}{8\,x^2}$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(b^3*c^2*x^2 - 2*b^3)*arcsec(c*x)^3 - 4*a^3 + 6*a*b^2 + 6*(a*b^2*c^2
*x^2 - 2*a*b^2)*arcsec(c*x)^2 + 3*((2*a^2*b - b^3)*c^2*x^2 - 4*a^2*b + 2*b^
3)*arcsec(c*x) + 3*(2*b^3*arcsec(c*x)^2 + 4*a*b^2*arcsec(c*x) + 2*a^2*b - b
^3)*sqrt(c^2*x^2 - 1))/x^2
```

Sympy [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^3}\,dx = \int \frac{(a + b\,\mathrm{asec}\,(cx))^3}{x^3}\,dx$$

```
[In] integrate((a+b*asec(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*asec(c*x))**3/x**3, x)
```

Maxima [F]

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^3}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/4*a^2*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
 c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) - 1/2*a^3/x
^2 - 1/8*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(a*b^2*c^2*(log(c*x + 1) + log(c
*x - 1) - 2*log(x))*log(c)^2 + 16*b^3*c^2*integrate(1/8*x^2*arctan(sqrt(c*x
 + 1)*sqrt(c*x - 1))/(c^2*x^5 - x^3), x)*log(c)^2 - 16*b^3*c^2*integrate(1/
8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^5 - x^3), x)*
log(c) + 32*b^3*c^2*integrate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*l
og(x)/(c^2*x^5 - x^3), x)*log(c) - 16*a*b^2*c^2*integrate(1/8*x^2*log(c^2*x
```

```
^2)/(c^2*x^5 - x^3), x)*log(c) + 32*a*b^2*c^2*integrate(1/8*x^2*log(x)/(c^2
*x^5 - x^3), x)*log(c) - 16*b^3*c^2*integrate(1/8*x^2*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) + 16*b^3*c^2*integra
te(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^5 - x^3), x)
 - 16*a*b^2*c^2*integrate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^
2*x^5 - x^3), x) + 8*b^3*c^2*integrate(1/8*x^2*arctan(sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c^2*x^2)/(c^2*x^5 - x^3), x) + 4*a*b^2*c^2*integrate(1/8*x^2*lo
g(c^2*x^2)^2/(c^2*x^5 - x^3), x) - 16*a*b^2*c^2*integrate(1/8*x^2*log(c^2*x
^2)*log(x)/(c^2*x^5 - x^3), x) + 16*a*b^2*c^2*integrate(1/8*x^2*log(x)^2/(c
^2*x^5 - x^3), x) - (c^2*log(c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1
/x^2)*a*b^2*log(c)^2 - 16*b^3*integrate(1/8*arctan(sqrt(c*x + 1)*sqrt(c*x -
 1))/(c^2*x^5 - x^3), x)*log(c)^2 + 16*b^3*integrate(1/8*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 32*b^3*integrat
e(1/8*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^5 - x^3), x)*log(c)
 + 16*a*b^2*integrate(1/8*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 32*a*b^
2*integrate(1/8*log(x)/(c^2*x^5 - x^3), x)*log(c) - 8*b^3*integrate(1/8*sqr
t(c*x + 1)*sqrt(c*x - 1)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^5 - x
^3), x) + 2*b^3*integrate(1/8*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2/(c
^2*x^5 - x^3), x) + 16*b^3*integrate(1/8*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)
)*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) - 16*b^3*integrate(1/8*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^5 - x^3), x) + 16*a*b^2*integrate
(1/8*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^5 - x^3), x) - 8*b^3*inte
grate(1/8*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^5 - x^3),
 x) - 4*a*b^2*integrate(1/8*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) + 16*a*b^2*i
ntegrate(1/8*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) - 16*a*b^2*integrate(1
/8*log(x)^2/(c^2*x^5 - x^3), x))*x^2)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(121) = 242.

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.03

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^3}\,dx$$
$$=\frac{1}{8}\left(2\,b^3c\arccos\left(\frac{1}{cx}\right)^3+6\,ab^2c\arccos\left(\frac{1}{cx}\right)^2+6\,a^2bc\arccos\left(\frac{1}{cx}\right)-3\,b^3c\arccos\left(\frac{1}{cx}\right)+\frac{6\,b^3\sqrt{-\frac{1}{c^2x^2}+\cdots}}{x}\right.$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^3*c*arccos(1/(c*x))^3 + 6*a*b^2*c*arccos(1/(c*x))^2 + 6*a^2*b*c*ar
ccos(1/(c*x)) - 3*b^3*c*arccos(1/(c*x)) + 6*b^3*sqrt(-1/(c^2*x^2) + 1)*arcc
os(1/(c*x))^2/x - 3*a*b^2*c + 12*a*b^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x
))/x - 4*b^3*arccos(1/(c*x))^3/(c*x^2) + 6*a^2*b*sqrt(-1/(c^2*x^2) + 1)/x -
```

```
3*b^3*sqrt(-1/(c^2*x^2) + 1)/x - 12*a*b^2*arccos(1/(c*x))^2/(c*x^2) - 12*a
^2*b*arccos(1/(c*x))/(c*x^2) + 6*b^3*arccos(1/(c*x))/(c*x^2) - 4*a^3/(c*x^2
) + 6*a*b^2/(c*x^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))^3/x^3,x)
[Out] int((a + b*acos(1/(c*x)))^3/x^3, x)
```

3.31 $\int \frac{(a+b\sec^{-1}(cx))^3}{x^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 170

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4} dx = -\frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}}+\frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}+\frac{2b^2(a+b\sec^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a+b\sec^{-1}(cx))}{3x}+\frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2 + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{3x^2}-\frac{(a+b\sec^{-1}(cx))^3}{3x^3}$$

```
[Out] 2/27*b^3*c^3*(1-1/c^2/x^2)^(3/2)+2/9*b^2*(a+b*arcsec(c*x))/x^3+4/3*b^2*c^2*
(a+b*arcsec(c*x))/x-1/3*(a+b*arcsec(c*x))^3/x^3-14/9*b^3*c^3*(1-1/c^2/x^2)^
(1/2)+2/3*b*c^3*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)+1/3*b*c*(a+b*arcsec
(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5330, 4490, 3392, 3377, 2717, 2713}

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx = \frac{4b^2c^2(a+b\sec^{-1}(cx))}{3x}+\frac{2b^2(a+b\sec^{-1}(cx))}{9x^3}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{3x^2}+\frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2-\frac{(a+b\sec^{-1}(cx))^3}{3x^3}+\frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}}$$

```
[In] Int[(a + b*ArcSec[c*x])^3/x^4,x]
```

```
[Out] (-14*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)])/9 + (2*b^3*c^3*(1 - 1/(c^2*x^2))^(3/2))
/27 + (2*b^2*(a + b*ArcSec[c*x]))/(9*x^3) + (4*b^2*c^2*(a + b*ArcSec[c*x]))
/(3*x) + (2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x])^2)/3 + (b*c*Sqr
t[1 - 1/(c^2*x^2)]*(a + b*ArcSec[c*x])^2)/(3*x^2) - (a + b*ArcSec[c*x])^3/(
3*x^3)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^3\text{Subst}\left(\int (a+bx)^3\cos^2(x)\sin(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= -\frac{(a+b\sec^{-1}(cx))^3}{3x^3} + \left(bc^3\right)\text{Subst}\left(\int (a+bx)^2\cos^3(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{2b^2(a+b\sec^{-1}(cx))}{9x^3} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{3x^2} - \frac{(a+b\sec^{-1}(cx))^3}{3x^3}\\
&\quad + \frac{1}{3}\left(2bc^3\right)\text{Subst}\left(\int (a+bx)^2\cos(x)\,dx, x, \sec^{-1}(cx)\right)\\
&\quad - \frac{1}{9}\left(2b^3c^3\right)\text{Subst}\left(\int \cos^3(x)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{2b^2(a+b\sec^{-1}(cx))}{9x^3} + \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2\\
&\quad + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{3x^2} - \frac{(a+b\sec^{-1}(cx))^3}{3x^3}\\
&\quad - \frac{1}{3}\left(4b^2c^3\right)\text{Subst}\left(\int (a+bx)\sin(x)\,dx, x, \sec^{-1}(cx)\right)\\
&\quad + \frac{1}{9}\left(2b^3c^3\right)\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sqrt{1-\frac{1}{c^2x^2}}\right)
\end{aligned}$$

$$= -\frac{2}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} + \frac{2b^2(a+b\sec^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a+b\sec^{-1}(cx))}{3x} + \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2 + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{3x^2} - \frac{\left(a+b\sec^{-1}(cx)\right)^3}{3x^3} - \frac{1}{3}(4b^3c^3)\,\text{Subst}\left(\int\cos(x)\,dx, x, \sec^{-1}(cx)\right)$$

$$= -\frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} + \frac{2b^2(a+b\sec^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a+b\sec^{-1}(cx))}{3x} + \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2 + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{3x^2} - \frac{\left(a+b\sec^{-1}(cx)\right)^3}{3x^3}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^4}\,dx$$

$$= \frac{-9a^3 + 9a^2bc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2) + 6ab^2(1+6c^2x^2) - 2b^3c\sqrt{1-\frac{1}{c^2x^2}}x(1+20c^2x^2) + 3b\Big(-9a^2 + 6ab$$

```
[In] Integrate[(a + b*ArcSec[c*x])^3/x^4,x]
```

```
[Out] (-9*a^3 + 9*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 6*a*b^2*(1 +
6*c^2*x^2) - 2*b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 20*c^2*x^2) + 3*b*(-9*a^2
 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c^2*x^2))
*ArcSec[c*x] + 9*b^2*(-3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*A
rcSec[c*x]^2 - 9*b^3*ArcSec[c*x]^3)/(27*x^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(148) = 296.

Time = 1.16 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.76

method	result
derivativedivides	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{3c^3x^3}+\frac{\operatorname{arcsec}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}-\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arcsec}(cx)}{3cx}+\frac{2\operatorname{arcsec}(}{9c^3x^3}\right.\right.$
default	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{3c^3x^3}+\frac{\operatorname{arcsec}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}-\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arcsec}(cx)}{3cx}+\frac{2\operatorname{arcsec}(}{9c^3x^3}\right.\right.$
parts	$-\frac{a^3}{3x^3}+b^3c^3\left(-\frac{\operatorname{arcsec}(cx)^3}{3c^3x^3}+\frac{\operatorname{arcsec}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}-\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arcsec}(cx)}{3cx}+\frac{2\operatorname{arcsec}(cx)}{9c^3x^3}\right.$

```
[In] int((a+b*arcsec(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arcsec(c*x)^3+1/3*arcsec(c*x)^2*(2*
c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-4/3*((c^2*x^2-1)/c^2/x^2)^(1
/2)+4/3/c/x*arcsec(c*x)+2/9/c^3/x^3*arcsec(c*x)-2/27*(2*c^2*x^2+1)/c^2/x^2*
((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a*b^2*(-1/3/c^3/x^3*arcsec(c*x)^2+2/9*arcsec
(c*x)*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/
x)+3*a^2*b*(-1/3/c^3/x^3*arcsec(c*x)+1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^
2-1)/c^2/x^2)^(1/2)/c^4/x^4))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^4}\,dx$$
$$=\frac{36\,ab^2c^2x^2-9\,b^3\operatorname{arcsec}(cx)^3-27\,ab^2\operatorname{arcsec}(cx)^2-9\,a^3+6\,ab^2+3\,(12\,b^3c^2x^2-9\,a^2b+2\,b^3)\operatorname{arcsec}(cx)+}{}$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(36*a*b^2*c^2*x^2 - 9*b^3*arcsec(c*x)^3 - 27*a*b^2*arcsec(c*x)^2 - 9*a
^3 + 6*a*b^2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b + 2*b^3)*arcsec(c*x) + (2*(9*a^2
*b - 20*b^3)*c^2*x^2 + 9*a^2*b - 2*b^3 + 9*(2*b^3*c^2*x^2 + b^3)*arcsec(c*x
)^2 + 18*(2*a*b^2*c^2*x^2 + a*b^2)*arcsec(c*x))*sqrt(c^2*x^2 - 1))/x^3
```

Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))^3}{x^4}\,dx$$

```
[In] integrate((a+b*asec(c*x))**3/x**4,x)
[Out] Integral((a + b*asec(c*x))**3/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(148) = 296$.

Time = 0.70 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.38

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^4}\,dx = -\frac{1}{216}\left(\frac{72\left(c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}\right)\mathrm{arcsec}\,(cx)^2}{c}+\frac{72\,c^4\left(\frac{c^2\arcsin\left(\frac{1}{c|x|}\right)+\frac{2\sqrt{c^2x^2-1}c}{x}-\frac{\sqrt{c^2x^2-1}}{x^2}}{c}-\cdots\right.}{\cdots}\right.$$
$$-\frac{1}{3}a^2b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\mathrm{arcsec}\,(cx)}{x^3}\right)$$
$$-\frac{b^3\,\mathrm{arcsec}\,(cx)^3}{3\,x^3}-\frac{ab^2\,\mathrm{arcsec}\,(cx)^2}{x^3}-\frac{a^3}{3\,x^3}$$
$$+\frac{2\left((6\,c^3x^2+c)\sqrt{cx+1}\sqrt{cx-1}+3\,(2\,c^5x^4-c^3x^2-c)\arctan\left(\sqrt{cx+1}\sqrt{cx-1}\right)\right)ab^2}{9\sqrt{cx+1}\sqrt{cx-1}cx^3}$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^4,x, algorithm="maxima")
[Out] -1/216*(72*(c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))*ar
csec(c*x)^2/c + (72*c^4*((c^2*arcsin(1/(c*abs(x))) + 2*sqrt(c^2*x^2 - 1)*c/
x - sqrt(c^2*x^2 - 1)/x^2)/c - (c^2*arcsin(1/(c*abs(x))) - 2*sqrt(c^2*x^2 -
 1)*c/x - sqrt(c^2*x^2 - 1)/x^2)/c - 4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/
x) + c^2*((9*c^4*arcsin(1/(c*abs(x))) + 16*sqrt(c^2*x^2 - 1)*c^3/x - 9*sqrt
(c^2*x^2 - 1)*c^2/x^2 + 8*sqrt(c^2*x^2 - 1)*c/x^3 - 6*sqrt(c^2*x^2 - 1)/x^4
)/c - (9*c^4*arcsin(1/(c*abs(x))) - 16*sqrt(c^2*x^2 - 1)*c^3/x - 9*sqrt(c^2
*x^2 - 1)*c^2/x^2 - 8*sqrt(c^2*x^2 - 1)*c/x^3 - 6*sqrt(c^2*x^2 - 1)/x^4)/c
- 48*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/x^3))/c^2)*b^3 - 1/3*a^2*b*((c^4*(
-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c*x)/x
^3) - 1/3*b^3*arcsec(c*x)^3/x^3 - a*b^2*arcsec(c*x)^2/x^3 - 1/3*a^3/x^3 + 2
/9*((6*c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c*x - 1) + 3*(2*c^5*x^4 - c^3*x^2 -
c)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*
c*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(148) = 296.

Time = 0.32 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.98

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^4}\,dx$$
$$= \frac{1}{27}\left(18\,b^3c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right)^2 + 36\,ab^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arccos\left(\frac{1}{cx}\right) + 18\,a^2bc^2\sqrt{-\frac{1}{c^2x^2}+1} - 40\right.$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] 1/27*(18*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2 + 36*a*b^2*c^2*sq
rt(-1/(c^2*x^2) + 1)*arccos(1/(c*x)) + 18*a^2*b*c^2*sqrt(-1/(c^2*x^2) + 1)
- 40*b^3*c^2*sqrt(-1/(c^2*x^2) + 1) + 36*b^3*c*arccos(1/(c*x))/x + 9*b^3*sq
rt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2/x^2 + 36*a*b^2*c/x + 18*a*b^2*sqrt(-
1/(c^2*x^2) + 1)*arccos(1/(c*x))/x^2 - 9*b^3*arccos(1/(c*x))^3/(c*x^3) + 9*
a^2*b*sqrt(-1/(c^2*x^2) + 1)/x^2 - 2*b^3*sqrt(-1/(c^2*x^2) + 1)/x^2 - 27*a*
b^2*arccos(1/(c*x))^2/(c*x^3) - 27*a^2*b*arccos(1/(c*x))/(c*x^3) + 6*b^3*ar
ccos(1/(c*x))/(c*x^3) - 9*a^3/(c*x^3) + 6*a*b^2/(c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^4}\,dx = \int \frac{\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}{x^4}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))^3/x^4,x)
```

```
[Out] int((a + b*acos(1/(c*x)))^3/x^4, x)
```

3.32 $\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 208

$$\begin{aligned}\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^5}\,dx =& -\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3}-\frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x} \\ &-\frac{45}{256}b^3c^4\sec^{-1}(cx)+\frac{3b^2\left(a+b\sec^{-1}(cx)\right)}{32x^4} \\ &+\frac{9b^2c^2\left(a+b\sec^{-1}(cx)\right)}{32x^2}+\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{16x^3} \\ &+\frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{32x} \\ &+\frac{3}{32}c^4\left(a+b\sec^{-1}(cx)\right)^3-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{4x^4}\end{aligned}$$

```
[Out] -45/256*b^3*c^4*arcsec(c*x)+3/32*b^2*(a+b*arcsec(c*x))/x^4+9/32*b^2*c^2*(a+
b*arcsec(c*x))/x^2+3/32*c^4*(a+b*arcsec(c*x))^3-1/4*(a+b*arcsec(c*x))^3/x^4
-3/128*b^3*c*(1-1/c^2/x^2)^(1/2)/x^3-45/256*b^3*c^3*(1-1/c^2/x^2)^(1/2)/x+3
/16*b*c*(a+b*arcsec(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^3+9/32*b*c^3*(a+b*arcsec(
c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5330, 4490, 3392, 32, 2715, 8}

$$\int \frac{\left(a+b\sec^{-1}(cx)\right)^3}{x^5}\,dx=\frac{9b^2c^2(a+b\sec^{-1}(cx))}{32x^2}+\frac{3b^2(a+b\sec^{-1}(cx))}{32x^4}+\frac{3}{32}c^4\left(a+b\sec^{-1}(cx)\right)^3+\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{16x^3}+\frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{32x}-\frac{\left(a+b\sec^{-1}(cx)\right)^3}{4x^4}-\frac{45}{256}b^3c^4\sec^{-1}(cx)-\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3}-\frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x}$$

```
[In] Int[(a + b*ArcSec[c*x])^3/x^5,x]
```

```
[Out] (-3*b^3*c*Sqrt[1 - 1/(c^2*x^2)])/(128*x^3) - (45*b^3*c^3*Sqrt[1 - 1/(c^2*x^
2)])/(256*x) - (45*b^3*c^4*ArcSec[c*x])/256 + (3*b^2*(a + b*ArcSec[c*x]))/(
32*x^4) + (9*b^2*c^2*(a + b*ArcSec[c*x]))/(32*x^2) + (3*b*c*Sqrt[1 - 1/(c^2
*x^2)]*(a + b*ArcSec[c*x])^2)/(16*x^3) + (9*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a
+ b*ArcSec[c*x])^2)/(32*x) + (3*c^4*(a + b*ArcSec[c*x])^3)/32 - (a + b*ArcS
ec[c*x])^3/(4*x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
```

```
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4490

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1
), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\text{integral} = c^4\text{Subst}\left(\int (a+bx)^3\cos^3(x)\sin(x)\,dx, x, \sec^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\sec^{-1}(cx)\right)^3}{4x^4} + \frac{1}{4}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)^2\cos^4(x)\,dx, x, \sec^{-1}(cx)\right)$$

$$= \frac{3b^2\left(a+b\sec^{-1}(cx)\right)}{32x^4} + \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{16x^3} - \frac{\left(a+b\sec^{-1}(cx)\right)^3}{4x^4} + \frac{1}{16}\left(9bc^4\right)\text{Subst}\left(\int (a+bx)^2\cos^2(x)\,dx, x, \sec^{-1}(cx)\right) - \frac{1}{32}\left(3b^3c^4\right)\text{Subst}\left(\int \cos^4(x)\,dx, x, \sec^{-1}(cx)\right)$$

$$= -\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} + \frac{3b^2\left(a+b\sec^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a+b\sec^{-1}(cx)\right)}{32x^2} + \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{16x^3} + \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\sec^{-1}(cx)\right)^2}{32x} - \frac{\left(a+b\sec^{-1}(cx)\right)^3}{4x^4} + \frac{1}{32}\left(9bc^4\right)\text{Subst}\left(\int (a+bx)^2\,dx, x, \sec^{-1}(cx)\right) - \frac{1}{128}\left(9b^3c^4\right)\text{Subst}\left(\int \cos^2(x)\,dx, x, \sec^{-1}(cx)\right) - \frac{1}{32}\left(9b^3c^4\right)\text{Subst}\left(\int \cos^2(x)\,dx, x, \sec^{-1}(cx)\right)$$

$$
\begin{aligned}
&= -\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x} + \frac{3b^2(a+b\sec^{-1}(cx))}{32x^4} \\
&\quad + \frac{9b^2c^2(a+b\sec^{-1}(cx))}{32x^2} + \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{16x^3} \\
&\quad + \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{32x} + \frac{3}{32}c^4(a+b\sec^{-1}(cx))^3 \\
&\quad - \frac{(a+b\sec^{-1}(cx))^3}{4x^4} - \frac{1}{256}(9b^3c^4)\,\mathrm{Subst}\left(\int 1\,dx, x, \sec^{-1}(cx)\right) \\
&\quad - \frac{1}{64}(9b^3c^4)\,\mathrm{Subst}\left(\int 1\,dx, x, \sec^{-1}(cx)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x} - \frac{45}{256}b^3c^4\sec^{-1}(cx) + \frac{3b^2(a+b\sec^{-1}(cx))}{32x^4} \\
&\quad + \frac{9b^2c^2(a+b\sec^{-1}(cx))}{32x^2} + \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{16x^3} \\
&\quad + \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\sec^{-1}(cx))^2}{32x} + \frac{3}{32}c^4(a+b\sec^{-1}(cx))^3 - \frac{(a+b\sec^{-1}(cx))^3}{4x^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.36

$$
\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx
$$

$$
= \frac{-64a^3 + 24ab^2 + 48a^2bc\sqrt{1-\frac{1}{c^2x^2}}x - 6b^3c\sqrt{1-\frac{1}{c^2x^2}}x + 72ab^2c^2x^2 + 72a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}x^3 - 45b^3c^3\sqrt{1-}}{}
$$

```
[In] Integrate[(a + b*ArcSec[c*x])^3/x^5,x]
```

```
[Out] (-64*a^3 + 24*a*b^2 + 48*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 -
 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 + 72*a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3
 - 45*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + 24*b*(-8*a^2 + b^2*(1 + 3*c^2*x^2
) + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2))*ArcSec[c*x] + 24*b^2*(
b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2) + a*(-8 + 3*c^4*x^4))*ArcSec[c*
x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcSec[c*x]^3 + 9*b*(-8*a^2 + 5*b^2)*c^4*x^4*
ArcSin[1/(c*x)])/(256*x^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(182) = 364$.

Time = 1.10 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.26

method	result
parts	$-\frac{a^3}{4x^4}+b^3c^4\left(-\frac{\operatorname{arcsec}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arcsec}(cx)^2\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arcsec}(cx)}{32c^4x^4}\right.$
derivativedivides	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arcsec}(cx)^2\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arcsec}}{32c^4x}\right.\right.$
default	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arcsec}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arcsec}(cx)^2\left(3c^3x^3\operatorname{arcsec}(cx)+3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arcsec}}{32c^4x}\right.\right.$

```
[In] int((a+b*arcsec(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^3/x^4+b^3*c^4*(-1/4/c^4/x^4*arcsec(c*x)^3+3/32*arcsec(c*x)^2*(3*c^3*
x^3*arcsec(c*x)+3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2*((c^2*x^2-1)/c^2/x^
2)^(1/2))/c^3/x^3+3/32*arcsec(c*x)/c^4/x^4-3/256*(3*c^2*x^2+2)/c^3/x^3*((c^
2*x^2-1)/c^2/x^2)^(1/2)-45/256*arcsec(c*x)+9/32/c^2/x^2*arcsec(c*x)-9/64*((
c^2*x^2-1)/c^2/x^2)^(1/2)/c/x-3/16*arcsec(c*x)^3)+3*a*b^2*c^4*(-1/4/c^4/x^4
*arcsec(c*x)^2+1/16*arcsec(c*x)*(3*c^3*x^3*arcsec(c*x)+3*c^2*x^2*((c^2*x^2-
1)/c^2/x^2)^(1/2)+2*((c^2*x^2-1)/c^2/x^2)^(1/2))/c^3/x^3-3/32*arcsec(c*x)^2
+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-3/4*a^2*b/x^4*arcsec(c*x)-9/32*a^2*b*c^3*(c
^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))+9
/32*a^2*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^3+3/16*a^2*b/c*(c^2*x
^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx = \frac{72\,ab^2c^2x^2+8\,(3\,b^3c^4x^4-8\,b^3)\operatorname{arcsec}(cx)^3-64\,a^3+24\,ab^2+24\,(3\,ab^2c^4x^4-8\,ab^2)\operatorname{arcsec}(cx)^2+3\,(3\,(8}{}$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*arcsec(c*x)^3 - 64*a^3
+ 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8*a*b^2)*arcsec(c*x)^2 + 3*(3*(8*a^2*b -
 5*b^3)*c^4*x^4 + 24*b^3*c^2*x^2 - 64*a^2*b + 8*b^3)*arcsec(c*x) + 3*(3*(8*
```

```
a^2*b - 5*b^3)*c^2*x^2 + 16*a^2*b - 2*b^3 + 8*(3*b^3*c^2*x^2 + 2*b^3)*arcse
c(c*x)^2 + 16*(3*a*b^2*c^2*x^2 + 2*a*b^2)*arcsec(c*x))*sqrt(c^2*x^2 - 1))/x
^4
```

Sympy [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx = \int \frac{(a+b\operatorname{asec}(cx))^3}{x^5}\,dx$$

```
[In] integrate((a+b*asec(c*x))**3/x**5,x)
[Out] Integral((a + b*asec(c*x))**3/x**5, x)
```

Maxima [F]

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)^3}{x^5}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^5,x, algorithm="maxima")
[Out] 3/32*a^2*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2
*x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) -
1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 8*arcsec(c*x)/x^4) - 1/4*a^3/x
^4 - 1/16*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan(sqrt(
c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(2*(c^2*log(c*x + 1) + c^2*log(
c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*c^2*log(c)^2 + 64*b^3*c^2*integrate(
1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^7 - x^5), x)*log(c)^2 -
 64*b^3*c^2*integrate(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*
x^2)/(c^2*x^7 - x^5), x)*log(c) + 128*b^3*c^2*integrate(1/16*x^2*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^7 - x^5), x)*log(c) - 64*a*b^2*c^2*
integrate(1/16*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 128*a*b^2*c^2*
integrate(1/16*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) - 64*b^3*c^2*integrate
(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)*log(x)/(c^2*x^7
- x^5), x) + 64*b^3*c^2*integrate(1/16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x -
1))*log(x)^2/(c^2*x^7 - x^5), x) - 64*a*b^2*c^2*integrate(1/16*x^2*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^7 - x^5), x) + 16*b^3*c^2*integrate(1/
16*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^7 - x^5), x)
 + 16*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)^2/(c^2*x^7 - x^5), x) - 64*
a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 64*a
*b^2*c^2*integrate(1/16*x^2*log(x)^2/(c^2*x^7 - x^5), x) - (2*c^4*log(c*x +
 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x) + (2*c^2*x^2 + 1)/x^4)*a*b^2*log(c)
^2 - 64*b^3*integrate(1/16*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*x^7 - x
```

```
^5), x)*log(c)^2 + 64*b^3*integrate(1/16*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)
)*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) - 128*b^3*integrate(1/16*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))*log(x)/(c^2*x^7 - x^5), x)*log(c) + 64*a*b^2*in
tegrate(1/16*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) - 128*a*b^2*integrate(
1/16*log(x)/(c^2*x^7 - x^5), x)*log(c) - 16*b^3*integrate(1/16*sqrt(c*x + 1
)*sqrt(c*x - 1)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^7 - x^5), x) +
 4*b^3*integrate(1/16*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2/(c^2*x^7 -
 x^5), x) + 64*b^3*integrate(1/16*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c
^2*x^2)*log(x)/(c^2*x^7 - x^5), x) - 64*b^3*integrate(1/16*arctan(sqrt(c*x
+ 1)*sqrt(c*x - 1))*log(x)^2/(c^2*x^7 - x^5), x) + 64*a*b^2*integrate(1/16*
arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^2*x^7 - x^5), x) - 16*b^3*integrat
e(1/16*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)/(c^2*x^7 - x^5), x)
 - 16*a*b^2*integrate(1/16*log(c^2*x^2)^2/(c^2*x^7 - x^5), x) + 64*a*b^2*in
tegrate(1/16*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) - 64*a*b^2*integrate(1
/16*log(x)^2/(c^2*x^7 - x^5), x))*x^4)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(182) = 364$.

Time = 0.31 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.05

$$\int \frac{(a + b\sec^{-1}(cx))^3}{x^5}\,dx$$

$$= \frac{1}{256}\left(24\,b^3c^3\arccos\left(\frac{1}{cx}\right)^3 + 72\,ab^2c^3\arccos\left(\frac{1}{cx}\right)^2 + 72\,a^2bc^3\arccos\left(\frac{1}{cx}\right) - 45\,b^3c^3\arccos\left(\frac{1}{cx}\right) + \frac{72}{}\right.$$

```
[In] integrate((a+b*arcsec(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] 1/256*(24*b^3*c^3*arccos(1/(c*x))^3 + 72*a*b^2*c^3*arccos(1/(c*x))^2 + 72*a
^2*b*c^3*arccos(1/(c*x)) - 45*b^3*c^3*arccos(1/(c*x)) + 72*b^3*c^2*sqrt(-1/
(c^2*x^2) + 1)*arccos(1/(c*x))^2/x - 45*a*b^2*c^3 + 144*a*b^2*c^2*sqrt(-1/(
c^2*x^2) + 1)*arccos(1/(c*x))/x + 72*a^2*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x - 4
5*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 72*b^3*c*arccos(1/(c*x))/x^2 + 48*b^3*
sqrt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))^2/x^3 + 72*a*b^2*c/x^2 + 96*a*b^2*sq
rt(-1/(c^2*x^2) + 1)*arccos(1/(c*x))/x^3 - 64*b^3*arccos(1/(c*x))^3/(c*x^4)
 + 48*a^2*b*sqrt(-1/(c^2*x^2) + 1)/x^3 - 6*b^3*sqrt(-1/(c^2*x^2) + 1)/x^3 -
 192*a*b^2*arccos(1/(c*x))^2/(c*x^4) - 192*a^2*b*arccos(1/(c*x))/(c*x^4) +
24*b^3*arccos(1/(c*x))/(c*x^4) - 64*a^3/(c*x^4) + 24*a*b^2/(c*x^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec^{-1}(cx))^3}{x^5}\,dx = \int \frac{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))^3/x^5,x)
[Out] int((a + b*acos(1/(c*x)))^3/x^5, x)
```

3.33 $\int \frac{x}{a+b\sec^{-1}(cx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx = \text{Int}\left(\frac{x}{a+b\sec^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x/(a+b*arcsec(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx = \int \frac{x}{a+b\sec^{-1}(cx)}\,dx$$

```
[In] Int[x/(a + b*ArcSec[c*x]),x]
```

```
[Out] Defer[Int][x/(a + b*ArcSec[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{a+b\sec^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx=\int \frac{x}{a+b\sec^{-1}(cx)}\,dx$$

```
[In] Integrate[x/(a + b*ArcSec[c*x]),x]
[Out] Integrate[x/(a + b*ArcSec[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\ \mathrm{arcsec}\left(cx\right)}dx$$

```
[In] int(x/(a+b*arcsec(c*x)),x)
[Out] int(x/(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx=\int \frac{x}{b\,\mathrm{arcsec}\left(cx\right)+a}\,dx$$

```
[In] integrate(x/(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral(x/(b*arcsec(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+b\sec^{-1}(cx)}\,dx=\int \frac{x}{a+b\,\mathrm{asec}\left(cx\right)}\,dx$$

```
[In] integrate(x/(a+b*asec(c*x)),x)
[Out] Integral(x/(a + b*asec(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\sec^{-1}(cx)}\,dx = \int \frac{x}{b\,\mathrm{arcsec}\,(cx) + a}\,dx$$

```
[In] integrate(x/(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] integrate(x/(b*arcsec(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 33.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\sec^{-1}(cx)}\,dx = \int \frac{x}{b\,\mathrm{arcsec}\,(cx) + a}\,dx$$

```
[In] integrate(x/(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate(x/(b*arcsec(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{a + b\sec^{-1}(cx)}\,dx = \int \frac{x}{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}\,dx$$

```
[In] int(x/(a + b*acos(1/(c*x))),x)
[Out] int(x/(a + b*acos(1/(c*x))), x)
```

3.34 $\int \frac{1}{a+b\sec^{-1}(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx = \text{Int}\left(\frac{1}{a+b\sec^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsec(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx = \int \frac{1}{a+b\sec^{-1}(cx)}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])^(-1),x]
```

```
[Out] Defer[Int][(a + b*ArcSec[c*x])^(-1), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{a+b\sec^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx=\int \frac{1}{a+b\sec^{-1}(cx)}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])^(-1),x]
[Out] Integrate[(a + b*ArcSec[c*x])^(-1), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\ \operatorname{arcsec}\left(cx\right)}dx$$

```
[In] int(1/(a+b*arcsec(c*x)),x)
[Out] int(1/(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx=\int \frac{1}{b\operatorname{arcsec}\left(cx\right)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arcsec(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx=\int \frac{1}{a+b\operatorname{asec}\left(cx\right)}\,dx$$

```
[In] integrate(1/(a+b*asec(c*x)),x)
[Out] Integral(1/(a + b*asec(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx = \int \frac{1}{b\,\mathrm{arcsec}\,(cx)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsec(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 11.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx = \int \frac{1}{b\,\mathrm{arcsec}\,(cx)+a}\,dx$$

```
[In] integrate(1/(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arcsec(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{a+b\sec^{-1}(cx)}\,dx = \int \frac{1}{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}\,dx$$

```
[In] int(1/(a + b*acos(1/(c*x))),x)
[Out] int(1/(a + b*acos(1/(c*x))), x)
```

3.35 $\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\sec^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsec(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*ArcSec[c*x])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcSec[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcSec[c*x])),x]
[Out] Integrate[1/(x*(a + b*ArcSec[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \mathrm{arcsec}\left(cx\right)\right)}dx$$

```
[In] int(1/x/(a+b*arcsec(c*x)),x)
[Out] int(1/x/(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arcsec}\left(cx\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x*arcsec(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{asec}\left(cx\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*asec(c*x)),x)
[Out] Integral(1/(x*(a + b*asec(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\,(a+b\sec^{-1}(cx))}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arcsec(c*x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\,(a+b\sec^{-1}(cx))}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arcsec(c*x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\,(a+b\sec^{-1}(cx))}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

```
[In] int(1/(x*(a + b*acos(1/(c*x)))),x)
[Out] int(1/(x*(a + b*acos(1/(c*x)))), x)
```

3.36 $\int \frac{1}{x^2(a+b\sec^{-1}(cx))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx=-\frac{c\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}+\frac{c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}$$

```
[Out] c*cos(a/b)*Si(a/b+arcsec(c*x))/b-c*Ci(a/b+arcsec(c*x))*sin(a/b)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5330, 3384, 3380, 3383}

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx=\frac{c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}-\frac{c\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}$$

```
[In] Int[1/(x^2*(a + b*ArcSec[c*x])),x]
```

```
[Out] -((c*CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/b) + (c*Cos[a/b]*SinIntegral[
a/b + ArcSec[c*x]])/b
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c\text{Subst}\left(\int \frac{\sin(x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)\\
&= \left(c\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)\\
&\quad -\left(c\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)\\
&= -\frac{c\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}+\frac{c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx = \frac{c\left(-\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)+\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)}{b}$$

```
[In] Integrate[1/(x^2*(a + b*ArcSec[c*x])),x]
[Out] (c*(-(CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b]) + Cos[a/b]*SinIntegral[a/b +
 ArcSec[c*x]]))/b
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$c\left(\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}\right)$	47
default	$c\left(\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}\right)$	47

```
[In] int(1/x^2/(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
[Out] c*(Si(a/b+arcsec(c*x))*cos(a/b)/b-Ci(a/b+arcsec(c*x))*sin(a/b)/b)
```

Fricas [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*arcsec(c*x) + a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\left(a+b\,\mathrm{asec}\,(cx)\right)}\,dx$$

```
[In] integrate(1/x**2/(a+b*asec(c*x)),x)
[Out] Integral(1/(x**2*(a + b*asec(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arcsec(c*x) + a)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 (a + b \sec^{-1}(cx))} dx = -c \left(\frac{\text{Ci}\left(\frac{a}{b} + \arccos\left(\frac{1}{cx}\right)\right) \sin\left(\frac{a}{b}\right)}{b} - \frac{\cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \arccos\left(\frac{1}{cx}\right)\right)}{b} \right)$$

```
[In] integrate(1/x^2/(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] -c*(cos_integral(a/b + arccos(1/(c*x)))*sin(a/b)/b - cos(a/b)*sin_integral(
a/b + arccos(1/(c*x)))/b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + b \sec^{-1}(cx))} dx = \int \frac{1}{x^2 \left(a + b \operatorname{acos}\left(\frac{1}{cx}\right)\right)} dx$$

```
[In] int(1/(x^2*(a + b*acos(1/(c*x)))),x)
[Out] int(1/(x^2*(a + b*acos(1/(c*x)))), x)
```

3.37 $\int \frac{1}{x^3(a+b\sec^{-1}(cx))} dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{1}{x^3(a+b\sec^{-1}(cx))} dx = -\frac{c^2 \operatorname{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b} + \frac{c^2\cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{2b}$$

```
[Out] 1/2*c^2*cos(2*a/b)*Si(2*a/b+2*arcsec(c*x))/b-1/2*c^2*Ci(2*a/b+2*arcsec(c*x)
)*sin(2*a/b)/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5330, 4491, 12, 3384, 3380, 3383}

$$\int \frac{1}{x^3(a+b\sec^{-1}(cx))} dx = \frac{c^2\cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{2b} - \frac{c^2\sin\left(\frac{2a}{b}\right)\operatorname{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{2b}$$

```
[In] Int[1/(x^3*(a + b*ArcSec[c*x])),x]
```

```
[Out] -1/2*(c^2*CosIntegral[(2*a)/b + 2*ArcSec[c*x]]*Sin[(2*a)/b])/b + (c^2*Cos[(
2*a)/b]*SinIntegral[(2*a)/b + 2*ArcSec[c*x]])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c^2\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)\\
&= c^2\text{Subst}\left(\int \frac{\sin(2x)}{2(a+bx)}\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{1}{2}c^2\text{Subst}\left(\int \frac{\sin(2x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{1}{2}\left(c^2\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)\\
&\quad -\frac{1}{2}\left(c^2\sin\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)
\end{aligned}
$$

$$= -\frac{c^2 \operatorname{CosIntegral}\left(\frac{2a}{b} + 2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b} + \frac{c^2\cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2a}{b} + 2\sec^{-1}(cx)\right)}{2b}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3 (a + b\sec^{-1}(cx))}\,dx = \frac{c^2\left(-\operatorname{CosIntegral}\left(\frac{2a}{b} + 2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right) + \cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2a}{b} + 2\sec^{-1}(cx)\right)\right)}{2b}$$

```
[In] Integrate[1/(x^3*(a + b*ArcSec[c*x])),x]
[Out] (c^2*(-(CosIntegral[(2*a)/b + 2*ArcSec[c*x]]*Sin[(2*a)/b]) + Cos[(2*a)/b]*S
inIntegral[(2*a)/b + 2*ArcSec[c*x]]))/(2*b)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$c^2\left(\frac{\operatorname{Si}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b} - \frac{\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$	58
default	$c^2\left(\frac{\operatorname{Si}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b} - \frac{\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arcsec}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$	58

```
[In] int(1/x^3/(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
[Out] c^2*(1/2*Si(2*a/b+2*arcsec(c*x))*cos(2*a/b)/b-1/2*Ci(2*a/b+2*arcsec(c*x))*s
in(2*a/b)/b)
```

Fricas [F]

$$\int \frac{1}{x^3 (a + b\sec^{-1}(cx))}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx) + a)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*arcsec(c*x) + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3 (a+b\sec^{-1}(cx))}\,dx=\int \frac{1}{x^3 (a+b\,\mathrm{asec}\,(cx))}\,dx$$

[In] integrate(1/x**3/(a+b*asec(c*x)),x)

[Out] Integral(1/(x**3*(a + b*asec(c*x))), x)

Maxima [F]

$$\int \frac{1}{x^3 (a+b\sec^{-1}(cx))}\,dx=\int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x^3}\,dx$$

[In] integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="maxima")

[Out] integrate(1/((b*arcsec(c*x) + a)*x^3), x)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^3 (a+b\sec^{-1}(cx))}\,dx = -\frac{1}{2}\left(\frac{2\,c\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{2\,a}{b}+2\,\arccos\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{2\,c\cos\left(\frac{a}{b}\right)^2\mathrm{Si}\left(\frac{2\,a}{b}+2\,\arccos\left(\frac{1}{cx}\right)\right)}{b}+\frac{c\,\mathrm{Si}\left(\frac{2\,a}{b}+2\,\mathrm{arc}\right.}{b}\right.$$

[In] integrate(1/x^3/(a+b*arcsec(c*x)),x, algorithm="giac")

[Out] -1/2*(2*c*cos(a/b)*cos_integral(2*a/b + 2*arccos(1/(c*x)))*sin(a/b)/b - 2*c*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)))/b + c*sin_integral(2*a/b + 2*arccos(1/(c*x)))/b)*c

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + b \sec^{-1}(cx))} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{acos}\left(\frac{1}{c x}\right)\right)} \, dx$$

```
[In] int(1/(x^3*(a + b*acos(1/(c*x)))),x)
[Out] int(1/(x^3*(a + b*acos(1/(c*x)))), x)
```

3.38 $\int \frac{1}{x^4(a+b\sec^{-1}(cx))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx=-\frac{c^3\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b}-\frac{c^3\,\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}+\frac{c^3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b}+\frac{c^3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b}$$

```
[Out] 1/4*c^3*cos(a/b)*Si(a/b+arcsec(c*x))/b+1/4*c^3*cos(3*a/b)*Si(3*a/b+3*arcsec
(c*x))/b-1/4*c^3*Ci(a/b+arcsec(c*x))*sin(a/b)/b-1/4*c^3*Ci(3*a/b+3*arcsec(c
*x))*sin(3*a/b)/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5330, 4491, 3384, 3380, 3383}

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx=-\frac{c^3\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b}-\frac{c^3\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b}+\frac{c^3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b}+\frac{c^3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b}$$

```
[In] Int[1/(x^4*(a + b*ArcSec[c*x])),x]
```

```
[Out] -1/4*(c^3*CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/b - (c^3*CosIntegral[(3*
a)/b + 3*ArcSec[c*x]]*Sin[(3*a)/b])/(4*b) + (c^3*Cos[a/b]*SinIntegral[a/b +
 ArcSec[c*x]])/(4*b) + (c^3*Cos[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcSec[c*x
]])/(4*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^3\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)\\
&= c^3\text{Subst}\left(\int \left(\frac{\sin(x)}{4(a+bx)}+\frac{\sin(3x)}{4(a+bx)}\right)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{1}{4}c^3\text{Subst}\left(\int \frac{\sin(x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)+\frac{1}{4}c^3\text{Subst}\left(\int \frac{\sin(3x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)
\end{aligned}$$

$$= \frac{1}{4}\left(c^3\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)$$
$$+ \frac{1}{4}\left(c^3\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)$$
$$- \frac{1}{4}\left(c^3\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)$$
$$- \frac{1}{4}\left(c^3\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)$$
$$= -\frac{c^3\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b}$$
$$- \frac{c^3\,\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}$$
$$+ \frac{c^3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b} + \frac{c^3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx$$
$$= \frac{c^3\left(-\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right) - \text{CosIntegral}\left(3\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)\sin\left(\frac{3a}{b}\right) + \cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\text{sec}\right.\right.}{4b}$$

```
[In] Integrate[1/(x^4*(a + b*ArcSec[c*x])),x]
```

```
[Out] (c^3*(-(CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b]) - CosIntegral[3*(a/b + Arc
Sec[c*x])]*Sin[(3*a)/b] + Cos[a/b]*SinIntegral[a/b + ArcSec[c*x]] + Cos[(3*
a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])]))/(4*b)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result
derivativedivides	$c^3\left(\frac{\text{Si}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4b} - \frac{\text{Ci}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b} + \frac{\text{Si}\left(\frac{a}{b}+\text{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b} - \frac{\text{Ci}\left(\frac{a}{b}+\text{arcsec}(c\right.}{4b}\right.$
default	$c^3\left(\frac{\text{Si}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4b} - \frac{\text{Ci}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b} + \frac{\text{Si}\left(\frac{a}{b}+\text{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b} - \frac{\text{Ci}\left(\frac{a}{b}+\text{arcsec}(c\right.}{4b}\right.$

```
[In] int(1/x^4/(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(1/4*Si(3*a/b+3*arcsec(c*x))*cos(3*a/b)/b-1/4*Ci(3*a/b+3*arcsec(c*x))*s
in(3*a/b)/b+1/4*Si(a/b+arcsec(c*x))*cos(a/b)/b-1/4*Ci(a/b+arcsec(c*x))*sin(
a/b)/b)
```

Fricas [F]

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^4*arcsec(c*x) + a*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{x^4\left(a+b\,\mathrm{asec}\,(cx)\right)}\,dx$$

```
[In] integrate(1/x**4/(a+b*asec(c*x)),x)
[Out] Integral(1/(x**4*(a + b*asec(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arcsec(c*x) + a)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)}\,dx =$$
$$-\frac{1}{4}\left(\frac{4\,c^2\cos\left(\frac{a}{b}\right)^2\mathrm{Ci}\left(\frac{3\,a}{b}+3\,\arccos\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{4\,c^2\cos\left(\frac{a}{b}\right)^3\mathrm{Si}\left(\frac{3\,a}{b}+3\,\arccos\left(\frac{1}{cx}\right)\right)}{b}-\frac{c^2\,\mathrm{Ci}\left(\frac{3\,a}{b}+3\,a\right.}{}\right.$$

```
[In] integrate(1/x^4/(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] -1/4*(4*c^2*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/b -
 4*c^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(1/(c*x)))/b - c^2*cos_integ
ral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/b + c^2*cos_integral(a/b + arccos(1
/(c*x)))*sin(a/b)/b + 3*c^2*cos(a/b)*sin_integral(3*a/b + 3*arccos(1/(c*x))
)/b - c^2*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/b)*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + b\sec^{-1}(cx))}\,dx = \int \frac{1}{x^4\,\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

```
[In] int(1/(x^4*(a + b*acos(1/(c*x)))),x)
```

```
[Out] int(1/(x^4*(a + b*acos(1/(c*x)))), x)
```

3.39 $\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \text{Int}\left(\frac{x}{(a+b\sec^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable(x/(a+b*arcsec(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx$$

```
[In] Int[x/(a + b*ArcSec[c*x])^2,x]
```

```
[Out] Defer[Int][x/(a + b*ArcSec[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.91 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx$$

```
[In] Integrate[x/(a + b*ArcSec[c*x])^2,x]
[Out] Integrate[x/(a + b*ArcSec[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\ \mathrm{arcsec}\,(cx))^2}dx$$

```
[In] int(x/(a+b*arcsec(c*x))^2,x)
[Out] int(x/(a+b*arcsec(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{x}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
[Out] integral(x/(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{x}{(a+b\,\mathrm{asec}\,(cx))^2}\,dx$$

```
[In] integrate(x/(a+b*asec(c*x))**2,x)
[Out] Integral(x/(a + b*asec(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 583, normalized size of antiderivative = 48.58

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{x}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(4*(b*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*x^2)*sqrt(c*x + 1)*sqrt(
c*x - 1) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^
2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^
2*x^2))*integrate(-4*(3*a*c^2*x^3 - 2*a*x + (3*b*c^2*x^3 - 2*b*x)*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1)))*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3*log(c)^2 + 4
*a^2*b - 4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2 - b^3)*arcta
n(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)^2 - 4*(
b^3*c^2*x^2 - b^3)*log(x)^2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*x^2 - b^3)
*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(x)), x))/(4
*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b^3*log
(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))
```

Giac [N/A]

Not integrable

Time = 105.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{x}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsec(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{(a+b\sec^{-1}(cx))^2}\,dx=\int \frac{x}{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int(x/(a + b*acos(1/(c*x)))^2,x)
[Out] int(x/(a + b*acos(1/(c*x)))^2, x)
```

3.40 $\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \operatorname{Int}\left(\frac{1}{\left(a+b\sec^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsec(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])^(-2),x]
```

```
[Out] Defer[Int][(a + b*ArcSec[c*x])^(-2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 23.96 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])^(-2),x]
[Out] Integrate[(a + b*ArcSec[c*x])^(-2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\,\mathrm{arcsec}\,(cx))^2}dx$$

```
[In] int(1/(a+b*arcsec(c*x))^2,x)
[Out] int(1/(a+b*arcsec(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{asec}\,(cx))^2}\,dx$$

```
[In] integrate(1/(a+b*asec(c*x))**2,x)
[Out] Integral((a + b*asec(c*x))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 577, normalized size of antiderivative = 57.70

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(4*(b*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*x)*sqrt(c*x + 1)*sqrt(c*x
- 1) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 +
4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt
(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^
2))*integrate(-4*(2*a*c^2*x^2 + (2*b*c^2*x^2 - b)*arctan(sqrt(c*x + 1)*sqrt
(c*x - 1)) - a)*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3*log(c)^2 + 4*a^2*b - 4*(
b^3*c^2*log(c)^2 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2 - b^3)*arctan(sqrt(c*x +
 1)*sqrt(c*x - 1))^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)^2 - 4*(b^3*c^2*x^2
- b^3)*log(x)^2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arctan(sqrt(c*x + 1)*sqrt(c*x -
 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*x^2 - b^3)*log(x))*log
(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(x)), x))/(4*b^3*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^
3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x -
1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))
```

Giac [N/A]

Not integrable

Time = 23.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^{-1}(cx))^2}\,dx=\int \frac{1}{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int(1/(a + b*acos(1/(c*x)))^2,x)
[Out] int(1/(a + b*acos(1/(c*x)))^2, x)
```

3.41 $\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2} dx = \text{Int}\left(\frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}, x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsec(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2} dx = \int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2} dx$$

```
[In] Int[1/(x*(a + b*ArcSec[c*x])^2),x]
[Out] Defer[Int][1/(x*(a + b*ArcSec[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcSec[c*x])^2),x]
[Out] Integrate[1/(x*(a + b*ArcSec[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \mathrm{arcsec}\left(cx\right)\right)^2}dx$$

```
[In] int(1/x/(a+b*arcsec(c*x))^2,x)
[Out] int(1/x/(a+b*arcsec(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*arcsec(c*x)^2 + 2*a*b*x*arcsec(c*x) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{asec}\left(cx\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*asec(c*x))**2,x)
[Out] Integral(1/(x*(a + b*asec(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 560, normalized size of antiderivative = 40.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{arcsec}\left(cx\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a)
 - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b^
3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x
 + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*
integrate(-4*(b*c^2*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*c^2*x)*sqrt(c
*x + 1)*sqrt(c*x - 1)/(4*b^3*log(c)^2 + 4*a^2*b - 4*(b^3*c^2*log(c)^2 + a^2
*b*c^2)*x^2 - 4*(b^3*c^2*x^2 - b^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 -
 (b^3*c^2*x^2 - b^3)*log(c^2*x^2)^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^2 - 8*(a
*b^2*c^2*x^2 - a*b^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*c^2*x^2*
log(c) - b^3*log(c) + (b^3*c^2*x^2 - b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2
*x^2*log(c) - b^3*log(c))*log(x)), x))/(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^
3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3
*log(c) + b^3*log(x))*log(c^2*x^2))
```

Giac [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{arcsec}\left(cx\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsec(c*x) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int(1/(x*(a + b*acos(1/(c*x)))^2),x)
[Out] int(1/(x*(a + b*acos(1/(c*x)))^2), x)
```

3.42 $\int \frac{1}{x^2(a+b\sec^{-1}(cx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx = -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)}+\frac{c\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b^2}+\frac{c\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b^2}$$

```
[Out] c*Ci(a/b+arcsec(c*x))*cos(a/b)/b^2+c*Si(a/b+arcsec(c*x))*sin(a/b)/b^2-c*(1-
1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5330, 3378, 3384, 3380, 3383}

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{c\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b^2}+\frac{c\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b^2}-\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)}$$

```
[In] Int[1/(x^2*(a + b*ArcSec[c*x])^2),x]
```

```
[Out] -((c*Sqrt[1 - 1/(c^2*x^2)])/(b*(a + b*ArcSec[c*x]))) + (c*Cos[a/b]*CosInteg
ral[a/b + ArcSec[c*x]])/b^2 + (c*Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]])/b
^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c\text{Subst}\left(\int \frac{\sin(x)}{(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)\\
&= -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)} + \frac{c\text{Subst}\left(\int \frac{\cos(x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{b}\\
&= -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)} + \frac{\left(c\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{b}\\
&\quad + \frac{\left(c\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{b}\\
&= -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{b\left(a+b\sec^{-1}(cx)\right)} + \frac{c\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b^2} + \frac{c\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{c\left(-\frac{b\sqrt{1-\frac{1}{c^2x^2}}}{a+b\sec^{-1}(cx)}+\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)}{b^2}$$

```
[In] Integrate[1/(x^2*(a + b*ArcSec[c*x])^2),x]
[Out] (c*(-((b*Sqrt[1 - 1/(c^2*x^2)])/(a + b*ArcSec[c*x])) + Cos[a/b]*CosIntegral
[a/b + ArcSec[c*x]] + Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]]))/b^2
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{(a+b\,\mathrm{arcsec}(cx))b}+\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)+\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b^2}\right)$	78
default	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{(a+b\,\mathrm{arcsec}(cx))b}+\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\sin\left(\frac{a}{b}\right)+\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arcsec}(cx)\right)\cos\left(\frac{a}{b}\right)}{b^2}\right)$	78

```
[In] int(1/x^2/(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
[Out] c*(-((c^2*x^2-1)/c^2/x^2)^(1/2)/(a+b*arcsec(c*x))/b+(Si(a/b+arcsec(c*x))*si
n(a/b)+Ci(a/b+arcsec(c*x))*cos(a/b))/b^2)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2 x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*arcsec(c*x)^2 + 2*a*b*x^2*arcsec(c*x) + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\,\mathrm{asec}\,(cx)\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*asec(c*x))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*asec(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\,(cx)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a)
 - (4*b^3*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x*log(c^2*x^2)^2 +
8*b^3*x*log(c)*log(x) + 4*b^3*x*log(x)^2 + 8*a*b^2*x*arctan(sqrt(c*x + 1)*s
qrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x - 4*(b^3*x*log(c) + b^3*x*log(x)
)*log(c^2*x^2))*integrate(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x
+ 1)*sqrt(c*x - 1)) + a)/(4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^4 - 4*(b^3*log
(c)^2 + a^2*b)*x^2 + 4*(b^3*c^2*x^4 - b^3*x^2)*arctan(sqrt(c*x + 1)*sqrt(c*
x - 1))^2 + (b^3*c^2*x^4 - b^3*x^2)*log(c^2*x^2)^2 + 4*(b^3*c^2*x^4 - b^3*x
^2)*log(x)^2 + 8*(a*b^2*c^2*x^4 - a*b^2*x^2)*arctan(sqrt(c*x + 1)*sqrt(c*x
- 1)) - 4*(b^3*c^2*x^4*log(c) - b^3*x^2*log(c) + (b^3*c^2*x^4 - b^3*x^2)*lo
g(x))*log(c^2*x^2) + 8*(b^3*c^2*x^4*log(c) - b^3*x^2*log(c))*log(x)), x))/(
4*b^3*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x*log(c^2*x^2)^2 + 8*b^
3*x*log(c)*log(x) + 4*b^3*x*log(x)^2 + 8*a*b^2*x*arctan(sqrt(c*x + 1)*sqrt(
c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x - 4*(b^3*x*log(c) + b^3*x*log(x))*lo
g(c^2*x^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(73) = 146$.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.01

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

$$=\left(\frac{b\arccos\left(\frac{1}{cx}\right)\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)}{b^3\arccos\left(\frac{1}{cx}\right)+ab^2}+\frac{b\arccos\left(\frac{1}{cx}\right)\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)}{b^3\arccos\left(\frac{1}{cx}\right)+ab^2}+\frac{a\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}\right.}{b^3\arccos\left(\right.}\right.$$

```
[In] integrate(1/x^2/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

```
[Out] (b*arccos(1/(c*x))*cos(a/b)*cos_integral(a/b + arccos(1/(c*x)))/(b^3*arccos
(1/(c*x)) + a*b^2) + b*arccos(1/(c*x))*sin(a/b)*sin_integral(a/b + arccos(1
/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + a*cos(a/b)*cos_integral(a/b + arcc
os(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + a*sin(a/b)*sin_integral(a/b +
arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - b*sqrt(-1/(c^2*x^2) + 1)/(
b^3*arccos(1/(c*x)) + a*b^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + b\sec^{-1}(cx))^2} \, dx = \int \frac{1}{x^2 \left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2} \, dx$$

```
[In] int(1/(x^2*(a + b*acos(1/(c*x)))^2),x)
```

```
[Out] int(1/(x^2*(a + b*acos(1/(c*x)))^2), x)
```

3.43 $\int \frac{1}{x^3 (a+b \sec^{-1}(cx))^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 84

$$\int \frac{1}{x^3 (a+b \sec^{-1}(cx))^2} \, dx = \frac{c^2 \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^2} - \frac{c^2 \sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)} + \frac{c^2 \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^2}$$

```
[Out] c^2*Ci(2*a/b+2*arcsec(c*x))*cos(2*a/b)/b^2+c^2*Si(2*a/b+2*arcsec(c*x))*sin(
2*a/b)/b^2-1/2*c^2*sin(2*arcsec(c*x))/b/(a+b*arcsec(c*x))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5330, 4491, 12, 3378, 3384, 3380, 3383}

$$\int \frac{1}{x^3 (a+b \sec^{-1}(cx))^2} \, dx = \frac{c^2 \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^2} + \frac{c^2 \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^2} - \frac{c^2 \sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)}$$

```
[In] Int[1/(x^3*(a + b*ArcSec[c*x])^2),x]
```

```
[Out] (c^2*Cos[(2*a)/b]*CosIntegral[(2*a)/b + 2*ArcSec[c*x]])/b^2 - (c^2*Sin[2*Ar
cSec[c*x]])/(2*b*(a + b*ArcSec[c*x])) + (c^2*Sin[(2*a)/b]*SinIntegral[(2*a)
/b + 2*ArcSec[c*x]])/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\text{integral} = c^2\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)$$

$$= c^2\text{Subst}\left(\int \frac{\sin(2x)}{2(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)$$

$$= \frac{1}{2}c^2\text{Subst}\left(\int \frac{\sin(2x)}{(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)$$

$$= -\frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)} + \frac{c^2\text{Subst}\left(\int \frac{\cos(2x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{b}$$

$$= -\frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)} + \frac{\left(c^2\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{b} + \frac{\left(c^2\sin\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{b}$$

$$= \frac{c^2\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^2} - \frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{2b\left(a+b\sec^{-1}(cx)\right)} + \frac{c^2\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^2}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{c\left(-\frac{b\sqrt{1-\frac{1}{c^2x^2}}}{ax+bx\sec^{-1}(cx)} + c\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right) + c\sin\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)\right)}{b^2}$$

```
[In] Integrate[1/(x^3*(a + b*ArcSec[c*x])^2),x]
[Out] (c*(-((b*Sqrt[1 - 1/(c^2*x^2)])/(a*x + b*x*ArcSec[c*x])) + c*Cos[(2*a)/b]*C
osIntegral[2*(a/b + ArcSec[c*x])] + c*Sin[(2*a)/b]*SinIntegral[2*(a/b + Arc
Sec[c*x])]))/b^2
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$c^2\left(-\frac{\sin(2\,\text{arcsec}(cx))}{2(a+b\,\text{arcsec}(cx))b} + \frac{\text{Si}\left(\frac{2a}{b}+2\,\text{arcsec}(cx)\right)\sin\left(\frac{2a}{b}\right)+\text{Ci}\left(\frac{2a}{b}+2\,\text{arcsec}(cx)\right)\cos\left(\frac{2a}{b}\right)}{b^2}\right)$	77
default	$c^2\left(-\frac{\sin(2\,\text{arcsec}(cx))}{2(a+b\,\text{arcsec}(cx))b} + \frac{\text{Si}\left(\frac{2a}{b}+2\,\text{arcsec}(cx)\right)\sin\left(\frac{2a}{b}\right)+\text{Ci}\left(\frac{2a}{b}+2\,\text{arcsec}(cx)\right)\cos\left(\frac{2a}{b}\right)}{b^2}\right)$	77

```
[In] int(1/x^3/(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*sin(2*arcsec(c*x))/(a+b*arcsec(c*x))/b+(Si(2*a/b+2*arcsec(c*x))*s
in(2*a/b)+Ci(2*a/b+2*arcsec(c*x))*cos(2*a/b))/b^2)
```

Fricas [F]

$$\int \frac{1}{x^3 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2 x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x^3*arcsec(c*x)^2 + 2*a*b*x^3*arcsec(c*x) + a^2*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^3 \left(a+b\,\mathrm{asec}\left(cx\right)\right)^2}\,dx$$

```
[In] integrate(1/x**3/(a+b*asec(c*x))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*asec(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2 x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a)
 + (4*b^3*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^2*log(c^2*x^2)^
2 + 8*b^3*x^2*log(c)*log(x) + 4*b^3*x^2*log(x)^2 + 8*a*b^2*x^2*arctan(sqrt(
c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^2 - 4*(b^3*x^2*log(c)
+ b^3*x^2*log(x))*log(c^2*x^2))*integrate(4*(a*c^2*x^2 + (b*c^2*x^2 - 2*b)*
arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 2*a)*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*(
b^3*c^2*log(c)^2 + a^2*b*c^2)*x^5 - 4*(b^3*log(c)^2 + a^2*b)*x^3 + 4*(b^3*c
^2*x^5 - b^3*x^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + (b^3*c^2*x^5 - b^
3*x^3)*log(c^2*x^2)^2 + 4*(b^3*c^2*x^5 - b^3*x^3)*log(x)^2 + 8*(a*b^2*c^2*x
^5 - a*b^2*x^3)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*c^2*x^5*log(c)
 - b^3*x^3*log(c) + (b^3*c^2*x^5 - b^3*x^3)*log(x))*log(c^2*x^2) + 8*(b^3*c
^2*x^5*log(c) - b^3*x^3*log(c))*log(x)), x))/(4*b^3*x^2*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1))^2 + b^3*x^2*log(c^2*x^2)^2 + 8*b^3*x^2*log(c)*log(x) + 4*b
^3*x^2*log(x)^2 + 8*a*b^2*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*
log(c)^2 + a^2*b)*x^2 - 4*(b^3*x^2*log(c) + b^3*x^2*log(x))*log(c^2*x^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(82) = 164.

Time = 0.27 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.25

$$\int \frac{1}{x^3 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \left(\frac{2\,bc\arccos\left(\frac{1}{cx}\right)\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos\left(\frac{1}{cx}\right)\right)}{b^3\arccos\left(\frac{1}{cx}\right)+ab^2} + \frac{2\,bc\arccos\left(\frac{1}{cx}\right)\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos\left(\frac{1}{cx}\right)\right.}{b^3\arccos\left(\frac{1}{cx}\right)+ab^2}\right.$$

```
[In] integrate(1/x^3/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

```
[Out] (2*b*c*arccos(1/(c*x))*cos(a/b)^2*cos_integral(2*a/b + 2*arccos(1/(c*x)))/(
b^3*arccos(1/(c*x)) + a*b^2) + 2*b*c*arccos(1/(c*x))*cos(a/b)*sin(a/b)*sin_
integral(2*a/b + 2*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + 2*a*c*c
os(a/b)^2*cos_integral(2*a/b + 2*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*
b^2) + 2*a*c*cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(1/(c*x)))/(b^3
*arccos(1/(c*x)) + a*b^2) - b*c*arccos(1/(c*x))*cos_integral(2*a/b + 2*arcc
os(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - a*c*cos_integral(2*a/b + 2*arc
cos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - b*sqrt(-1/(c^2*x^2) + 1)/((b^
3*arccos(1/(c*x)) + a*b^2)*x))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^3\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int(1/(x^3*(a + b*acos(1/(c*x)))^2),x)
```

```
[Out] int(1/(x^3*(a + b*acos(1/(c*x)))^2), x)
```

3.44 $\int \frac{1}{x^4(a+b\sec^{-1}(cx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 178

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^2}\,dx = -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{4b\left(a+b\sec^{-1}(cx)\right)}+\frac{c^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2}+\frac{3c^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2}-\frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)}+\frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2}+\frac{3c^3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2}$$

```
[Out] 1/4*c^3*Ci(a/b+arcsec(c*x))*cos(a/b)/b^2+3/4*c^3*Ci(3*a/b+3*arcsec(c*x))*co
s(3*a/b)/b^2+1/4*c^3*Si(a/b+arcsec(c*x))*sin(a/b)/b^2+3/4*c^3*Si(3*a/b+3*ar
csec(c*x))*sin(3*a/b)/b^2-1/4*c^3*sin(3*arcsec(c*x))/b/(a+b*arcsec(c*x))-1/
4*c^3*(1-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{5330, 4491, 3378, 3384, 3380, 3383\}$

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{c^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2} + \frac{3c^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2} + \frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2} + \frac{3c^3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2} - \frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)} - \frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{4b\left(a+b\sec^{-1}(cx)\right)}$$

```
[In] Int[1/(x^4*(a + b*ArcSec[c*x])^2),x]
```

```
[Out] -1/4*(c^3*Sqrt[1 - 1/(c^2*x^2)])/(b*(a + b*ArcSec[c*x])) + (c^3*Cos[a/b]*Co
sIntegral[a/b + ArcSec[c*x]])/(4*b^2) + (3*c^3*Cos[(3*a)/b]*CosIntegral[(3*
a)/b + 3*ArcSec[c*x]])/(4*b^2) - (c^3*Sin[3*ArcSec[c*x]])/(4*b*(a + b*ArcSe
c[c*x])) + (c^3*Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]])/(4*b^2) + (3*c^3*S
in[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcSec[c*x]])/(4*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^3\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)\\
&= c^3\text{Subst}\left(\int \left(\frac{\sin(x)}{4(a+bx)^2}+\frac{\sin(3x)}{4(a+bx)^2}\right)\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{1}{4}c^3\text{Subst}\left(\int \frac{\sin(x)}{(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)+\frac{1}{4}c^3\text{Subst}\left(\int \frac{\sin(3x)}{(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)\\
&= -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{4b\left(a+b\sec^{-1}(cx)\right)}-\frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)}\\
&\quad+\frac{c^3\text{Subst}\left(\int \frac{\cos(x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{4b}+\frac{(3c^3)\,\text{Subst}\left(\int \frac{\cos(3x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{4b}\\
&= -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{4b\left(a+b\sec^{-1}(cx)\right)}-\frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)}\\
&\quad+\frac{\left(c^3\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{4b}\\
&\quad+\frac{\left(3c^3\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{4b}\\
&\quad+\frac{\left(c^3\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{4b}\\
&\quad+\frac{\left(3c^3\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{4b}
\end{aligned}$$

$$= -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{4b\left(a+b\sec^{-1}(cx)\right)} + \frac{c^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2} + \frac{3c^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2} - \frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)} + \frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{4b^2} + \frac{3c^3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{4b^2}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \frac{-4bc\sqrt{1-\frac{1}{c^2x^2}} + c^3x^2(a+b\sec^{-1}(cx))\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right) + 3c^3x^2(a+b\sec^{-1}(cx))\cos}{}$$

```
[In] Integrate[1/(x^4*(a + b*ArcSec[c*x])^2),x]
```

```
[Out] (-4*b*c*Sqrt[1 - 1/(c^2*x^2)] + c^3*x^2*(a + b*ArcSec[c*x])*Cos[a/b]*CosInt
egral[a/b + ArcSec[c*x]] + 3*c^3*x^2*(a + b*ArcSec[c*x])*Cos[(3*a)/b]*CosIn
tegral[3*(a/b + ArcSec[c*x])] + a*c^3*x^2*Sin[a/b]*SinIntegral[a/b + ArcSec
[c*x]] + b*c^3*x^2*ArcSec[c*x]*Sin[a/b]*SinIntegral[a/b + ArcSec[c*x]] + 3*
a*c^3*x^2*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])] + 3*b*c^3*x^2*Arc
Sec[c*x]*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSec[c*x])])/(4*b^2*x^2*(a + b
*ArcSec[c*x]))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.86

method	result
derivativedivides	$c^3\left(-\frac{\sin(3\,\text{arcsec}(cx))}{4(a+b\,\text{arcsec}(cx))b} + \frac{\frac{3\,\text{Si}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4}+\frac{3\,\text{Ci}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4}}{b^2} - \frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4(a+b\,\text{arcsec}(cx))b}\right.$
default	$c^3\left(-\frac{\sin(3\,\text{arcsec}(cx))}{4(a+b\,\text{arcsec}(cx))b} + \frac{\frac{3\,\text{Si}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4}+\frac{3\,\text{Ci}\left(\frac{3a}{b}+3\,\text{arcsec}(cx)\right)\cos\left(\frac{3a}{b}\right)}{4}}{b^2} - \frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4(a+b\,\text{arcsec}(cx))b}\right.$

```
[In] int(1/x^4/(a+b*arcsec(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/4*sin(3*arcsec(c*x))/(a+b*arcsec(c*x))/b+3/4*(Si(3*a/b+3*arcsec(c*x
))*sin(3*a/b)+Ci(3*a/b+3*arcsec(c*x))*cos(3*a/b))/b^2-1/4*((c^2*x^2-1)/c^2/
x^2)^(1/2)/(a+b*arcsec(c*x))/b+1/4*(Si(a/b+arcsec(c*x))*sin(a/b)+Ci(a/b+arc
sec(c*x))*cos(a/b))/b^2)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\mathrm{arcsec}\,(cx)+a\right)^2 x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^4*arcsec(c*x)^2 + 2*a*b*x^4*arcsec(c*x) + a^2*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{x^4 \left(a+b\,\mathrm{asec}\,(cx)\right)^2}\,dx$$

```
[In] integrate(1/x**4/(a+b*asec(c*x))**2,x)
[Out] Integral(1/(x**4*(a + b*asec(c*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(b\,\mathrm{arcsec}\,(cx)+a\right)^2 x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
[Out] -(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a)
 + (4*b^3*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*x^3*log(c^2*x^2)^
2 + 8*b^3*x^3*log(c)*log(x) + 4*b^3*x^3*log(x)^2 + 8*a*b^2*x^3*arctan(sqrt(
c*x + 1)*sqrt(c*x - 1)) + 4*(b^3*log(c)^2 + a^2*b)*x^3 - 4*(b^3*x^3*log(c)
+ b^3*x^3*log(x))*log(c^2*x^2))*integrate(4*(2*a*c^2*x^2 + (2*b*c^2*x^2 - 3
*b)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*a)*sqrt(c*x + 1)*sqrt(c*x - 1)/
(4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^6 - 4*(b^3*log(c)^2 + a^2*b)*x^4 + 4*(b
^3*c^2*x^6 - b^3*x^4)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + (b^3*c^2*x^6
- b^3*x^4)*log(c^2*x^2)^2 + 4*(b^3*c^2*x^6 - b^3*x^4)*log(x)^2 + 8*(a*b^2*c
^2*x^6 - a*b^2*x^4)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*c^2*x^6*lo
g(c) - b^3*x^4*log(c) + (b^3*c^2*x^6 - b^3*x^4)*log(x))*log(c^2*x^2) + 8*(b
^3*c^2*x^6*log(c) - b^3*x^4*log(c))*log(x)), x))/(4*b^3*x^3*arctan(sqrt(c*x
 + 1)*sqrt(c*x - 1))^2 + b^3*x^3*log(c^2*x^2)^2 + 8*b^3*x^3*log(c)*log(x) +
 4*b^3*x^3*log(x)^2 + 8*a*b^2*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*(
b^3*log(c)^2 + a^2*b)*x^3 - 4*(b^3*x^3*log(c) + b^3*x^3*log(x))*log(c^2*x^2
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(164) = 328$.

Time = 0.30 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.90

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))^2} \, dx$$
$$= \frac{1}{4} \left(\frac{12 \, bc^2 \arccos\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right)^3 \operatorname{Ci}\left(\frac{3a}{b} + 3 \arccos\left(\frac{1}{cx}\right)\right)}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2} + \frac{12 \, bc^2 \arccos\left(\frac{1}{cx}\right) \cos\left(\frac{a}{b}\right)^2 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{3a}{b} + 3 \operatorname{arcc}\right.}{b^3 \arccos\left(\frac{1}{cx}\right) + ab^2} \right.$$

```
[In] integrate(1/x^4/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/4*(12*b*c^2*arccos(1/(c*x))*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(1/(c
*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + 12*b*c^2*arccos(1/(c*x))*cos(a/b)^2*s
in(a/b)*sin_integral(3*a/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^
2) + 12*a*c^2*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(1/(c*x)))/(b^3*arcco
s(1/(c*x)) + a*b^2) + 12*a*c^2*cos(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*a
rccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - 9*b*c^2*arccos(1/(c*x))*cos
(a/b)*cos_integral(3*a/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2)
 + b*c^2*arccos(1/(c*x))*cos(a/b)*cos_integral(a/b + arccos(1/(c*x)))/(b^3*
arccos(1/(c*x)) + a*b^2) - 3*b*c^2*arccos(1/(c*x))*sin(a/b)*sin_integral(3*
a/b + 3*arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + b*c^2*arccos(1/(c*
x))*sin(a/b)*sin_integral(a/b + arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b
^2) - 9*a*c^2*cos(a/b)*cos_integral(3*a/b + 3*arccos(1/(c*x)))/(b^3*arccos(
1/(c*x)) + a*b^2) + a*c^2*cos(a/b)*cos_integral(a/b + arccos(1/(c*x)))/(b^3
*arccos(1/(c*x)) + a*b^2) - 3*a*c^2*sin(a/b)*sin_integral(3*a/b + 3*arccos(
1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) + a*c^2*sin(a/b)*sin_integral(a/b +
 arccos(1/(c*x)))/(b^3*arccos(1/(c*x)) + a*b^2) - 4*b*sqrt(-1/(c^2*x^2) + 1
)/((b^3*arccos(1/(c*x)) + a*b^2)*x^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))^2} \, dx = \int \frac{1}{x^4 \left(a + b \operatorname{acos}\left(\frac{1}{c x}\right)\right)^2} \, dx$$

```
[In] int(1/(x^4*(a + b*acos(1/(c*x)))^2),x)
```

```
[Out] int(1/(x^4*(a + b*acos(1/(c*x)))^2), x)
```

3.45 $\int \frac{x}{\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \operatorname{Int}\left(\frac{x}{(a+b\sec^{-1}(cx))^3},x\right)$$

```
[Out] Unintegrable(x/(a+b*arcsec(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx$$

```
[In] Int[x/(a + b*ArcSec[c*x])^3,x]
```

```
[Out] Defer[Int][x/(a + b*ArcSec[c*x])^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx=\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx$$

```
[In] Integrate[x/(a + b*ArcSec[c*x])^3,x]
[Out] Integrate[x/(a + b*ArcSec[c*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\,\mathrm{arcsec}\,(cx))^3}dx$$

```
[In] int(x/(a+b*arcsec(c*x))^3,x)
[Out] int(x/(a+b*arcsec(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx=\int \frac{x}{(b\,\mathrm{arcsec}\,(cx)+a)^3}\,dx$$

```
[In] integrate(x/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
[Out] integral(x/(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x)
 + a^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx=\int \frac{x}{(a+b\,\mathrm{asec}\,(cx))^3}\,dx$$

```
[In] integrate(x/(a+b*asec(c*x))**3,x)
[Out] Integral(x/(a + b*asec(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 29.31 (sec) , antiderivative size = 1790, normalized size of antiderivative = 149.17

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx=\int \frac{x}{(b\,\mathrm{arcsec}\,(cx)+a)^3}\,dx$$

```
[In] integrate(x/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
[Out] -(24*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^4 + 8*(3*b^3*c^2*x^4 - 2*b^3*x^2)*arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 16*(a*b^2*log(c)^2 + a^3)*x^2 + 24*(3*
a*b^2*c^2*x^4 - 2*a*b^2*x^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(3*a
*b^2*c^2*x^4 - 2*a*b^2*x^2)*log(c^2*x^2)^2 + 8*(3*a*b^2*c^2*x^4 - 2*a*b^2*x
^2)*log(x)^2 + 2*(4*b^3*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*x^2
*log(c^2*x^2)^2 - 8*b^3*x^2*log(c)*log(x) - 4*b^3*x^2*log(x)^2 + 8*a*b^2*x^
2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*log(c)^2 - a^2*b)*x^2 + 4*(b
^3*x^2*log(c) + b^3*x^2*log(x))*log(c^2*x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) +
 2*(12*(b^3*c^2*log(c)^2 + 3*a^2*b*c^2)*x^4 - 8*(b^3*log(c)^2 + 3*a^2*b)*x^
2 + (3*b^3*c^2*x^4 - 2*b^3*x^2)*log(c^2*x^2)^2 + 4*(3*b^3*c^2*x^4 - 2*b^3*x
^2)*log(x)^2 - 4*(3*b^3*c^2*x^4*log(c) - 2*b^3*x^2*log(c) + (3*b^3*c^2*x^4
- 2*b^3*x^2)*log(x))*log(c^2*x^2) + 8*(3*b^3*c^2*x^4*log(c) - 2*b^3*x^2*log
(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*arctan(sqrt(c*x
+ 1)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b^6*log(c
)*log(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
^3 + 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^6*log(x))*log(c^2
*x^2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*log(c)*log(x) + 4*
b^6*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*log(c^2*x^2))*arcta
n(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6*b^6*log(c)*log(x)
+ 3*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*
log(x)^2 + 16*(a*b^5*log(c^2*x^2)^2 + 4*a*b^5*log(c)^2 + 8*a*b^5*log(c)*log
(x) + 4*a*b^5*log(x)^2 + 4*a^3*b^3 - 4*(a*b^5*log(c) + a*b^5*log(x))*log(c^
```

```
2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(b^6*log(c)^3 + 3*b^6*log(
c)*log(x)^2 + b^6*log(x)^3 + a^2*b^4*log(c) + (3*b^6*log(c)^2 + a^2*b^4)*lo
g(x))*log(c^2*x^2) + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*log(x))*integrate(8
*(3*a*c^2*x^3 - a*x + (3*b*c^2*x^3 - b*x)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
)))/(4*b^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^4*log(c^2*x^2)^2 + 4*b
^4*log(c)^2 + 8*b^4*log(c)*log(x) + 4*b^4*log(x)^2 + 8*a*b^3*arctan(sqrt(c*
x + 1)*sqrt(c*x - 1)) + 4*a^2*b^2 - 4*(b^4*log(c) + b^4*log(x))*log(c^2*x^2
)), x) - 8*(3*a*b^2*c^2*x^4*log(c) - 2*a*b^2*x^2*log(c) + (3*a*b^2*c^2*x^4
- 2*a*b^2*x^2)*log(x))*log(c^2*x^2) + 16*(3*a*b^2*c^2*x^4*log(c) - 2*a*b^2*
x^2*log(c))*log(x))/(16*b^6*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*log
(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 +
64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*a
^4*b^2 - 8*(b^6*log(c) + b^6*log(x))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^2
 + 4*b^6*log(c)^2 + 8*b^6*log(c)*log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*(
b^6*log(c) + b^6*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^
2 + 8*(3*b^6*log(c)^2 + 6*b^6*log(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*log
(c^2*x^2)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*log(x)^2 + 16*(a*b^5*log(c^2*x^
2)^2 + 4*a*b^5*log(c)^2 + 8*a*b^5*log(c)*log(x) + 4*a*b^5*log(x)^2 + 4*a^3*
b^3 - 4*(a*b^5*log(c) + a*b^5*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1)) - 32*(b^6*log(c)^3 + 3*b^6*log(c)*log(x)^2 + b^6*log(x)^3 + a^
2*b^4*log(c) + (3*b^6*log(c)^2 + a^2*b^4)*log(x))*log(c^2*x^2) + 64*(b^6*lo
g(c)^3 + a^2*b^4*log(c))*log(x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(a+b*arcsec(c*x))^3,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Not invertible Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{x}{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

```
[In] int(x/(a + b*acos(1/(c*x)))^3,x)
[Out] int(x/(a + b*acos(1/(c*x)))^3, x)
```

3.46 $\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \operatorname{Int}\left(\frac{1}{(a+b\sec^{-1}(cx))^3},x\right)$$

```
[Out] Unintegrable(1/(a+b*arcsec(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])^(-3),x]
```

```
[Out] Defer[Int][(a + b*ArcSec[c*x])^(-3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 9.57 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])^(-3),x]
[Out] Integrate[(a + b*ArcSec[c*x])^(-3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\ \mathrm{arcsec}\,(cx))^3}dx$$

```
[In] int(1/(a+b*arcsec(c*x))^3,x)
[Out] int(1/(a+b*arcsec(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 4.00

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x)
 + a^3), x)
```

Sympy [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(a+b\operatorname{asec}(cx))^3}\,dx$$

```
[In] integrate(1/(a+b*asec(c*x))**3,x)
```

```
[Out] Integral((a + b*asec(c*x))**(-3), x)
```

Maxima [N/A]

Not integrable

Time = 28.50 (sec) , antiderivative size = 1744, normalized size of antiderivative = 174.40

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(b\operatorname{arcsec}(cx)+a)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

```
[Out] -(16*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^3 + 8*(2*b^3*c^2*x^3 - b^3*x)*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 24*(2*a*b^2*c^2*x^3 - a*b^2*x)*arctan(sqrt
(c*x + 1)*sqrt(c*x - 1))^2 + 2*(2*a*b^2*c^2*x^3 - a*b^2*x)*log(c^2*x^2)^2 +
 8*(2*a*b^2*c^2*x^3 - a*b^2*x)*log(x)^2 + 2*(4*b^3*x*arctan(sqrt(c*x + 1)*s
qrt(c*x - 1))^2 - b^3*x*log(c^2*x^2)^2 - 8*b^3*x*log(c)*log(x) - 4*b^3*x*lo
g(x)^2 + 8*a*b^2*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(b^3*log(c)^2 -
a^2*b)*x + 4*(b^3*x*log(c) + b^3*x*log(x))*log(c^2*x^2))*sqrt(c*x + 1)*sqrt
(c*x - 1) - 8*(a*b^2*log(c)^2 + a^3)*x + 2*(8*(b^3*c^2*log(c)^2 + 3*a^2*b*c
^2)*x^3 + (2*b^3*c^2*x^3 - b^3*x)*log(c^2*x^2)^2 + 4*(2*b^3*c^2*x^3 - b^3*x
)*log(x)^2 - 4*(b^3*log(c)^2 + 3*a^2*b)*x - 4*(2*b^3*c^2*x^3*log(c) - b^3*x
*log(c) + (2*b^3*c^2*x^3 - b^3*x)*log(x))*log(c^2*x^2) + 8*(2*b^3*c^2*x^3*l
og(c) - b^3*x*log(c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6
*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)
^4 + 64*b^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^
6*log(x))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*l
og(c)*log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*lo
g(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6*b
^6*log(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b^6*log
(c)^2 + a^2*b^4)*log(x)^2 + 16*(a*b^5*log(c^2*x^2)^2 + 4*a*b^5*log(c)^2 + 8
*a*b^5*log(c)*log(x) + 4*a*b^5*log(x)^2 + 4*a^3*b^3 - 4*(a*b^5*log(c) + a*b
^5*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(b^6*log(
```

```
c)^3 + 3*b^6*log(c)*log(x)^2 + b^6*log(x)^3 + a^2*b^4*log(c) + (3*b^6*log(c
)^2 + a^2*b^4)*log(x))*log(c^2*x^2) + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*lo
g(x))*integrate(2*(6*a*c^2*x^2 + (6*b*c^2*x^2 - b)*arctan(sqrt(c*x + 1)*sqr
t(c*x - 1)) - a)/(4*b^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^4*log(c^2
*x^2)^2 + 4*b^4*log(c)^2 + 8*b^4*log(c)*log(x) + 4*b^4*log(x)^2 + 8*a*b^3*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b^2 - 4*(b^4*log(c) + b^4*log(x)
)*log(c^2*x^2)), x) - 8*(2*a*b^2*c^2*x^3*log(c) - a*b^2*x*log(c) + (2*a*b^2
*c^2*x^3 - a*b^2*x)*log(x))*log(c^2*x^2) + 16*(2*a*b^2*c^2*x^3*log(c) - a*b
^2*x*log(c))*log(x))/(16*b^6*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*lo
g(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b^6*log(c)*log(x)^3 + 16*b^6*log(x)^4 +
 64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*a^2*b^4*log(c)^2 + 16*
a^4*b^2 - 8*(b^6*log(c) + b^6*log(x))*log(c^2*x^2)^3 + 8*(b^6*log(c^2*x^2)^
2 + 4*b^6*log(c)^2 + 8*b^6*log(c)*log(x) + 4*b^6*log(x)^2 + 12*a^2*b^4 - 4*
(b^6*log(c) + b^6*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
^2 + 8*(3*b^6*log(c)^2 + 6*b^6*log(c)*log(x) + 3*b^6*log(x)^2 + a^2*b^4)*lo
g(c^2*x^2)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*log(x)^2 + 16*(a*b^5*log(c^2*x
^2)^2 + 4*a*b^5*log(c)^2 + 8*a*b^5*log(c)*log(x) + 4*a*b^5*log(x)^2 + 4*a^3
*b^3 - 4*(a*b^5*log(c) + a*b^5*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*s
qrt(c*x - 1)) - 32*(b^6*log(c)^3 + 3*b^6*log(c)*log(x)^2 + b^6*log(x)^3 + a
^2*b^4*log(c) + (3*b^6*log(c)^2 + a^2*b^4)*log(x))*log(c^2*x^2) + 64*(b^6*l
og(c)^3 + a^2*b^4*log(c))*log(x))
```

Giac [N/A]

Not integrable

Time = 52.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)^3}\,dx$$

```
[In] integrate(1/(a+b*arcsec(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)^(-3), x)
```

Mupad [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

```
[In] int(1/(a + b*acos(1/(c*x)))^3,x)
[Out] int(1/(a + b*acos(1/(c*x)))^3, x)
```

3.47 $\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3} dx = \text{Int}\left(\frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}, x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arcsec(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3} dx = \int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3} dx$$

```
[In] Int[1/(x*(a + b*ArcSec[c*x])^3),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcSec[c*x])^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3} dx$$

Mathematica [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcSec[c*x])^3),x]
[Out] Integrate[1/(x*(a + b*ArcSec[c*x])^3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \mathrm{arcsec}\left(cx\right)\right)^{3}}dx$$

```
[In] int(1/x/(a+b*arcsec(c*x))^3,x)
[Out] int(1/x/(a+b*arcsec(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.21

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^{3}x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*x*arcsec(c*x)^3 + 3*a*b^2*x*arcsec(c*x)^2 + 3*a^2*b*x*arcse
c(c*x) + a^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\left(a+b\operatorname{asec}\left(cx\right)\right)^3}\,dx$$

```
[In] integrate(1/x/(a+b*asec(c*x))**3,x)
[Out] Integral(1/(x*(a + b*asec(c*x))**3), x)
```

Maxima [N/A]

Not integrable

Time = 21.87 (sec) , antiderivative size = 1568, normalized size of antiderivative = 112.00

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\operatorname{arcsec}\left(cx\right)+a\right)^3x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
[Out] -(8*b^3*c^2*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 24*a*b^2*c^2*x^2*ar
ctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*a*b^2*c^2*x^2*log(c^2*x^2)^2 + 16*a
*b^2*c^2*x^2*log(c)*log(x) + 8*a*b^2*c^2*x^2*log(x)^2 + 8*(a*b^2*c^2*log(c)
^2 + a^3*c^2)*x^2 + 2*(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*lo
g(c^2*x^2)^2 - 4*b^3*log(c)^2 - 8*b^3*log(c)*log(x) - 4*b^3*log(x)^2 + 8*a*
b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b + 4*(b^3*log(c) + b^3*log
(x))*log(c^2*x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*(b^3*c^2*x^2*log(c^2*x^2
)^2 + 8*b^3*c^2*x^2*log(c)*log(x) + 4*b^3*c^2*x^2*log(x)^2 + 4*(b^3*c^2*log
(c)^2 + 3*a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2*log(c) + b^3*c^2*x^2*log(x))*log(
c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*arctan(sqrt(c*x + 1
)*sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b^6*log(c)*l
og(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3
+ 32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^6*log(x))*log(c^2*x^
2)^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*log(c)*log(x) + 4*b^6
*log(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*log(c^2*x^2))*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6*b^6*log(c)*log(x) + 3
*b^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*log
(x)^2 + 16*(a*b^5*log(c^2*x^2)^2 + 4*a*b^5*log(c)^2 + 8*a*b^5*log(c)*log(x)
 + 4*a*b^5*log(x)^2 + 4*a^3*b^3 - 4*(a*b^5*log(c) + a*b^5*log(x))*log(c^2*x
^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(b^6*log(c)^3 + 3*b^6*log(c)*
log(x)^2 + b^6*log(x)^3 + a^2*b^4*log(c) + (3*b^6*log(c)^2 + a^2*b^4)*log(x
))*log(c^2*x^2) + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*log(x))*integrate(4*(b
```

```
*c^2*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*c^2*x)/(4*b^4*arctan(sqrt(c*
x + 1)*sqrt(c*x - 1))^2 + b^4*log(c^2*x^2)^2 + 4*b^4*log(c)^2 + 8*b^4*log(c
)*log(x) + 4*b^4*log(x)^2 + 8*a*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4
*a^2*b^2 - 4*(b^4*log(c) + b^4*log(x))*log(c^2*x^2)), x) - 8*(a*b^2*c^2*x^2
*log(c) + a*b^2*c^2*x^2*log(x))*log(c^2*x^2))/(16*b^6*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^4 + b^6*log(c^2*x^2)^4 + 16*b^6*log(c)^4 + 64*b^6*log(c)*log
(x)^3 + 16*b^6*log(x)^4 + 64*a*b^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 +
32*a^2*b^4*log(c)^2 + 16*a^4*b^2 - 8*(b^6*log(c) + b^6*log(x))*log(c^2*x^2)
^3 + 8*(b^6*log(c^2*x^2)^2 + 4*b^6*log(c)^2 + 8*b^6*log(c)*log(x) + 4*b^6*l
og(x)^2 + 12*a^2*b^4 - 4*(b^6*log(c) + b^6*log(x))*log(c^2*x^2))*arctan(sqr
t(c*x + 1)*sqrt(c*x - 1))^2 + 8*(3*b^6*log(c)^2 + 6*b^6*log(c)*log(x) + 3*b
^6*log(x)^2 + a^2*b^4)*log(c^2*x^2)^2 + 32*(3*b^6*log(c)^2 + a^2*b^4)*log(x
)^2 + 16*(a*b^5*log(c^2*x^2)^2 + 4*a*b^5*log(c)^2 + 8*a*b^5*log(c)*log(x) +
 4*a*b^5*log(x)^2 + 4*a^3*b^3 - 4*(a*b^5*log(c) + a*b^5*log(x))*log(c^2*x^2
))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(b^6*log(c)^3 + 3*b^6*log(c)*lo
g(x)^2 + b^6*log(x)^3 + a^2*b^4*log(c) + (3*b^6*log(c)^2 + a^2*b^4)*log(x))
*log(c^2*x^2) + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*log(x))
```

Giac [N/A]

Not integrable

Time = 7.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\operatorname{arcsec}\left(cx\right)+a\right)^3x}\,dx$$

```
[In] integrate(1/x/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsec(c*x) + a)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x\,{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}^3}\,dx$$

```
[In] int(1/(x*(a + b*acos(1/(c*x)))^3),x)
```

```
[Out] int(1/(x*(a + b*acos(1/(c*x)))^3), x)
```

3.48 $\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 103

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx=-\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{1}{2b^2x\left(a+b\sec^{-1}(cx)\right)}+\frac{c\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{2b^3}-\frac{c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{2b^3}$$

```
[Out] -1/2/b^2/x/(a+b*arcsec(c*x))-1/2*c*cos(a/b)*Si(a/b+arcsec(c*x))/b^3+1/2*c*C
i(a/b+arcsec(c*x))*sin(a/b)/b^3-1/2*c*(1-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x
))^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5330, 3378, 3384, 3380, 3383}

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\frac{c\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{2b^3}-\frac{c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{2b^3}-\frac{1}{2b^2x\left(a+b\sec^{-1}(cx)\right)}-\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2}$$

```
[In] Int[1/(x^2*(a + b*ArcSec[c*x])^3),x]
```

```
[Out] -1/2*(c*Sqrt[1 - 1/(c^2*x^2)])/(b*(a + b*ArcSec[c*x])^2) - 1/(2*b^2*x*(a +
b*ArcSec[c*x])) + (c*CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/(2*b^3) - (c*
Cos[a/b]*SinIntegral[a/b + ArcSec[c*x]])/(2*b^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$\text{integral} = c\text{Subst}\left(\int \frac{\sin(x)}{(a+bx)^3}\,dx, x, \sec^{-1}(cx)\right)$$

$$= -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2} + \frac{c\text{Subst}\left(\int \frac{\cos(x)}{(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)}{2b}$$

$$= -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{1}{2b^2x\left(a+b\sec^{-1}(cx)\right)} - \frac{c\text{Subst}\left(\int \frac{\sin(x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{2b^2}$$

$$= -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{1}{2b^2x\left(a+b\sec^{-1}(cx)\right)} - \frac{\left(c\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sec^{-1}(cx)\right)}{2b^2} + \frac{\left(c\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sec^{-1}(cx)\right)}{2b^2}$$

$$= -\frac{c\sqrt{1-\frac{1}{c^2x^2}}}{2b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{1}{2b^2x\left(a+b\sec^{-1}(cx)\right)} + \frac{c\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{2b^3} - \frac{c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{2b^3}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int\frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx = -\frac{\frac{b\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}x+b\sec^{-1}(cx)\right)}{x\left(a+b\sec^{-1}(cx)\right)^2} - c\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right) + c\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{2b^3}$$

```
[In] Integrate[1/(x^2*(a + b*ArcSec[c*x])^3),x]
```

```
[Out] -1/2*((b*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b*ArcSec[c*x]))/(x*(a + b*ArcSe
c[c*x])^2) - c*CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b] + c*Cos[a/b]*SinInte
gral[a/b + ArcSec[c*x]])/b^3
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

method	result
derivativedivides	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2(a+b\,\text{arcsec}(cx))^2b} - \frac{\text{arcsec}(cx)\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\text{arcsec}(cx)\right)bcx-\text{arcsec}(cx)\sin\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\text{arcsec}(cx)\right)bcx+\cos\left(\frac{a}{b}\right.}{2cx(a+b\,\text{arcsec}(cx))b^3}\right.$
default	$c\left(-\frac{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2(a+b\,\text{arcsec}(cx))^2b} - \frac{\text{arcsec}(cx)\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\text{arcsec}(cx)\right)bcx-\text{arcsec}(cx)\sin\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\text{arcsec}(cx)\right)bcx+\cos\left(\frac{a}{b}\right.}{2cx(a+b\,\text{arcsec}(cx))b^3}\right.$

```
[In] int(1/x^2/(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/2*((c^2*x^2-1)/c^2/x^2)^(1/2)/(a+b*arcsec(c*x))^2/b-1/2*(arcsec(c*x)*
cos(a/b)*Si(a/b+arcsec(c*x))*b*c*x-arcsec(c*x)*sin(a/b)*Ci(a/b+arcsec(c*x))
```

```
*b*c*x+cos(a/b)*Si(a/b+arcsec(c*x))*a*c*x-sin(a/b)*Ci(a/b+arcsec(c*x))*a*c*
x+b)/c/x/(a+b*arcsec(c*x))/b^3)
```

Fricas [F]

$$\int \frac{1}{x^2 (a + b \sec^{-1}(cx))^3} dx = \int \frac{1}{(b \operatorname{arcsec}(cx) + a)^3 x^2} dx$$

```
[In] integrate(1/x^2/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^2*arcsec(c*x)^3 + 3*a*b^2*x^2*arcsec(c*x)^2 + 3*a^2*b*x^2
*arcsec(c*x) + a^3*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a + b \sec^{-1}(cx))^3} dx = \int \frac{1}{x^2 (a + b \operatorname{asec}(cx))^3} dx$$

```
[In] integrate(1/x**2/(a+b*asec(c*x))**3,x)
```

```
[Out] Integral(1/(x**2*(a + b*asec(c*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + b \sec^{-1}(cx))^3} dx = \int \frac{1}{(b \operatorname{arcsec}(cx) + a)^3 x^2} dx$$

```
[In] integrate(1/x^2/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

```
[Out] -(8*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 24*a*b^2*arctan(sqrt(c*x +
1)*sqrt(c*x - 1))^2 + 2*a*b^2*log(c^2*x^2)^2 + 8*a*b^2*log(c)^2 + 16*a*b^2*
log(c)*log(x) + 8*a*b^2*log(x)^2 + 8*a^3 + 2*(4*b^3*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1))^2 - b^3*log(c^2*x^2)^2 - 4*b^3*log(c)^2 - 8*b^3*log(c)*log(x)
- 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b +
4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*(
b^3*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2
+ 12*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1)) + (16*b^6*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*x*l
og(c^2*x^2)^4 + 64*b^6*x*log(c)*log(x)^3 + 16*b^6*x*log(x)^4 + 64*a*b^5*x*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 8*(b^6*x*log(c) + b^6*x*log(x))*log(
c^2*x^2)^3 + 32*(3*b^6*log(c)^2 + a^2*b^4)*x*log(x)^2 + 8*(b^6*x*log(c^2*x^
```

```
2)^2 + 8*b^6*x*log(c)*log(x) + 4*b^6*x*log(x)^2 + 4*(b^6*log(c)^2 + 3*a^2*b
^4)*x - 4*(b^6*x*log(c) + b^6*x*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*
sqrt(c*x - 1))^2 + 8*(6*b^6*x*log(c)*log(x) + 3*b^6*x*log(x)^2 + (3*b^6*log
(c)^2 + a^2*b^4)*x)*log(c^2*x^2)^2 + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*x*l
og(x) + 16*(b^6*log(c)^4 + 2*a^2*b^4*log(c)^2 + a^4*b^2)*x + 16*(a*b^5*x*lo
g(c^2*x^2)^2 + 8*a*b^5*x*log(c)*log(x) + 4*a*b^5*x*log(x)^2 + 4*(a*b^5*log(
c)^2 + a^3*b^3)*x - 4*(a*b^5*x*log(c) + a*b^5*x*log(x))*log(c^2*x^2))*arcta
n(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(3*b^6*x*log(c)*log(x)^2 + b^6*x*log(x)
^3 + (3*b^6*log(c)^2 + a^2*b^4)*x*log(x) + (b^6*log(c)^3 + a^2*b^4*log(c))*
x)*log(c^2*x^2))*integrate(2*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a)/(4
*b^4*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^4*x^2*log(c^2*x^2)^2 + 8
*b^4*x^2*log(c)*log(x) + 4*b^4*x^2*log(x)^2 + 8*a*b^3*x^2*arctan(sqrt(c*x +
 1)*sqrt(c*x - 1)) + 4*(b^4*log(c)^2 + a^2*b^2)*x^2 - 4*(b^4*x^2*log(c) + b
^4*x^2*log(x))*log(c^2*x^2)), x) - 8*(a*b^2*log(c) + a*b^2*log(x))*log(c^2*
x^2))/(16*b^6*x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*x*log(c^2*x^2)^
4 + 64*b^6*x*log(c)*log(x)^3 + 16*b^6*x*log(x)^4 + 64*a*b^5*x*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1))^3 - 8*(b^6*x*log(c) + b^6*x*log(x))*log(c^2*x^2)^3 +
 32*(3*b^6*log(c)^2 + a^2*b^4)*x*log(x)^2 + 8*(b^6*x*log(c^2*x^2)^2 + 8*b^6
*x*log(c)*log(x) + 4*b^6*x*log(x)^2 + 4*(b^6*log(c)^2 + 3*a^2*b^4)*x - 4*(b
^6*x*log(c) + b^6*x*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
))^2 + 8*(6*b^6*x*log(c)*log(x) + 3*b^6*x*log(x)^2 + (3*b^6*log(c)^2 + a^2*
b^4)*x)*log(c^2*x^2)^2 + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*x*log(x) + 16*(
b^6*log(c)^4 + 2*a^2*b^4*log(c)^2 + a^4*b^2)*x + 16*(a*b^5*x*log(c^2*x^2)^2
 + 8*a*b^5*x*log(c)*log(x) + 4*a*b^5*x*log(x)^2 + 4*(a*b^5*log(c)^2 + a^3*b
^3)*x - 4*(a*b^5*x*log(c) + a*b^5*x*log(x))*log(c^2*x^2))*arctan(sqrt(c*x +
 1)*sqrt(c*x - 1)) - 32*(3*b^6*x*log(c)*log(x)^2 + b^6*x*log(x)^3 + (3*b^6*
log(c)^2 + a^2*b^4)*x*log(x) + (b^6*log(c)^3 + a^2*b^4*log(c))*x)*log(c^2*x
^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(93) = 186$.

Time = 0.31 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.63

$$\int \frac{1}{x^2 \left(a+b\sec^{-1}(cx)\right)^3}\,dx$$
$$= \frac{1}{2}\left(\frac{b^2 \arccos\left(\frac{1}{cx}\right)^2 \operatorname{Ci}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b^5 \arccos\left(\frac{1}{cx}\right)^2+2\,ab^4\arccos\left(\frac{1}{cx}\right)+a^2b^3} - \frac{b^2 \arccos\left(\frac{1}{cx}\right)^2 \cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos\left(\frac{1}{cx}\right)\right)}{b^5 \arccos\left(\frac{1}{cx}\right)^2+2\,ab^4\arccos\left(\frac{1}{cx}\right)+a^2b^3} + \frac{2\,ab\arccos}{b^5\arccos}\right.$$

```
[In] integrate(1/x^2/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*arccos(1/(c*x))^2*cos_integral(a/b + arccos(1/(c*x)))*sin(a/b)/(b^
5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - b^2*arccos(1/(c*
```

```
x))^2*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 +
 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 2*a*b*arccos(1/(c*x))*cos_integral(a/
b + arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*
x)) + a^2*b^3) - 2*a*b*arccos(1/(c*x))*cos(a/b)*sin_integral(a/b + arccos(1
/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + a^2*
cos_integral(a/b + arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b
^4*arccos(1/(c*x)) + a^2*b^3) - a^2*cos(a/b)*sin_integral(a/b + arccos(1/(c
*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - b^2*sqr
t(-1/(c^2*x^2) + 1)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*
b^3) - b^2*arccos(1/(c*x))/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)
) + a^2*b^3)*c*x) - a*b/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) +
 a^2*b^3)*c*x))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

```
[In] int(1/(x^2*(a + b*acos(1/(c*x)))^3),x)
[Out] int(1/(x^2*(a + b*acos(1/(c*x)))^3), x)
```

3.49 $\int \frac{1}{x^3(a+b\sec^{-1}(cx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 112

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx = -\frac{c^2\cos\left(2\sec^{-1}(cx)\right)}{2b^2\left(a+b\sec^{-1}(cx)\right)} + \frac{c^2\,\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{b^3} - \frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^3}$$

```
[Out] -1/2*c^2*cos(2*arcsec(c*x))/b^2/(a+b*arcsec(c*x))-c^2*cos(2*a/b)*Si(2*a/b+2
*arcsec(c*x))/b^3+c^2*Ci(2*a/b+2*arcsec(c*x))*sin(2*a/b)/b^3-1/4*c^2*sin(2*
arcsec(c*x))/b/(a+b*arcsec(c*x))^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5330, 4491, 12, 3378, 3384, 3380, 3383}

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \frac{c^2\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^3} - \frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^3} - \frac{c^2\cos\left(2\sec^{-1}(cx)\right)}{2b^2\left(a+b\sec^{-1}(cx)\right)} - \frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)^2}$$

```
[In] Int[1/(x^3*(a + b*ArcSec[c*x])^3),x]
```

```
[Out] -1/2*(c^2*Cos[2*ArcSec[c*x]])/(b^2*(a + b*ArcSec[c*x])) + (c^2*CosIntegral[
(2*a)/b + 2*ArcSec[c*x]]*Sin[(2*a)/b])/b^3 - (c^2*Sin[2*ArcSec[c*x]])/(4*b*
(a + b*ArcSec[c*x])^2) - (c^2*Cos[(2*a)/b]*SinIntegral[(2*a)/b + 2*ArcSec[c
*x]])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
```

```
LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^2\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{(a+bx)^3}\,dx, x, \sec^{-1}(cx)\right)\\
&= c^2\text{Subst}\left(\int \frac{\sin(2x)}{2(a+bx)^3}\,dx, x, \sec^{-1}(cx)\right)\\
&= \frac{1}{2}c^2\text{Subst}\left(\int \frac{\sin(2x)}{(a+bx)^3}\,dx, x, \sec^{-1}(cx)\right)\\
&= -\frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)^2} + \frac{c^2\text{Subst}\left(\int \frac{\cos(2x)}{(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)}{2b}\\
&= -\frac{c^2\cos\left(2\sec^{-1}(cx)\right)}{2b^2\left(a+b\sec^{-1}(cx)\right)} - \frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{c^2\text{Subst}\left(\int \frac{\sin(2x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{b^2}\\
&= -\frac{c^2\cos\left(2\sec^{-1}(cx)\right)}{2b^2\left(a+b\sec^{-1}(cx)\right)} - \frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)^2}\\
&\quad - \frac{\left(c^2\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{b^2}\\
&\quad + \frac{\left(c^2\sin\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{b^2}\\
&= -\frac{c^2\cos\left(2\sec^{-1}(cx)\right)}{2b^2\left(a+b\sec^{-1}(cx)\right)} + \frac{c^2\,\text{CosIntegral}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{b^3}\\
&\quad - \frac{c^2\sin\left(2\sec^{-1}(cx)\right)}{4b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\sec^{-1}(cx)\right)}{b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \frac{-\frac{b^2c\sqrt{1-\frac{1}{c^2x^2}}}{x\left(a+b\sec^{-1}(cx)\right)^2} + \frac{b\left(-2+c^2x^2\right)}{x^2\left(a+b\sec^{-1}(cx)\right)} + 2c^2\left(\text{CosIntegral}\left(2\left(\frac{a}{b}+\sec^{-1}(cx)\right)\right)\sin\left(\frac{2a}{b}\right) - \cos\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\sec^{-1}\right.\right.\right.}{2b^3}$$

```
[In] Integrate[1/(x^3*(a + b*ArcSec[c*x])^3),x]
```

```
[Out] (-((b^2*c*Sqrt[1 - 1/(c^2*x^2)])/(x*(a + b*ArcSec[c*x])^2)) + (b*(-2 + c^2*
x^2))/(x^2*(a + b*ArcSec[c*x])) + 2*c^2*(CosIntegral[2*(a/b + ArcSec[c*x])]
*Sin[(2*a)/b] - Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSec[c*x])]))/(2*b^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.40

method	result
derivativedivides	$c^2\left(-\frac{\sin(2\,\mathrm{arcsec}(cx))}{4(a+b\,\mathrm{arcsec}(cx))^2 b}-\frac{2\cos\left(\frac{2a}{b}\right)\mathrm{Si}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\mathrm{arcsec}(cx)b-2\sin\left(\frac{2a}{b}\right)\mathrm{Ci}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\mathrm{arcsec}(cx)}{2(a+b\,\mathrm{ar}}\right.$
default	$c^2\left(-\frac{\sin(2\,\mathrm{arcsec}(cx))}{4(a+b\,\mathrm{arcsec}(cx))^2 b}-\frac{2\cos\left(\frac{2a}{b}\right)\mathrm{Si}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\mathrm{arcsec}(cx)b-2\sin\left(\frac{2a}{b}\right)\mathrm{Ci}\left(\frac{2a}{b}+2\,\mathrm{arcsec}(cx)\right)\mathrm{arcsec}(cx)}{2(a+b\,\mathrm{ar}}\right.$

```
[In] int(1/x^3/(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/4*sin(2*arcsec(c*x))/(a+b*arcsec(c*x))^2/b-1/2*(2*cos(2*a/b)*Si(2*a
/b+2*arcsec(c*x))*arcsec(c*x)*b-2*sin(2*a/b)*Ci(2*a/b+2*arcsec(c*x))*arcsec
(c*x)*b+2*cos(2*a/b)*Si(2*a/b+2*arcsec(c*x))*a-2*sin(2*a/b)*Ci(2*a/b+2*arcs
ec(c*x))*a+cos(2*arcsec(c*x))*b)/(a+b*arcsec(c*x))/b^3)
```

Fricas [F]

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\,(cx)+a\right)^3x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^3*arcsec(c*x)^3 + 3*a*b^2*x^3*arcsec(c*x)^2 + 3*a^2*b*x^3
*arcsec(c*x) + a^3*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^3\left(a+b\,\mathrm{asec}\,(cx)\right)^3}\,dx$$

```
[In] integrate(1/x**3/(a+b*asec(c*x))**3,x)
```

```
[Out] Integral(1/(x**3*(a + b*asec(c*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+b\sec^{-1}(cx))^3}\,dx = \int \frac{1}{(b\,\mathrm{arcsec}\,(cx)+a)^3 x^3}\,dx$$

[In] integrate(1/x^3/(a+b*arcsec(c*x))^3,x, algorithm="maxima")

```
[Out] -(16*a*b^2*log(c)^2 - 8*(b^3*c^2*x^2 - 2*b^3)*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1))^3 + 16*a^3 - 8*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^2 - 24*(a*b^2*c^2*x^
2 - 2*a*b^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - 2*(a*b^2*c^2*x^2 - 2*a
*b^2)*log(c^2*x^2)^2 - 8*(a*b^2*c^2*x^2 - 2*a*b^2)*log(x)^2 + 2*(4*b^3*arct
an(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*log(c^2*x^2)^2 - 4*b^3*log(c)^2 - 8
*b^3*log(c)*log(x) - 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x
 - 1)) + 4*a^2*b + 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*sqrt(c*x + 1)*
sqrt(c*x - 1) + 2*(8*b^3*log(c)^2 + 24*a^2*b - 4*(b^3*c^2*log(c)^2 + 3*a^2*
b*c^2)*x^2 - (b^3*c^2*x^2 - 2*b^3)*log(c^2*x^2)^2 - 4*(b^3*c^2*x^2 - 2*b^3)
*log(x)^2 + 4*(b^3*c^2*x^2*log(c) - 2*b^3*log(c) + (b^3*c^2*x^2 - 2*b^3)*lo
g(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - 2*b^3*log(c))*log(x))*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1)) + (16*b^6*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
))^4 + b^6*x^2*log(c^2*x^2)^4 + 64*b^6*x^2*log(c)*log(x)^3 + 16*b^6*x^2*log
(x)^4 + 64*a*b^5*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*(3*b^6*log(
c)^2 + a^2*b^4)*x^2*log(x)^2 - 8*(b^6*x^2*log(c) + b^6*x^2*log(x))*log(c^2*
x^2)^3 + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*x^2*log(x) + 16*(b^6*log(c)^4 +
 2*a^2*b^4*log(c)^2 + a^4*b^2)*x^2 + 8*(b^6*x^2*log(c^2*x^2)^2 + 8*b^6*x^2*
log(c)*log(x) + 4*b^6*x^2*log(x)^2 + 4*(b^6*log(c)^2 + 3*a^2*b^4)*x^2 - 4*(
b^6*x^2*log(c) + b^6*x^2*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*
x - 1))^2 + 8*(6*b^6*x^2*log(c)*log(x) + 3*b^6*x^2*log(x)^2 + (3*b^6*log(c)
^2 + a^2*b^4)*x^2)*log(c^2*x^2)^2 + 16*(a*b^5*x^2*log(c^2*x^2)^2 + 8*a*b^5*
x^2*log(c)*log(x) + 4*a*b^5*x^2*log(x)^2 + 4*(a*b^5*log(c)^2 + a^3*b^3)*x^2
 - 4*(a*b^5*x^2*log(c) + a*b^5*x^2*log(x))*log(c^2*x^2))*arctan(sqrt(c*x +
1)*sqrt(c*x - 1)) - 32*(3*b^6*x^2*log(c)*log(x)^2 + b^6*x^2*log(x)^3 + (3*b
^6*log(c)^2 + a^2*b^4)*x^2*log(x) + (b^6*log(c)^3 + a^2*b^4*log(c))*x^2)*lo
g(c^2*x^2))*integrate(8*(b*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a)/(4*b^4*
x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^4*x^3*log(c^2*x^2)^2 + 8*b^4*
x^3*log(c)*log(x) + 4*b^4*x^3*log(x)^2 + 8*a*b^3*x^3*arctan(sqrt(c*x + 1)*s
qrt(c*x - 1)) + 4*(b^4*log(c)^2 + a^2*b^2)*x^3 - 4*(b^4*x^3*log(c) + b^4*x^
3*log(x))*log(c^2*x^2)), x) + 8*(a*b^2*c^2*x^2*log(c) - 2*a*b^2*log(c) + (a
*b^2*c^2*x^2 - 2*a*b^2)*log(x))*log(c^2*x^2) - 16*(a*b^2*c^2*x^2*log(c) - 2
*a*b^2*log(c))*log(x))/(16*b^6*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 +
b^6*x^2*log(c^2*x^2)^4 + 64*b^6*x^2*log(c)*log(x)^3 + 16*b^6*x^2*log(x)^4 +
 64*a*b^5*x^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*(3*b^6*log(c)^2 +
a^2*b^4)*x^2*log(x)^2 - 8*(b^6*x^2*log(c) + b^6*x^2*log(x))*log(c^2*x^2)^3
+ 64*(b^6*log(c)^3 + a^2*b^4*log(c))*x^2*log(x) + 16*(b^6*log(c)^4 + 2*a^2*
b^4*log(c)^2 + a^4*b^2)*x^2 + 8*(b^6*x^2*log(c^2*x^2)^2 + 8*b^6*x^2*log(c)*
```

```
log(x) + 4*b^6*x^2*log(x)^2 + 4*(b^6*log(c)^2 + 3*a^2*b^4)*x^2 - 4*(b^6*x^2
*log(c) + b^6*x^2*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
^2 + 8*(6*b^6*x^2*log(c)*log(x) + 3*b^6*x^2*log(x)^2 + (3*b^6*log(c)^2 + a^
2*b^4)*x^2)*log(c^2*x^2)^2 + 16*(a*b^5*x^2*log(c^2*x^2)^2 + 8*a*b^5*x^2*log
(c)*log(x) + 4*a*b^5*x^2*log(x)^2 + 4*(a*b^5*log(c)^2 + a^3*b^3)*x^2 - 4*(a
*b^5*x^2*log(c) + a*b^5*x^2*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt
(c*x - 1)) - 32*(3*b^6*x^2*log(c)*log(x)^2 + b^6*x^2*log(x)^3 + (3*b^6*log(
c)^2 + a^2*b^4)*x^2*log(x) + (b^6*log(c)^3 + a^2*b^4*log(c))*x^2)*log(c^2*x
^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(108) = 216$.

Time = 0.30 (sec) , antiderivative size = 929, normalized size of antiderivative = 8.29

$$\int \frac{1}{x^3 (a + b \sec^{-1}(cx))^3} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^2*c*arccos(1/(c*x))^2*cos(a/b)*cos_integral(2*a/b + 2*arccos(1/(c*
x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) -
 4*b^2*c*arccos(1/(c*x))^2*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)
))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 8*a*b*c*ar
ccos(1/(c*x))*cos(a/b)*cos_integral(2*a/b + 2*arccos(1/(c*x)))*sin(a/b)/(b^
5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 8*a*b*c*arccos(1
/(c*x))*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)))/(b^5*arccos(1/(c
*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 4*a^2*c*cos(a/b)*cos_integral
(2*a/b + 2*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arcco
s(1/(c*x)) + a^2*b^3) + 2*b^2*c*arccos(1/(c*x))^2*sin_integral(2*a/b + 2*ar
ccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)
- 4*a^2*c*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(1/(c*x)))/(b^5*arccos(1/
(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 4*a*b*c*arccos(1/(c*x))*sin
_integral(2*a/b + 2*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arcco
s(1/(c*x)) + a^2*b^3) + b^2*c*arccos(1/(c*x))/(b^5*arccos(1/(c*x))^2 + 2*a*
b^4*arccos(1/(c*x)) + a^2*b^3) + 2*a^2*c*sin_integral(2*a/b + 2*arccos(1/(c
*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + a*b*c/(
b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - b^2*sqrt(-1/(c
^2*x^2) + 1)/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)*x
) - 2*b^2*arccos(1/(c*x))/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x))
 + a^2*b^3)*c*x^2) - 2*a*b/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)
) + a^2*b^3)*c*x^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a + b \sec^{-1}(cx)\right)^3} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{acos}\left(\frac{1}{c x}\right)\right)^3} \, dx$$

```
[In] int(1/(x^3*(a + b*acos(1/(c*x)))^3),x)
[Out] int(1/(x^3*(a + b*acos(1/(c*x)))^3), x)
```

3.50 $\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 228

$$
\begin{aligned}
\int \frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^3}\,dx = & -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{8b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{c^2}{8b^2x\left(a+b\sec^{-1}(cx)\right)} \\
& -\frac{3c^3\cos\left(3\sec^{-1}(cx)\right)}{8b^2\left(a+b\sec^{-1}(cx)\right)} \\
& +\frac{c^3\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{8b^3} \\
& +\frac{9c^3\,\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)\sin\left(\frac{3a}{b}\right)}{8b^3} \\
& -\frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{8b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{c^3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{8b^3} \\
& -\frac{9c^3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{8b^3}
\end{aligned}
$$

```
[Out] -1/8*c^2/b^2/x/(a+b*arcsec(c*x))-3/8*c^3*cos(3*arcsec(c*x))/b^2/(a+b*arcsec
(c*x))-1/8*c^3*cos(a/b)*Si(a/b+arcsec(c*x))/b^3-9/8*c^3*cos(3*a/b)*Si(3*a/b
+3*arcsec(c*x))/b^3+1/8*c^3*Ci(a/b+arcsec(c*x))*sin(a/b)/b^3+9/8*c^3*Ci(3*a
/b+3*arcsec(c*x))*sin(3*a/b)/b^3-1/8*c^3*sin(3*arcsec(c*x))/b/(a+b*arcsec(c
*x))^2-1/8*c^3*(1-1/c^2/x^2)^(1/2)/b/(a+b*arcsec(c*x))^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5330, 4491, 3378, 3384, 3380, 3383}

$$\int \frac{1}{x^4 (a+b\sec^{-1}(cx))^3}\,dx = \frac{c^3 \sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{8b^3} + \frac{9c^3 \sin\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{8b^3} - \frac{c^3 \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{8b^3} - \frac{9c^3 \cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{8b^3} - \frac{3c^3 \cos\left(3\sec^{-1}(cx)\right)}{8b^2 (a+b\sec^{-1}(cx))} - \frac{c^2}{8b^2 x (a+b\sec^{-1}(cx))} - \frac{c^3 \sin\left(3\sec^{-1}(cx)\right)}{8b (a+b\sec^{-1}(cx))^2} - \frac{c^3 \sqrt{1-\frac{1}{c^2x^2}}}{8b (a+b\sec^{-1}(cx))^2}$$

```
[In] Int[1/(x^4*(a + b*ArcSec[c*x])^3),x]
```

```
[Out] -1/8*(c^3*Sqrt[1 - 1/(c^2*x^2)])/(b*(a + b*ArcSec[c*x])^2) - c^2/(8*b^2*x*(
a + b*ArcSec[c*x])) - (3*c^3*Cos[3*ArcSec[c*x]])/(8*b^2*(a + b*ArcSec[c*x])
) + (c^3*CosIntegral[a/b + ArcSec[c*x]]*Sin[a/b])/(8*b^3) + (9*c^3*CosInteg
ral[(3*a)/b + 3*ArcSec[c*x]]*Sin[(3*a)/b])/(8*b^3) - (c^3*Sin[3*ArcSec[c*x]
])/(8*b*(a + b*ArcSec[c*x])^2) - (c^3*Cos[a/b]*SinIntegral[a/b + ArcSec[c*x
]])/(8*b^3) - (9*c^3*Cos[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcSec[c*x]])/(8*
b^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5330

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c^3\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{(a+bx)^3}\,dx, x, \sec^{-1}(cx)\right) \\
&= c^3\text{Subst}\left(\int \left(\frac{\sin(x)}{4(a+bx)^3}+\frac{\sin(3x)}{4(a+bx)^3}\right)\,dx, x, \sec^{-1}(cx)\right) \\
&= \frac{1}{4}c^3\text{Subst}\left(\int \frac{\sin(x)}{(a+bx)^3}\,dx, x, \sec^{-1}(cx)\right)+\frac{1}{4}c^3\text{Subst}\left(\int \frac{\sin(3x)}{(a+bx)^3}\,dx, x, \sec^{-1}(cx)\right) \\
&= -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{8b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{8b\left(a+b\sec^{-1}(cx)\right)^2} \\
&\quad+\frac{c^3\text{Subst}\left(\int \frac{\cos(x)}{(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)}{8b}+\frac{(3c^3)\,\text{Subst}\left(\int \frac{\cos(3x)}{(a+bx)^2}\,dx, x, \sec^{-1}(cx)\right)}{8b} \\
&= -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{8b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{c^2}{8b^2x\left(a+b\sec^{-1}(cx)\right)}-\frac{3c^3\cos\left(3\sec^{-1}(cx)\right)}{8b^2\left(a+b\sec^{-1}(cx)\right)} \\
&\quad-\frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{8b\left(a+b\sec^{-1}(cx)\right)^2}-\frac{c^3\text{Subst}\left(\int \frac{\sin(x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{8b^2} \\
&\quad-\frac{(9c^3)\,\text{Subst}\left(\int \frac{\sin(3x)}{a+bx}\,dx, x, \sec^{-1}(cx)\right)}{8b^2}
\end{aligned}
$$

$$= -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{8b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{c^2}{8b^2x\left(a+b\sec^{-1}(cx)\right)} - \frac{3c^3\cos\left(3\sec^{-1}(cx)\right)}{8b^2\left(a+b\sec^{-1}(cx)\right)} - \frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{8b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{\left(c^3\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sec^{-1}(cx)\right)}{8b^2} - \frac{\left(9c^3\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx,x,\sec^{-1}(cx)\right)}{8b^2} + \frac{\left(c^3\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sec^{-1}(cx)\right)}{8b^2} + \frac{\left(9c^3\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx,x,\sec^{-1}(cx)\right)}{8b^2}$$

$$= -\frac{c^3\sqrt{1-\frac{1}{c^2x^2}}}{8b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{c^2}{8b^2x\left(a+b\sec^{-1}(cx)\right)} - \frac{3c^3\cos\left(3\sec^{-1}(cx)\right)}{8b^2\left(a+b\sec^{-1}(cx)\right)} + \frac{c^3\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right)}{8b^3} + \frac{9c^3\,\text{CosIntegral}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)\sin\left(\frac{3a}{b}\right)}{8b^3} - \frac{c^3\sin\left(3\sec^{-1}(cx)\right)}{8b\left(a+b\sec^{-1}(cx)\right)^2} - \frac{c^3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\sec^{-1}(cx)\right)}{8b^3} - \frac{9c^3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\sec^{-1}(cx)\right)}{8b^3}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.74

$$\int\frac{1}{x^4\left(a+b\sec^{-1}(cx)\right)^3}\,dx = \frac{-\frac{4b^2c\sqrt{1-\frac{1}{c^2x^2}}}{x^2\left(a+b\sec^{-1}(cx)\right)^2} - \frac{12b}{x^3\left(a+b\sec^{-1}(cx)\right)} + \frac{8bc^2}{ax+bx\sec^{-1}(cx)} + c^3\,\text{CosIntegral}\left(\frac{a}{b}+\sec^{-1}(cx)\right)\sin\left(\frac{a}{b}\right) + 9c^3\,\text{CosIntegr}}{8b^3}$$

```
[In] Integrate[1/(x^4*(a + b*ArcSec[c*x])^3),x]
```

```
[Out] ((-4*b^2*c*Sqrt[1 - 1/(c^2*x^2)])/(x^2*(a + b*ArcSec[c*x])^2) - (12*b)/(x^3
*(a + b*ArcSec[c*x])) + (8*b*c^2)/(a*x + b*x*ArcSec[c*x]) + c^3*CosIntegral
[a/b + ArcSec[c*x]]*Sin[a/b] + 9*c^3*CosIntegral[3*(a/b + ArcSec[c*x])]*Sin
[(3*a)/b] - c^3*Cos[a/b]*SinIntegral[a/b + ArcSec[c*x]] - 9*c^3*Cos[(3*a)/b
]*SinIntegral[3*(a/b + ArcSec[c*x])])/(8*b^3)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.35

method	result
derivativedivides	$c^3\left(-\frac{\sin(3\,\mathrm{arcsec}(cx))}{8(a+b\,\mathrm{arcsec}(cx))^2 b}-\frac{3\left(3\,\mathrm{arcsec}(cx)\cos\left(\frac{3a}{b}\right)\mathrm{Si}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)b-3\,\mathrm{arcsec}(cx)\sin\left(\frac{3a}{b}\right)\mathrm{Ci}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(c\right.\right.}{8(a+b}\right.$
default	$c^3\left(-\frac{\sin(3\,\mathrm{arcsec}(cx))}{8(a+b\,\mathrm{arcsec}(cx))^2 b}-\frac{3\left(3\,\mathrm{arcsec}(cx)\cos\left(\frac{3a}{b}\right)\mathrm{Si}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(cx)\right)b-3\,\mathrm{arcsec}(cx)\sin\left(\frac{3a}{b}\right)\mathrm{Ci}\left(\frac{3a}{b}+3\,\mathrm{arcsec}(c\right.\right.}{8(a+b}\right.$

```
[In] int(1/x^4/(a+b*arcsec(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/8*sin(3*arcsec(c*x))/(a+b*arcsec(c*x))^2/b-3/8*(3*arcsec(c*x)*cos(3
*a/b)*Si(3*a/b+3*arcsec(c*x))*b-3*arcsec(c*x)*sin(3*a/b)*Ci(3*a/b+3*arcsec(
c*x))*b+3*cos(3*a/b)*Si(3*a/b+3*arcsec(c*x))*a-3*sin(3*a/b)*Ci(3*a/b+3*arcs
ec(c*x))*a+cos(3*arcsec(c*x))*b)/(a+b*arcsec(c*x))/b^3-1/8*((c^2*x^2-1)/c^2
/x^2)^(1/2)/(a+b*arcsec(c*x))^2/b-1/8*(arcsec(c*x)*cos(a/b)*Si(a/b+arcsec(c
*x))*b*c*x-arcsec(c*x)*sin(a/b)*Ci(a/b+arcsec(c*x))*b*c*x+cos(a/b)*Si(a/b+a
rcsec(c*x))*a*c*x-sin(a/b)*Ci(a/b+arcsec(c*x))*a*c*x+b)/c/x/(a+b*arcsec(c*x
))/b^3)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{\left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^3 x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arcsec(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^4*arcsec(c*x)^3 + 3*a*b^2*x^4*arcsec(c*x)^2 + 3*a^2*b*x^4
*arcsec(c*x) + a^3*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^4 \left(a+b\,\mathrm{asec}\left(cx\right)\right)^3}\,dx$$

```
[In] integrate(1/x**4/(a+b*asec(c*x))**3,x)
```

```
[Out] Integral(1/(x**4*(a + b*asec(c*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + b \sec^{-1}(cx))^3} dx = \int \frac{1}{(b \operatorname{arcsec}(cx) + a)^3 x^4} dx$$

```
[In] integrate(1/x^4/(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

```
[Out] -(24*a*b^2*log(c)^2 - 8*(2*b^3*c^2*x^2 - 3*b^3)*arctan(sqrt(c*x + 1)*sqrt(c
*x - 1))^3 + 24*a^3 - 16*(a*b^2*c^2*log(c)^2 + a^3*c^2)*x^2 - 24*(2*a*b^2*c
^2*x^2 - 3*a*b^2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - 2*(2*a*b^2*c^2*x^
2 - 3*a*b^2)*log(c^2*x^2)^2 - 8*(2*a*b^2*c^2*x^2 - 3*a*b^2)*log(x)^2 + 2*(4
*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*log(c^2*x^2)^2 - 4*b^3*log
(c)^2 - 8*b^3*log(c)*log(x) - 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*x + 1)
*sqrt(c*x - 1)) + 4*a^2*b + 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))*sqrt(
c*x + 1)*sqrt(c*x - 1) + 2*(12*b^3*log(c)^2 + 36*a^2*b - 8*(b^3*c^2*log(c)^
2 + 3*a^2*b*c^2)*x^2 - (2*b^3*c^2*x^2 - 3*b^3)*log(c^2*x^2)^2 - 4*(2*b^3*c^
2*x^2 - 3*b^3)*log(x)^2 + 4*(2*b^3*c^2*x^2*log(c) - 3*b^3*log(c) + (2*b^3*c
^2*x^2 - 3*b^3)*log(x))*log(c^2*x^2) - 8*(2*b^3*c^2*x^2*log(c) - 3*b^3*log(
c))*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (16*b^6*x^3*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1))^4 + b^6*x^3*log(c^2*x^2)^4 + 64*b^6*x^3*log(c)*log(x
)^3 + 16*b^6*x^3*log(x)^4 + 64*a*b^5*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)
)^3 + 32*(3*b^6*log(c)^2 + a^2*b^4)*x^3*log(x)^2 + 64*(b^6*log(c)^3 + a^2*b
^4*log(c))*x^3*log(x) + 16*(b^6*log(c)^4 + 2*a^2*b^4*log(c)^2 + a^4*b^2)*x^
3 - 8*(b^6*x^3*log(c) + b^6*x^3*log(x))*log(c^2*x^2)^3 + 8*(b^6*x^3*log(c^2
*x^2)^2 + 8*b^6*x^3*log(c)*log(x) + 4*b^6*x^3*log(x)^2 + 4*(b^6*log(c)^2 +
3*a^2*b^4)*x^3 - 4*(b^6*x^3*log(c) + b^6*x^3*log(x))*log(c^2*x^2))*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(6*b^6*x^3*log(c)*log(x) + 3*b^6*x^3*log(
x)^2 + (3*b^6*log(c)^2 + a^2*b^4)*x^3)*log(c^2*x^2)^2 + 16*(a*b^5*x^3*log(c
^2*x^2)^2 + 8*a*b^5*x^3*log(c)*log(x) + 4*a*b^5*x^3*log(x)^2 + 4*(a*b^5*log
(c)^2 + a^3*b^3)*x^3 - 4*(a*b^5*x^3*log(c) + a*b^5*x^3*log(x))*log(c^2*x^2)
)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32*(3*b^6*x^3*log(c)*log(x)^2 + b^6
*x^3*log(x)^3 + (3*b^6*log(c)^2 + a^2*b^4)*x^3*log(x) + (b^6*log(c)^3 + a^2
*b^4*log(c))*x^3)*log(c^2*x^2))*integrate(2*(2*a*c^2*x^2 + (2*b*c^2*x^2 - 9
*b)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 9*a)/(4*b^4*x^4*arctan(sqrt(c*x +
 1)*sqrt(c*x - 1))^2 + b^4*x^4*log(c^2*x^2)^2 + 8*b^4*x^4*log(c)*log(x) + 4
*b^4*x^4*log(x)^2 + 8*a*b^3*x^4*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*(b^
4*log(c)^2 + a^2*b^2)*x^4 - 4*(b^4*x^4*log(c) + b^4*x^4*log(x))*log(c^2*x^2
)), x) + 8*(2*a*b^2*c^2*x^2*log(c) - 3*a*b^2*log(c) + (2*a*b^2*c^2*x^2 - 3*
a*b^2)*log(x))*log(c^2*x^2) - 16*(2*a*b^2*c^2*x^2*log(c) - 3*a*b^2*log(c))*
log(x))/(16*b^6*x^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^4 + b^6*x^3*log(c^2
*x^2)^4 + 64*b^6*x^3*log(c)*log(x)^3 + 16*b^6*x^3*log(x)^4 + 64*a*b^5*x^3*a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 32*(3*b^6*log(c)^2 + a^2*b^4)*x^3*lo
g(x)^2 + 64*(b^6*log(c)^3 + a^2*b^4*log(c))*x^3*log(x) + 16*(b^6*log(c)^4 +
 2*a^2*b^4*log(c)^2 + a^4*b^2)*x^3 - 8*(b^6*x^3*log(c) + b^6*x^3*log(x))*lo
```

```
g(c^2*x^2)^3 + 8*(b^6*x^3*log(c^2*x^2)^2 + 8*b^6*x^3*log(c)*log(x) + 4*b^6*
x^3*log(x)^2 + 4*(b^6*log(c)^2 + 3*a^2*b^4)*x^3 - 4*(b^6*x^3*log(c) + b^6*x
^3*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 8*(6*b^6*x
^3*log(c)*log(x) + 3*b^6*x^3*log(x)^2 + (3*b^6*log(c)^2 + a^2*b^4)*x^3)*log
(c^2*x^2)^2 + 16*(a*b^5*x^3*log(c^2*x^2)^2 + 8*a*b^5*x^3*log(c)*log(x) + 4*
a*b^5*x^3*log(x)^2 + 4*(a*b^5*log(c)^2 + a^3*b^3)*x^3 - 4*(a*b^5*x^3*log(c)
 + a*b^5*x^3*log(x))*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 32
*(3*b^6*x^3*log(c)*log(x)^2 + b^6*x^3*log(x)^3 + (3*b^6*log(c)^2 + a^2*b^4)
*x^3*log(x) + (b^6*log(c)^3 + a^2*b^4*log(c))*x^3)*log(c^2*x^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. $2(210) = 420$.

Time = 0.31 (sec) , antiderivative size = 1640, normalized size of antiderivative = 7.19

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

```
[Out] 1/8*(36*b^2*c^2*arccos(1/(c*x))^2*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(
1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b
^3) - 36*b^2*c^2*arccos(1/(c*x))^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos
(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 72
*a*b*c^2*arccos(1/(c*x))*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))
*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 72*
a*b*c^2*arccos(1/(c*x))*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(1/(c*x)))/
(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 9*b^2*c^2*arc
cos(1/(c*x))^2*cos_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos
(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 36*a^2*c^2*cos(a/b)^2*co
s_integral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a
*b^4*arccos(1/(c*x)) + a^2*b^3) + b^2*c^2*arccos(1/(c*x))^2*cos_integral(a/
b + arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*
x)) + a^2*b^3) + 27*b^2*c^2*arccos(1/(c*x))^2*cos(a/b)*sin_integral(3*a/b +
 3*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*
b^3) - 36*a^2*c^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(1/(c*x)))/(b^5*a
rccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - b^2*c^2*arccos(1/(c
*x))^2*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2
+ 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 18*a*b*c^2*arccos(1/(c*x))*cos_integ
ral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*ar
ccos(1/(c*x)) + a^2*b^3) + 2*a*b*c^2*arccos(1/(c*x))*cos_integral(a/b + arc
cos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a
^2*b^3) + 54*a*b*c^2*arccos(1/(c*x))*cos(a/b)*sin_integral(3*a/b + 3*arccos
(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 2*
```

```
a*b*c^2*arccos(1/(c*x))*cos(a/b)*sin_integral(a/b + arccos(1/(c*x)))/(b^5*a
rccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - 9*a^2*c^2*cos_integ
ral(3*a/b + 3*arccos(1/(c*x)))*sin(a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*ar
ccos(1/(c*x)) + a^2*b^3) + a^2*c^2*cos_integral(a/b + arccos(1/(c*x)))*sin(
a/b)/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) + 27*a^2*c
^2*cos(a/b)*sin_integral(3*a/b + 3*arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2
+ 2*a*b^4*arccos(1/(c*x)) + a^2*b^3) - a^2*c^2*cos(a/b)*sin_integral(a/b +
arccos(1/(c*x)))/(b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3
) + 8*b^2*c*arccos(1/(c*x))/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x
)) + a^2*b^3)*x) + 8*a*b*c/((b^5*arccos(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)
) + a^2*b^3)*x) - 4*b^2*sqrt(-1/(c^2*x^2) + 1)/((b^5*arccos(1/(c*x))^2 + 2*
a*b^4*arccos(1/(c*x)) + a^2*b^3)*x^2) - 12*b^2*arccos(1/(c*x))/((b^5*arccos
(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)*c*x^3) - 12*a*b/((b^5*arcc
os(1/(c*x))^2 + 2*a*b^4*arccos(1/(c*x)) + a^2*b^3)*c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+b\sec^{-1}(cx)\right)^3}\,dx=\int \frac{1}{x^4\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3}\,dx$$

```
[In] int(1/(x^4*(a + b*acos(1/(c*x)))^3),x)
[Out] int(1/(x^4*(a + b*acos(1/(c*x)))^3), x)
```

3.51 $\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \mathrm{Int}\left((dx)^m \left(a+b\sec^{-1}(cx)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arcsec(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx$$

```
[In] Int[(d*x)^m*(a + b*ArcSec[c*x])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcSec[c*x])^3, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx$$

Mathematica [N/A]

Not integrable

Time = 4.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcSec[c*x])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcSec[c*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.76 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m (a+b \operatorname{arcsec}(cx))^3 \, dx$$

```
[In] int((d*x)^m*(a+b*arcsec(c*x))^3,x)
[Out] int((d*x)^m*(a+b*arcsec(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsec}(cx)+a)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsec(c*x))^3,x, algorithm="fricas")
[Out] integral((b^3*arcsec(c*x)^3 + 3*a*b^2*arcsec(c*x)^2 + 3*a^2*b*arcsec(c*x) +
 a^3)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 22.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx = \int (dx)^m (a+b \operatorname{asec}(cx))^3 \, dx$$

```
[In] integrate((d*x)**m*(a+b*asec(c*x))**3,x)
[Out] Integral((d*x)**m*(a + b*asec(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 14.78 (sec) , antiderivative size = 991, normalized size of antiderivative = 61.94

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx=\int (b\,\mathrm{arcsec}\,(cx)+a)^3(dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsec(c*x))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(4*b^3*d^m*x*x^m*arctan(sqrt(c*x + 1)*s
qrt(c*x - 1))^3 - 3*b^3*d^m*x*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(c
^2*x^2)^2 - 4*(m + 1)*integrate(3/4*(4*(a*b^2*d^m*m + a*b^2*d^m - (a*b^2*c^
2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - (
a*b^2*d^m*m + a*b^2*d^m - (a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*log(c^
2*x^2)^2 - 4*(a*b^2*d^m*m + a*b^2*d^m - (a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x
^2)*x^m*log(x)^2 - 8*(a*b^2*d^m*m*log(c) + a*b^2*d^m*log(c) - (a*b^2*c^2*d^
m*m*log(c) + a*b^2*c^2*d^m*log(c))*x^2)*x^m*log(x) + (4*b^3*d^m*x^m*arctan(
sqrt(c*x + 1)*sqrt(c*x - 1))^2 - b^3*d^m*x^m*log(c^2*x^2)^2)*sqrt(c*x + 1)*
sqrt(c*x - 1) - 4*(a*b^2*d^m*m*log(c)^2 + a*b^2*d^m*log(c)^2 - (a*b^2*c^2*d
^m*m*log(c)^2 + a*b^2*c^2*d^m*log(c)^2)*x^2)*x^m - 4*((b^3*d^m*m + b^3*d^m
- (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*log(x)^2 + 2*(b^3*d^m*m*log(c) + b
^3*d^m*log(c) - (b^3*c^2*d^m*m*log(c) + b^3*c^2*d^m*log(c))*x^2)*x^m*log(x)
 + ((b^3*log(c)^2 - a^2*b)*d^m*m - ((b^3*c^2*log(c)^2 - a^2*b*c^2)*d^m*m +
(b^3*c^2*log(c)^2 - a^2*b*c^2)*d^m)*x^2 + (b^3*log(c)^2 - a^2*b)*d^m)*x^m -
 ((b^3*d^m*m + b^3*d^m - (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*log(x) + (b
^3*d^m*m*log(c) - (b^3*c^2*d^m*m*log(c) + (b^3*c^2*log(c) + b^3*c^2)*d^m)*x
^2 + (b^3*log(c) + b^3)*d^m)*x^m)*log(c^2*x^2))*arctan(sqrt(c*x + 1)*sqrt(c
*x - 1)) + 4*((a*b^2*d^m*m + a*b^2*d^m - (a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*
x^2)*x^m*log(x) + (a*b^2*d^m*m*log(c) + a*b^2*d^m*log(c) - (a*b^2*c^2*d^m*m
*log(c) + a*b^2*c^2*d^m*log(c))*x^2)*x^m)*log(c^2*x^2))/((c^2*m + c^2)*x^2
- m - 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^3 \, dx=\int (b\,\mathrm{arcsec}\,(cx)+a)^3(dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsec(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)^3*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right)^3 \, dx = \int (d\,x)^m \left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int((d*x)^m*(a + b*acos(1/(c*x)))^3,x)
[Out] int((d*x)^m*(a + b*acos(1/(c*x)))^3, x)
```

3.52 $\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \, dx = \operatorname{Int}\left((dx)^m \left(a+b\sec^{-1}(cx)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arcsec(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \, dx$$

```
[In] Int[(d*x)^m*(a + b*ArcSec[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcSec[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \, dx$$

Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2\,dx=\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2\,dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcSec[c*x])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcSec[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.61 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\ \mathrm{arcsec}\left(cx\right)\right)^2 dx$$

```
[In] int((d*x)^m*(a+b*arcsec(c*x))^2,x)
[Out] int((d*x)^m*(a+b*arcsec(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2\,dx=\int \left(b\,\mathrm{arcsec}\left(cx\right)+a\right)^2(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arcsec(c*x))^2,x, algorithm="fricas")
[Out] integral((b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 10.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2\,dx=\int (dx)^m \left(a+b\,\mathrm{asec}\left(cx\right)\right)^2\,dx$$

```
[In] integrate((d*x)**m*(a+b*asec(c*x))**2,x)
[Out] Integral((d*x)**m*(a + b*asec(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 6.85 (sec) , antiderivative size = 512, normalized size of antiderivative = 32.00

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \,dx = \int (b\,\mathrm{arcsec}\,(cx)+a)^2(dx)^m \,dx$$

```
[In] integrate((d*x)^m*(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/4*(4*b^2*d^m*x*x^m*arctan(sqrt(c*x + 1)*s
qrt(c*x - 1))^2 - b^2*d^m*x*x^m*log(c^2*x^2)^2 - 4*(m + 1)*integrate((2*sqr
t(c*x + 1)*sqrt(c*x - 1)*b^2*d^m*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) -
(b^2*d^m*m + b^2*d^m - (b^2*c^2*d^m*m + b^2*c^2*d^m)*x^2)*x^m*log(x)^2 + 2*
(a*b*d^m*m + a*b*d^m - (a*b*c^2*d^m*m + a*b*c^2*d^m)*x^2)*x^m*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1)) - 2*(b^2*d^m*m*log(c) + b^2*d^m*log(c) - (b^2*c^2*d^
m*m*log(c) + b^2*c^2*d^m*log(c))*x^2)*x^m*log(x) - (b^2*d^m*m*log(c)^2 + b^
2*d^m*log(c)^2 - (b^2*c^2*d^m*m*log(c)^2 + b^2*c^2*d^m*log(c)^2)*x^2)*x^m +
 ((b^2*d^m*m + b^2*d^m - (b^2*c^2*d^m*m + b^2*c^2*d^m)*x^2)*x^m*log(x) + (b
^2*d^m*m*log(c) - (b^2*c^2*d^m*m*log(c) + (b^2*c^2*log(c) + b^2*c^2)*d^m)*x
^2 + (b^2*log(c) + b^2)*d^m)*x^m)*log(c^2*x^2))/((c^2*m + c^2)*x^2 - m - 1)
, x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \,dx = \int (b\,\mathrm{arcsec}\,(cx)+a)^2(dx)^m \,dx$$

```
[In] integrate((d*x)^m*(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)^2*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right)^2 \,dx = \int (d\,x)^m \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

```
[In] int((d*x)^m*(a + b*acos(1/(c*x)))^2,x)
```

```
[Out] int((d*x)^m*(a + b*acos(1/(c*x)))^2, x)
```

3.53 $\int (dx)^m \left(a+b\sec^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right) \, dx = \frac{(dx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{d(1+m)} - \frac{b(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arcsec(c*x))/d/(1+m)-b*(d*x)^m*hypergeom([1/2, -1/2*m],[1-
1/2*m],1/c^2/x^2)/c/m/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5328, 346, 371}

$$\int (dx)^m \left(a+b\sec^{-1}(cx)\right) \, dx = \frac{(dx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{d(m+1)} - \frac{b(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

```
[In] Int[(d*x)^m*(a + b*ArcSec[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcSec[c*x]))/(d*(1 + m)) - (b*(d*x)^m*Hypergeometric
2F1[1/2, -1/2*m, 1 - m/2, 1/(c^2*x^2)])/(c*m*(1 + m))
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
```

```
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(dx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{d(1+m)}-\frac{(bd)\int\frac{(dx)^{-1+m}}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{c(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{d(1+m)}+\frac{\left(b\left(\frac{1}{x}\right)^m(dx)^m\right)\text{Subst}\left(\int\frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{d(1+m)}-\frac{b(dx)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{c^2x^2}\right)}{cm(1+m)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int(dx)^m\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{x(dx)^m\left((1+m)\left(a+b\sec^{-1}(cx)\right)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{\sqrt{1-c^2x^2}}\right)}{(1+m)^2}$$

```
[In] Integrate[(d*x)^m*(a + b*ArcSec[c*x]),x]
```

```
[Out] (x*(d*x)^m*((1 + m)*(a + b*ArcSec[c*x]) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Hype
rgeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/Sqrt[1 - c^2*x^2]))/(1 +
 m)^2
```

Maple [F]

$$\int (dx)^m (a + b \operatorname{arcsec}(cx))\, dx$$

```
[In] int((d*x)^m*(a+b*arcsec(c*x)),x)
[Out] int((d*x)^m*(a+b*arcsec(c*x)),x)
```

Fricas [F]

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right)\, dx = \int (b \operatorname{arcsec}(cx) + a)(dx)^m\, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right)\, dx = \int (dx)^m \left(a + b \operatorname{asec}(cx)\right)\, dx$$

```
[In] integrate((d*x)**m*(a+b*asec(c*x)),x)
[Out] Integral((d*x)**m*(a + b*asec(c*x)), x)
```

Maxima [F]

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right)\, dx = \int (b \operatorname{arcsec}(cx) + a)(dx)^m\, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] (d^m*x*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (c^2*d^m*m + c^2*d^m)*inte
grate(-sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(c^2*m - (c^4*m + c^4)*x^2 + c^2), x
))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right) \, dx = \int (b \operatorname{arcsec}(cx) + a)(dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a + b\sec^{-1}(cx)\right) \, dx = \int (d\,x)^m \left(a + b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((d*x)^m*(a + b*acos(1/(c*x))),x)
[Out] int((d*x)^m*(a + b*acos(1/(c*x))), x)
```

3.54 $\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\sec^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arcsec(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcSec[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcSec[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcSec[c*x]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcSec[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\ \mathrm{arcsec}\left(cx\right)}dx$$

```
[In] int((d*x)^m/(a+b*arcsec(c*x)),x)
[Out] int((d*x)^m/(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\mathrm{arcsec}\left(cx\right)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arcsec(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\,\mathrm{asec}\left(cx\right)}\,dx$$

```
[In] integrate((d*x)**m/(a+b*asec(c*x)),x)
[Out] Integral((d*x)**m/(a + b*asec(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\mathrm{arcsec}\,(cx)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arcsec(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\mathrm{arcsec}\,(cx)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arcsec(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{a+b\sec^{-1}(cx)}\,dx=\int \frac{(d\,x)^m}{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}\,dx$$

```
[In] int((d*x)^m/(a + b*acos(1/(c*x))),x)
[Out] int((d*x)^m/(a + b*acos(1/(c*x))), x)
```

3.55 $\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \operatorname{Int}\left(\frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arcsec(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcSec[c*x])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcSec[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx=\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcSec[c*x])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcSec[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\ \mathrm{arcsec}\,(cx))^2}dx$$

```
[In] int((d*x)^m/(a+b*arcsec(c*x))^2,x)
[Out] int((d*x)^m/(a+b*arcsec(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx=\int \frac{(dx)^m}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsec(c*x))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arcsec(c*x)^2 + 2*a*b*arcsec(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx=\int \frac{(dx)^m}{(a+b\,\mathrm{asec}\,(cx))^2}\,dx$$

```
[In] integrate((d*x)**m/(a+b*asec(c*x))**2,x)
[Out] Integral((d*x)**m/(a + b*asec(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 643, normalized size of antiderivative = 40.19

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsec(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(4*(b*d^m*x*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + a*d^m*x*x^m)*sqrt(c*
x + 1)*sqrt(c*x - 1) - (4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*l
og(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a
*b^2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*lo
g(x))*log(c^2*x^2))*integrate(4*((b*d^m*m - (b*c^2*d^m*m + 2*b*c^2*d^m)*x^2
 + b*d^m)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*d^m*m - (a*c^2*d^m*m
 + 2*a*c^2*d^m)*x^2 + a*d^m)*x^m)*sqrt(c*x + 1)*sqrt(c*x - 1)/(4*b^3*log(c)
^2 + 4*a^2*b - 4*(b^3*c^2*log(c)^2 + a^2*b*c^2)*x^2 - 4*(b^3*c^2*x^2 - b^3)
*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)^2
 - 4*(b^3*c^2*x^2 - b^3)*log(x)^2 - 8*(a*b^2*c^2*x^2 - a*b^2)*arctan(sqrt(c
*x + 1)*sqrt(c*x - 1)) + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) + (b^3*c^2*x^2
- b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(c))*log(x)),
x))/(4*b^3*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(c^2*x^2)^2 + 4*b
^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^2*arctan(sqrt(c*
x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*log(x))*log(c^2*x^2))
```

Giac [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{(a+b\sec^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\,\mathrm{arcsec}\,(cx)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arcsec(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arcsec(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{\left(a+b\sec^{-1}(cx)\right)^2}\,dx=\int \frac{{\left(d\,x\right)}^m}{{\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}^2}\,dx$$

```
[In] int((d*x)^m/(a + b*acos(1/(c*x)))^2,x)
[Out] int((d*x)^m/(a + b*acos(1/(c*x)))^2, x)
```

3.56 $\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 167

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = -\frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3} - \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{2c} - \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c} + \frac{bd^4\csc^{-1}(cx)}{4e} + \frac{(d+ex)^4\left(a+b\sec^{-1}(cx)\right)}{4e} - \frac{bd(2c^2d^2+e^2)\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{2c^3}$$

```
[Out] 1/4*b*d^4*arccsc(c*x)/e+1/4*(e*x+d)^4*(a+b*arcsec(c*x))/e-1/2*b*d*(2*c^2*d^
2+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3-1/6*b*e*(9*c^2*d^2+e^2)*x*(1-1/c^2/
x^2)^(1/2)/c^3-1/2*b*d*e^2*x^2*(1-1/c^2/x^2)^(1/2)/c-1/12*b*e^3*x^3*(1-1/c^
2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used

$= \{5334, 1582, 1489, 1821, 858, 222, 272, 65, 214\}$

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \frac{(d+ex)^4 \left(a+b\sec^{-1}(cx)\right)}{4e} - \frac{bd\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)(2c^2d^2+e^2)}{2c^3} - \frac{bde^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{2c} - \frac{be^3x^3\sqrt{1-\frac{1}{c^2x^2}}}{12c} - \frac{bex\sqrt{1-\frac{1}{c^2x^2}}(9c^2d^2+e^2)}{6c^3} + \frac{bd^4\csc^{-1}(cx)}{4e}$$

```
[In] Int[(d + e*x)^3*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/6*(b*e*(9*c^2*d^2 + e^2)*Sqrt[1 - 1/(c^2*x^2)]*x)/c^3 - (b*d*e^2*Sqrt[1
- 1/(c^2*x^2)]*x^2)/(2*c) - (b*e^3*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(12*c) + (b*d
^4*ArcCsc[c*x])/(4*e) + ((d + e*x)^4*(a + b*ArcSec[c*x]))/(4*e) - (b*d*(2*c
^2*d^2 + e^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(2*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^4\,(a+b\sec^{-1}(cx))}{4e} - \frac{b\int\frac{(d+ex)^4}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{4ce}$$

$$= \frac{(d+ex)^4\,(a+b\sec^{-1}(cx))}{4e} - \frac{b\int\frac{\left(e+\frac{d}{x}\right)^4x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{4ce}$$

$$= \frac{(d+ex)^4\,(a+b\sec^{-1}(cx))}{4e} + \frac{b\text{Subst}\left(\int\frac{(e+dx)^4}{x^4\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce}$$

$$= -\frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4\,(a+b\sec^{-1}(cx))}{4e} - \frac{b\text{Subst}\left(\int \frac{-12de^3-2e^2\left(9d^2+\frac{e^2}{c^2}\right)x-12d^3ex^2-3d^4x^3}{x^3\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{12ce}$$

$$= -\frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} - \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4\,(a+b\sec^{-1}(cx))}{4e} + \frac{b\text{Subst}\left(\int \frac{4e^2\left(9d^2+\frac{e^2}{c^2}\right)+12de\left(2d^2+\frac{e^2}{c^2}\right)x+6d^4x^2}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{24ce}$$

$$= -\frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} - \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4\,(a+b\sec^{-1}(cx))}{4e} - \frac{b\text{Subst}\left(\int \frac{-12de\left(2d^2+\frac{e^2}{c^2}\right)-6d^4x}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{24ce}$$

$$= -\frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} - \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4\,(a+b\sec^{-1}(cx))}{4e} + \frac{(bd^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce} + \frac{(bd(2c^2d^2+e^2))\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c^3}$$

$$= -\frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} - \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{bd^4\csc^{-1}(cx)}{4e} + \frac{(d+ex)^4\,(a+b\sec^{-1}(cx))}{4e} + \frac{(bd(2c^2d^2+e^2))\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{4c^3}$$

$$= -\frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} - \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{bd^4\csc^{-1}(cx)}{4e} + \frac{(d+ex)^4\,(a+b\sec^{-1}(cx))}{4e} - \frac{(bd(2c^2d^2+e^2))\,\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)}{2c}$$

$$= -\frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} - \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{bd^4\csc^{-1}(cx)}{4e} + \frac{(d+ex)^4\,(a+b\sec^{-1}(cx))}{4e} - \frac{bd(2c^2d^2+e^2)\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{3ac^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)-be\sqrt{1-\frac{1}{c^2x^2}}x(2e^2+c^2(18d^2+6dex+e^2x^2))+3bc^3x(4d^3+6d^2e\ldots}{12c^3}$$

[In] Integrate[(d + e*x)^3*(a + b*ArcSec[c*x]),x]

[Out] (3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcSec[c*x] - 6*b*d*(2*c^2*d^2 + e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(12*c^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(147) = 294.

Time = 0.39 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.41

method	result
parts	$\frac{a(ex+d)^4}{4e}+\frac{b\,e^3\operatorname{arcsec}(cx)x^4}{4}+b\,e^2\operatorname{arcsec}(cx)\,x^3d+\frac{3be\operatorname{arcsec}(cx)x^2d^2}{2}+b\operatorname{arcsec}(cx)\,x\,d^3+\ldots$
derivativedivides	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{bc\operatorname{arcsec}(cx)d^4}{4e}+b\operatorname{arcsec}(cx)d^3cx+\frac{3bce\operatorname{arcsec}(cx)d^2x^2}{2}+bc\,e^2\operatorname{arcsec}(cx)d\,x^3+\frac{bc\,e^3\operatorname{arcsec}(cx)x^4}{4}+\frac{b\sqrt{c^2x^2-1}\ldots}{4\ldots}}{\ldots}$
default	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{bc\operatorname{arcsec}(cx)d^4}{4e}+b\operatorname{arcsec}(cx)d^3cx+\frac{3bce\operatorname{arcsec}(cx)d^2x^2}{2}+bc\,e^2\operatorname{arcsec}(cx)d\,x^3+\frac{bc\,e^3\operatorname{arcsec}(cx)x^4}{4}+\frac{b\sqrt{c^2x^2-1}\ldots}{4\ldots}}{\ldots}$

[In] int((e*x+d)^3*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/4*a*(e*x+d)^4/e+1/4*b*e^3*arcsec(c*x)*x^4+b*e^2*arcsec(c*x)*x^3*d+3/2*b*e*arcsec(c*x)*x^2*d^2+b*arcsec(c*x)*x*d^3+1/4*b/e*arcsec(c*x)*d^4-1/12*b/c^3*e^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x+1/4*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^4*arctan(1/(c^2*x^2-1)^(1/2))-1/2*b/c^3*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d-b/c^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*ln(c*x+(c^2*x^2-1)^(1/2))-3/2*b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2-1/2*b/c^4*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))-1/6*b/c^5*e^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.74

$$\int (d+ex)^3\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{3\,ac^4e^3x^4+12\,ac^4de^2x^3+18\,ac^4d^2ex^2+12\,ac^4d^3x+3\left(bc^4e^3x^4+4\,bc^4de^2x^3+6\,bc^4d^2ex^2+4\,bc^4d^3x-4\,bc\right.}{}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] 1/12*(3*a*c^4*e^3*x^4 + 12*a*c^4*d*e^2*x^3 + 18*a*c^4*d^2*e*x^2 + 12*a*c^4*
d^3*x + 3*(b*c^4*e^3*x^4 + 4*b*c^4*d*e^2*x^3 + 6*b*c^4*d^2*e*x^2 + 4*b*c^4*
d^3*x - 4*b*c^4*d^3 - 6*b*c^4*d^2*e - 4*b*c^4*d*e^2 - b*c^4*e^3)*arcsec(c*x
) + 6*(4*b*c^4*d^3 + 6*b*c^4*d^2*e + 4*b*c^4*d*e^2 + b*c^4*e^3)*arctan(-c*x
 + sqrt(c^2*x^2 - 1)) + 6*(2*b*c^3*d^3 + b*c*d*e^2)*log(-c*x + sqrt(c^2*x^2
 - 1)) - (b*c^2*e^3*x^2 + 6*b*c^2*d*e^2*x + 18*b*c^2*d^2*e + 2*b*e^3)*sqrt(
c^2*x^2 - 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.17

$$\begin{aligned}\int (d+ex)^3\left(a+b\sec^{-1}(cx)\right)\,dx &= ad^3x+\frac{3ad^2ex^2}{2}+ade^2x^3+\frac{ae^3x^4}{4}+bd^3x\operatorname{asec}(cx)\\&+\frac{3bd^2ex^2\operatorname{asec}(cx)}{2}+bde^2x^3\operatorname{asec}(cx)+\frac{be^3x^4\operatorname{asec}(cx)}{4}\\&-\frac{bd^3\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}-\frac{3bd^2e\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}\\&-\frac{bde^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{c}\\&-\frac{be^3\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}\end{aligned}$$

```
[In] integrate((e*x+d)**3*(a+b*asec(c*x)),x)
```

```
[Out] a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b*d**3*x*ase
c(c*x) + 3*b*d**2*e*x**2*asec(c*x)/2 + b*d*e**2*x**3*asec(c*x) + b*e**3*x**
4*asec(c*x)/4 - b*d**3*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c - 3*b*d**2*e*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**
2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c) - b*d*e**2*Piecewise((x*sq
rt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x
**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)
/(2*c**2), True))/c - b*e**3*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*
sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2
+ 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.63

$$\begin{aligned}
&\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx \\
&= \frac{1}{4}\,ae^3x^4 + ade^2x^3 + \frac{3}{2}\,ad^2ex^2 + \frac{3}{2}\left(x^2\,\mathrm{arcsec}\,(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2e \\
&\quad + \frac{1}{4}\left(4\,x^3\,\mathrm{arcsec}\,(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde^2 \\
&\quad + \frac{1}{12}\left(3\,x^4\,\mathrm{arcsec}\,(cx) - \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be^3 + ad^3x \\
&\quad + \frac{\left(2\,cx\,\mathrm{arcsec}\,(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^3}{2\,c}
\end{aligned}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arcsec(c*x) - x*sq
rt(-1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/4*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*
x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c
^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e^2 + 1/12*(3*x^4*arcsec(c
*x) - (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*
b*e^3 + a*d^3*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1)
+ log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^3/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9430 vs. $2(147) = 294$.

Time = 3.07 (sec) , antiderivative size = 9430, normalized size of antiderivative = 56.47

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/12*(12*b*c^3*d^3*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^3*d
^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^5 + 4*c^5*(1/(c^2*x^2)
 - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^
8) + 12*b*c^3*d^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^5 + 4*c
^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8) + 12*a*c^3*d^3/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 18*b*c^2*d^2
*e*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(
1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 24*b*c^3*d^3*(1/(c^2*x^2)
- 1)*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^
5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 48*b*c
^3*d^3*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c
^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2
) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 48*b*c^3*d^3*(1/(c^2*x^2) - 1)
*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1
/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8
)*(1/(c*x) + 1)^2) + 18*a*c^2*d^2*e/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 24*a*c^3*
d^3*(1/(c^2*x^2) - 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c
^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x)
 + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 12*b*
c*d*e^2*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*
c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x
) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 6*b*c*d*e^2*log(abs(s
qrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*
```

x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 72*b*c^
3*d^3*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((
c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 6*b*c*d*e^2*log(abs(sqrt(-1/(
c^2*x^2) + 1) - 1/(c*x) - 1))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 72*b*c^3*d^3*(1
/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^5 + 4*
c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 36*b*c^2*d^2*e*sqrt(-1/(c^2*x^2) + 1)
/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2
*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 12*a*c*d*e^2/(c^5 + 4*c^5*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8) + 3*b*e^3*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 24*b*c
*d*e^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c
*x) + 1)^2) - 36*b*c^2*d^2*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^5 + 4*
c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 24*b*c^3*d^3*(1/(c^2*x^2) - 1)^3*arcc
os(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 24*b*c*d*e^2*(1
/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^5 + 4*c^
5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/
(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 48*b*c^3*d^3*(1/(c^2*x^2) - 1)^3*log(ab
s(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c
*x) + 1)^6) + 24*b*c*d*e^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1)
 - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1
/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 48*b*c^3*d
^3*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^5
 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(
c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2)

- 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 12*b*c*d*e^2*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 108*b*c^2*d^2*e*(-1/(c^2*x^2) + 1)^(3/2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 3*a*e^3/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 24*a*c*d*e^2*(1/(c^2*x^2) - 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 36*a*c^2*d^2*e*(1/(c^2*x^2) - 1)^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 24*a*c^3*d^3*(1/(c^2*x^2) - 1)^3/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 12*b*e^3*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 12*b*c^3*d^3*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 36*b*c*d*e^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 12*b*c^3*d^3*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 36*b*c*d*e^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 12*b*c^3*d^3*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 6*b*e^3*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) - 108*b*c^2*d^2*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +

```
4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8)*(1/(c*x) + 1)^5) + 12*b*c*d*e^2*(-1/(c^2*x^2) + 1)^(3/2)/((c^5 +
4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) - 12*a*e^3*(1/(c^2*x^2) - 1)/((c^5 +
4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 12*a*c^3*d^3*(1/(c^2*x^2) - 1)^4/((
c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 18*b*e^3*(1/(c^2*x^2) - 1)^2*
arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/
(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^
6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 24*b*c*d*e^
2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x)
 + 1)^6) + 18*b*c^2*d^2*e*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^5 + 4*c^5
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(
1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 24*b*c*d*e^2*(1/(c^2*x^2) - 1)^3*log(abs
(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*
x) + 1)^6) + 24*b*c*d*e^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1
) - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(
1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 12*b*c*d*
e^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)
*(1/(c*x) + 1)^5) - 36*b*c^2*d^2*e*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) +
1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c
^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) + 10*b*e^3*(-1/(c^2*x^2) +
 1)^(3/2)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^
5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 18*a*e^3*(1/(c^2*
x^2) - 1)^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 24*a*c*d*e^2*(1
/(c^2*x^2) - 1)^3/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(
1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 18*a*c^2*
d^2*e*(1/(c^2*x^2) - 1)^4/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
```

6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 12*b*e^3*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 12*b*c*d*e^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 6*b*c*d*e^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 6*b*c*d*e^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 10*b*e^3*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) + 12*b*c*d*e^2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) - 12*a*e^3*(1/(c^2*x^2) - 1)^3/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 12*a*c*d*e^2*(1/(c^2*x^2) - 1)^4/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 3*b*e^3*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 6*b*e^3*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) + 3*a*e^3*(1/(c^2*x^2) - 1)^4/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8))*c

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \left(d+e\,x\right)^3 dx$$

```
[In] int((a + b*acos(1/(c*x)))*(d + e*x)^3,x)
[Out] int((a + b*acos(1/(c*x)))*(d + e*x)^3, x)
```

3.57 $\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) dx = -\frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} + \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{b(6c^2d^2+e^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

```
[Out] 1/3*b*d^3*arccsc(c*x)/e+1/3*(e*x+d)^3*(a+b*arcsec(c*x))/e-1/6*b*(6*c^2*d^2+
e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3-b*d*e*x*(1-1/c^2/x^2)^(1/2)/c-1/6*b*e
^2*x^2*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5334, 1582, 1489, 1821, 858, 222, 272, 65, 214}

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) dx = \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)(6c^2d^2+e^2)}{6c^3} - \frac{bdex\sqrt{1-\frac{1}{c^2x^2}}}{c} - \frac{be^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{6c} + \frac{bd^3\csc^{-1}(cx)}{3e}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] -((b*d*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c) - (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(6
*c) + (b*d^3*ArcCsc[c*x])/(3*e) + ((d + e*x)^3*(a + b*ArcSec[c*x]))/(3*e) -
 (b*(6*c^2*d^2 + e^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(6*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
```

```
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{b\int \frac{(d+ex)^3}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{3ce}$$

$$= \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{b\int \frac{\left(e+\frac{d}{x}\right)^3 x}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{3ce}$$

$$= \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} + \frac{b\text{Subst}\left(\int \frac{(e+dx)^3}{x^3\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{3ce}$$

$$= -\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{b\text{Subst}\left(\int \frac{-6de^2-e\left(6d^2+\frac{e^2}{c^2}\right)x-2d^3x^2}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6ce}$$

$$= -\frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} + \frac{b\text{Subst}\left(\int \frac{e\left(6d^2+\frac{e^2}{c^2}\right)+2d^3x}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6ce}$$

$$= -\frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} + \frac{(bd^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{3ce} + \frac{(b(6c^2d^2+e^2))\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6c^3}$$

$$= -\frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} + \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} + \frac{(b(6c^2d^2+e^2))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{12c^3}$$

$$= -\frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} + \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{(b(6c^2d^2+e^2))\operatorname{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}$$

$$= -\frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} + \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{b(6c^2d^2+e^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

$$\int (d+ex)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= \frac{c^2x\left(-be\sqrt{1-\frac{1}{c^2x^2}}(6d+ex)+2ac(3d^2+3dex+e^2x^2)\right)+2bc^3x(3d^2+3dex+e^2x^2)\sec^{-1}(cx)-b(6c^2d^2}{6c^3}$$

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSec[c*x]),x]
[Out] (c^2*x*(-(b*e*Sqrt[1 - 1/(c^2*x^2)]*(6*d + e*x)) + 2*a*c*(3*d^2 + 3*d*e*x +
 e^2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcSec[c*x] - b*(6*c^2*d
^2 + e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(110) = 220.

Time = 0.38 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.47

method	result
parts	$\frac{a(ex+d)^3}{3e}+\frac{b\,e^2\,\mathrm{arcsec}(cx)x^3}{3}+be\ \mathrm{arcsec}\,(cx)\,x^2d+b\ \mathrm{arcsec}\,(cx)\,x\,d^2+\frac{b\ \mathrm{arcsec}(cx)d^3}{3e}+\frac{b\sqrt{c^2x^2-1}}{3}$
derivativedivides	$\frac{\frac{a(cex+cd)^3}{3c^2e}+\frac{bc\ \mathrm{arcsec}(cx)d^3}{3e}+b\ \mathrm{arcsec}(cx)d^2cx+bce\ \mathrm{arcsec}(cx)d\,x^2+\frac{bc\,e^2\ \mathrm{arcsec}(cx)x^3}{3}+\frac{b\sqrt{c^2x^2-1}\,d^3\ \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-}{c}$
default	$\frac{\frac{a(cex+cd)^3}{3c^2e}+\frac{bc\ \mathrm{arcsec}(cx)d^3}{3e}+b\ \mathrm{arcsec}(cx)d^2cx+bce\ \mathrm{arcsec}(cx)d\,x^2+\frac{bc\,e^2\ \mathrm{arcsec}(cx)x^3}{3}+\frac{b\sqrt{c^2x^2-1}\,d^3\ \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-}{c}$

```
[In] int((e*x+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(e*x+d)^3/e+1/3*b*e^2*arcsec(c*x)*x^3+b*e*arcsec(c*x)*x^2*d+b*arcsec(
c*x)*x*d^2+1/3*b/e*arcsec(c*x)*d^3+1/3*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*d^3*arctan(1/(c^2*x^2-1)^(1/2))-1/6*b/c^3*e^2*(c^2*x^2-1)
/((c^2*x^2-1)/c^2/x^2)^(1/2)-b/c^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/
x^2)^(1/2)/x*d-1/6*b/c^4*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.69

$$\int (d+ex)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{2\,ac^3e^2x^3+6\,ac^3dex^2+6\,ac^3d^2x+2\left(bc^3e^2x^3+3\,bc^3dex^2+3\,bc^3d^2x-3\,bc^3d^2-3\,bc^3de-bc^3e^2\right)\mathrm{arcsec}\,(cx}{}$$

```
[In] integrate((e*x+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(b*c^3*e^2*x^3 +
 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 - 3*b*c^3*d*e - b*c^3*e^2)*a
rcsec(c*x) + 4*(3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*arctan(-c*x + sqrt(c
^2*x^2 - 1)) + (6*b*c^2*d^2 + b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - (b*c*e
^2*x + 6*b*c*d*e)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.84

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{asec}(cx) + bdex^2\operatorname{asec}(cx) + \frac{be^2x^3\operatorname{asec}(cx)}{3} - \frac{bd^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} - \frac{bde\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{c} - \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((e*x+d)**2*(a+b*asec(c*x)),x)
```

```
[Out] a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*asec(c*x) + b*d*e*x**2*ase
c(c*x) + b*e**2*x**3*asec(c*x)/3 - b*d**2*Piecewise((acosh(c*x), Abs(c**2*x
**2) > 1), (-I*asin(c*x), True))/c - b*d*e*Piecewise((sqrt(c**2*x**2 - 1)/c
, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/c - b*e**2*Piecewi
se((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1),
 (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*a
sin(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.61

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{1}{3}ae^2x^3 + adex^2 + \left(x^2\operatorname{arcsec}(cx) - \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bde + \frac{1}{12}\left(4x^3\operatorname{arcsec}(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 + ad^2x + \frac{\left(2\,cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2\,c}$$

```
[In] integrate((e*x+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1)/c)*
b*d*e + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^
2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2
) + 1) - 1)/c^2)/c)*b*e^2 + a*d^2*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/
(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^2/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6416 vs. $2(110) = 220$.

Time = 2.89 (sec) , antiderivative size = 6416, normalized size of antiderivative = 51.74

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] -1/6*(6*b*c^3*d*e*x^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c
^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*a*c^3*d*e*x^2*(1/(c^2*x^2) - 1)
/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 18*b*c^3*d*e*
x^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 18*a*c^3*d*e*x^2*(1/(c^2*x^2) -
 1)^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x)
+ 1)^2) + 18*b*c^3*d*e*x^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^4 + 3*c^
4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 18*a*c^3
*d*e*x^2*(1/(c^2*x^2) - 1)^3/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 6*b*c^3*d*e*x^2*(1/(c^2*x^2) - 1)^4*arccos
(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c
*x) + 1)^6) + 6*b*c^2*d*e*x*sqrt(-1/(c^2*x^2) + 1)/(c^4 + 3*c^4*(1/(c^2*x^2
) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1
/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*a*c^3*d*e*x^2*(1/(c^2*x^2) - 1)^4/((
c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6)
- 6*b*c^2*d^2*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6) + 6*b*c^2*d^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/
(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/
```

(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 6*b*c^2*d^2*lo
g(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*
x^2) - 1)^3/(1/(c*x) + 1)^6) - 18*b*c^2*d*e*x*(-1/(c^2*x^2) + 1)^(3/2)/((c^
4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/
(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) -
6*a*c^2*d^2/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 6*
b*c*d*e*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*
c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6) - 6*b*c^2*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 18*b*c^2*d^2*(1
/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^
4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 18*b*c^2
d^2(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4
+ 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(
c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 1
8*b*c^2*d*e*x*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 3*c^4*(1/(
c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 6*a*c*d*e/(c^4
+ 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(
c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 6*a*c^2*d^2*(1/(c^
2*x^2) - 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/
(c*x) + 1)^2) - 2*b*e^2*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(
c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6) - 18*b*c*d*e*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^
4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/
(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) +
6*b*c^2*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + b*e^2*log(abs(sqrt(-1/(
c^2*x^2) + 1) + 1/(c*x) + 1))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6) + 18*b*c^2*d^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2)
+ 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^
4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6)*(1/(c*x) + 1)^4) - b*e^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1
))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 18*b*c^2*d^
2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4
+ 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c
x) + 1)^4 + c^4(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 6*

```
b*c^2*d*e*x*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 3*c^4*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c
^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) - 2*a*e^2/(c^4 + 3
*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 18*a*c*d*e*(1/(c^2*x^2
) - 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x)
 + 1)^2) + 6*a*c^2*d^2*(1/(c^2*x^2) - 1)^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 6*b*e^2*(1/(c^2*x^2) - 1)*ar
ccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(
1/(c*x) + 1)^2) - 18*b*c*d*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^4 + 3*
c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 6*b*c^
2*d^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(
1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + 3*b*e^2*(1/(c^2*x^2) - 1)*log
(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*
x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 6*b*c^2*d^2*(1/(c^2*x^2) -
1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) - 3*b*e^2*(1/(c^2*x^2
) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^
4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 6*b*c^2*d^2*(1/(c
^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + 2*b*e^2*s
qrt(-1/(c^2*x^2) + 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c
^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
 1)^6)*(1/(c*x) + 1)) + 6*a*e^2*(1/(c^2*x^2) - 1)/((c^4 + 3*c^4*(1/(c^2*x^2
) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1
/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 18*a*c*d*e*(1/(c^2*x^
2) - 1)^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c
*x) + 1)^4) + 6*a*c^2*d^2*(1/(c^2*x^2) - 1)^3/((c^4 + 3*c^4*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) - 6*b*e^2*(1/(c^2*x^2) - 1)
^2*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*
(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6)*(1/(c*x) + 1)^4) - 6*b*c*d*e*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^4
+ 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + 3*
```

```
b*e^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((
c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4)
- 3*b*e^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1)
)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)
^4) - 6*a*e^2*(1/(c^2*x^2) - 1)^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 6*a*c*d*e*(1/(c^2*x^2) - 1)^3/((c^4 +
 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + 2*b
*e^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + b*e^2*(1/(c^2*x^2) - 1)^3*log(
abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) - b*e^2*(1/(c^2*x^2) - 1)^3*lo
g(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) - 2*b*e^2*(1/(c^2*x^2) - 1)^
2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x
) + 1)^6)*(1/(c*x) + 1)^5) + 2*a*e^2*(1/(c^2*x^2) - 1)^3/((c^4 + 3*c^4*(1/(
c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
 c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^6))*c
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \left(d+e\,x\right)^2 dx$$

```
[In] int((a + b*acos(1/(c*x)))*(d + e*x)^2,x)
[Out] int((a + b*acos(1/(c*x)))*(d + e*x)^2, x)
```

3.58 $\int (d+ex) \left(a+b\sec^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 84

$$\int (d+ex) \left(a+b\sec^{-1}(cx)\right) \, dx = -\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{bd^2\csc^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e} - \frac{bd\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] 1/2*b*d^2*arccsc(c*x)/e+1/2*(e*x+d)^2*(a+b*arcsec(c*x))/e-b*d*arctanh((1-1/
c^2/x^2)^(1/2))/c-1/2*b*e*x*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5334, 1582, 1410, 1821, 858, 222, 272, 65, 214}

$$\int (d+ex) \left(a+b\sec^{-1}(cx)\right) \, dx = \frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e} - \frac{bd\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c} - \frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{bd^2\csc^{-1}(cx)}{2e}$$

```
[In] Int[(d + e*x)*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/2*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c + (b*d^2*ArcCsc[c*x])/(2*e) + ((d + e*
x)^2*(a + b*ArcSec[c*x]))/(2*e) - (b*d*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1410

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[(d + e/x^n)^q*((a + c/x^(2*n))^p/x^2), x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}-\frac{b\int\frac{(d+ex)^2}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{2ce}\\
&=\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}-\frac{b\int\frac{\left(e+\frac{d}{x}\right)^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{2ce}\\
&=\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}+\frac{b\text{Subst}\left(\int\frac{(e+dx)^2}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2ce}\\
&=-\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}-\frac{b\text{Subst}\left(\int\frac{-2de-d^2x}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2ce}\\
&=-\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}\\
&\quad+\frac{(bd)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c}+\frac{\left(bd^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2ce}\\
&=-\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}\\
&\quad+\frac{(bd)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c}\\
&=-\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\sec^{-1}(cx)\right)}{2e}\\
&\quad-(bcd)\text{Subst}\left(\int\frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)
\end{aligned}$$

$$= -\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{bd^2\csc^{-1}(cx)}{2e} + \frac{(d+ex)^2(a+b\sec^{-1}(cx))}{2e} - \frac{bd\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = adx + \frac{1}{2}aex^2 - \frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c} + bdx\sec^{-1}(cx) + \frac{1}{2}bex^2\sec^{-1}(cx) - \frac{bd\sqrt{1-\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcSec[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 - (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x*
ArcSec[c*x] + (b*e*x^2*ArcSec[c*x])/2 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTan
h[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

method	result	size
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b\left(\frac{c\,\text{arcsec}(cx)x^2e}{2}+\text{arcsec}(cx)xcd-\frac{\sqrt{c^2x^2-1}\left(2dc\ln\left(cx+\sqrt{c^2x^2-1}\right)+e\sqrt{c^2x^2-1}\right)}{2c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}\right)}{c}$	110
derivativedivides	$\frac{\frac{a\left(dx\,c^2+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\text{arcsec}(cx)d\,c^2x+\frac{\text{arcsec}(cx)e\,c^2x^2}{2}-\frac{\sqrt{c^2x^2-1}\left(2dc\ln\left(cx+\sqrt{c^2x^2-1}\right)+e\sqrt{c^2x^2-1}\right)}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c}}{c}$	127
default	$\frac{\frac{a\left(dx\,c^2+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\text{arcsec}(cx)d\,c^2x+\frac{\text{arcsec}(cx)e\,c^2x^2}{2}-\frac{\sqrt{c^2x^2-1}\left(2dc\ln\left(cx+\sqrt{c^2x^2-1}\right)+e\sqrt{c^2x^2-1}\right)}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c}}{c}$	127

```
[In] int((e*x+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arcsec(c*x)*x^2*e+arcsec(c*x)*x*c*d-1/2/c^2/((
c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c^2*x^2-1)^(1/2)*(2*d*c*ln(c*x+(c^2*x^2-1)^(1/
2))+e*(c^2*x^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.55

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{ac^2ex^2+2\,ac^2dx+2\,bcd\log\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}be+\left(bc^2ex^2+2\,bc^2dx-2\,bc^2d-bc^2e\right)\operatorname{arcsec}}{2\,c^2}$$

```
[In] integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*e*x^2 + 2*a*c^2*d*x + 2*b*c*d*log(-c*x + sqrt(c^2*x^2 - 1)) - sq
rt(c^2*x^2 - 1)*b*e + (b*c^2*e*x^2 + 2*b*c^2*d*x - 2*b*c^2*d - b*c^2*e)*arc
sec(c*x) + 2*(2*b*c^2*d + b*c^2*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/c^2
```

Sympy [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.24

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx = adx+\frac{aex^2}{2}+bdx\operatorname{asec}(cx)+\frac{bex^2\operatorname{asec}(cx)}{2}-\frac{bd\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}-\frac{be\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

```
[In] integrate((e*x+d)*(a+b*asec(c*x)),x)
```

```
[Out] a*d*x + a*e*x**2/2 + b*d*x*asec(c*x) + b*e*x**2*asec(c*x)/2 - b*d*Piecewise
((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - b*e*Piecewise(
(sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, Tru
e))/(2*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{2}aex^2+\frac{1}{2}\left(x^2\,\mathrm{arcsec}\,(cx)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)be+adx$$
$$+\frac{\left(2\,cx\,\mathrm{arcsec}\,(cx)-\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}$$

```
[In] integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/2*(x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1)/c)*b*e + a*d*
x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1
/(c^2*x^2) + 1) + 1))*b*d/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1547 vs. 2(74) = 148.

Time = 0.53 (sec) , antiderivative size = 1547, normalized size of antiderivative = 18.42

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c*d*arccos(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b*c*d*log(abs(sqrt(-1/(c^2*
x^2) + 1) + 1/(c*x) + 1))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b*c*d*log(abs(sqrt(-1/(c^2*x^2
) + 1) - 1/(c*x) - 1))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*a*c*d/(c^3 + 2*c^3*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + b*e*arccos
(1/(c*x))/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4) - 4*b*c*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x
^2) + 1) + 1/(c*x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + 4*b*c*d*(1/(c^2
*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^3 + 2*c^3*(1/
(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(
1/(c*x) + 1)^2) + a*e/(c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*
(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - 2*b*e*(1/(c^2*x^2) - 1)*arccos(1/(c*
```

```
x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - 2*b*c*d*(1/(c^2*x^2) - 1)^2*arccos(1
/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 2*b*c*d*(1/(c^2*x^2) - 1)^2*log(
abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4)
 + 2*b*c*d*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1
))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - 2*b*e*sqrt(-1/(c^2*x^2) + 1)/((c^3 +
2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4)*(1/(c*x) + 1)) - 2*a*e*(1/(c^2*x^2) - 1)/((c^3 + 2*c^3*(1/(c^2*x^2)
 - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) +
 1)^2) - 2*a*c*d*(1/(c^2*x^2) - 1)^2/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + b*e
*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) + 2*b
*e*(-1/(c^2*x^2) + 1)^(3/2)/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + c^3*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^3) + a*e*(1/(c^2*
x^2) - 1)^2/((c^3 + 2*c^3*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + c^3*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4))*c
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int (d+ex)\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{a\,x\,(2\,d+e\,x)}{2}-\frac{b\,d\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}+b\,d\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right)-\frac{b\,e\,x\,\left(\sqrt{1-\frac{1}{c^2\,x^2}}-c\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{2\,c}$$

```
[In] int((a + b*acos(1/(c*x)))*(d + e*x),x)
```

```
[Out] (a*x*(2*d + e*x))/2 - (b*d*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c + b*d*x*acos
(1/(c*x)) - (b*e*x*((1 - 1/(c^2*x^2))^(1/2) - c*x*acos(1/(c*x))))/(2*c)
```

3.59 $\int (a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int (a+b\sec^{-1}(cx))\,dx=ax+bx\sec^{-1}(cx)-\frac{b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] a*x+b*x*arcsec(c*x)-b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5322, 272, 65, 214}

$$\int (a+b\sec^{-1}(cx))\,dx=ax-\frac{b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+bx\sec^{-1}(cx)$$

```
[In] Int[a + b*ArcSec[c*x],x]
```

```
[Out] a*x + b*x*ArcSec[c*x] - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5322

```
Int[ArcSec[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSec[c*x], x] - Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \sec^{-1}(cx)\,dx \\
&= ax + bx\sec^{-1}(cx) - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x}\,dx}{c} \\
&= ax + bx\sec^{-1}(cx) + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2c} \\
&= ax + bx\sec^{-1}(cx) - (bc)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right) \\
&= ax + bx\sec^{-1}(cx) - \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int \left(a+b\sec^{-1}(cx)\right)\,dx = ax + bx\sec^{-1}(cx) - \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

```
[In] Integrate[a + b*ArcSec[c*x],x]
```

```
[Out] a*x + b*x*ArcSec[c*x] - (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

method	result	size
default	$ax + bx\ \text{arcsec}\left(cx\right) - \frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$	38
parts	$ax + bx\ \text{arcsec}\left(cx\right) - \frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$	38
derivativedivides	$\frac{acx+b\left(cx\ \text{arcsec}(cx)-\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{c}$	42

```
[In] int(a+b*arcsec(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b*x*arcsec(c*x)-b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \left(a + b\sec^{-1}(cx)\right)\,dx = \frac{acx + 2\,bc\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + (bcx - bc)\,\text{arcsec}\left(cx\right) + b\log\left(-cx + \sqrt{c^2x^2 - 1}\right)}{c}$$

```
[In] integrate(a+b*arcsec(c*x),x, algorithm="fricas")
[Out] (a*c*x + 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arcsec(c*x)
 + b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \left(a + b\sec^{-1}(cx)\right)\,dx = ax + b\left(x\,\text{asec}\left(cx\right) - \frac{\begin{cases} \text{acosh}\left(cx\right) & \text{for}\ |c^2x^2| > 1 \\ -i\,\text{asin}\left(cx\right) & \text{otherwise} \end{cases}}{c}\right)$$

```
[In] integrate(a+b*asec(c*x),x)
[Out] a*x + b*(x*asec(c*x) - Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \left(a + b\sec^{-1}(cx)\right)\,dx = ax + \frac{\left(2\,cx\,\mathrm{arcsec}\left(cx\right) - \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)\right)b}{2\,c}$$

```
[In] integrate(a+b*arcsec(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(30) = 60.

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int \left(a + b\sec^{-1}(cx)\right)\,dx = \frac{1}{2}\,bc\left(\frac{2\,x\arccos\left(\frac{1}{cx}\right)}{c} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)}{c^2}\right) + ax$$

```
[In] integrate(a+b*arcsec(c*x),x, algorithm="giac")
```

```
[Out] 1/2*b*c*(2*x*arccos(1/(c*x))/c - (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sq
rt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \left(a + b\sec^{-1}(cx)\right)\,dx = a\,x + b\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right) - \frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

```
[In] int(a + b*acos(1/(c*x)),x)
```

```
[Out] a*x + b*x*acos(1/(c*x)) - (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

3.60 $\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 247

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx &= \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}\\
&+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}\\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e}\\
&-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}\\
&-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}\end{aligned}$$

```
[Out] -(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e+(a+b*arcsec(c*x)
)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e+(a+b*a
rcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/d
)/e+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e-I*b*polylog(2,-(1
/c/x+I*(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e-I*b*polylog(2,-
(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/d)/e
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5332, 2598}

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} - \frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))}{e} - \frac{ib\,\text{PolyLog}\left(2,-\frac{\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} - \frac{ib\,\text{PolyLog}\left(2,-\frac{\left(e+\sqrt{e^2-c^2d^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x),x]
```

```
[Out] ((a + b*ArcSec[c*x])*Log[1 + ((e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcSec[c*x]
))/(c*d)]])/e + ((a + b*ArcSec[c*x])*Log[1 + ((e + Sqrt[-(c^2*d^2) + e^2])*E
^(I*ArcSec[c*x]))/(c*d)]])/e - ((a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcSec[
c*x])])/e - (I*b*PolyLog[2, -(((e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcSec[c*x
]))/(c*d))])/e - (I*b*PolyLog[2, -(((e + Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcSe
c[c*x]))/(c*d))])/e + ((I/2)*b*PolyLog[2, -E^((2*I)*ArcSec[c*x])])/e
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rule 5332

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :> S
imp[(a + b*ArcSec[c*x])*(Log[1 + (e - Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcSec[
c*x])/(c*d))]/e), x] + (-Dist[b/(c*e), Int[Log[1 + (e - Sqrt[(-c^2)*d^2 + e
^2])*(E^(I*ArcSec[c*x])/(c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] - Dist[
b/(c*e), Int[Log[1 + (e + Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcSec[c*x])/(c*d))
]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] + Dist[b/(c*e), Int[Log[1 + E^(2*I*Ar
```

```
cSec[c*x])]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] + Simp[(a + b*ArcSec[c*x])*
(Log[1 + (e + Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcSec[c*x])/(c*d))]/e), x] - S
imp[(a + b*ArcSec[c*x])*(Log[1 + E^(2*I*ArcSec[c*x])]/e), x]) /; FreeQ[{a,
b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} \\
&\quad+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} \\
&\quad-\frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e}-\frac{b\int\frac{\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce} \\
&\quad-\frac{b\int\frac{\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}+\frac{b\int\frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce} \\
&= \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} \\
&\quad+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} \\
&\quad-\frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e} \\
&\quad-\frac{ib\,\text{PolyLog}\left(2,-\frac{\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e} \\
&\quad-\frac{ib\,\text{PolyLog}\left(2,-\frac{\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\sec^{-1}(cx)}}{cd}\right)}{e}+\frac{ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.35

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx = \frac{a\log(d+ex)}{e} + \frac{b\left(4i\arcsin\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\arctan\left(\frac{(-cd+e)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{-c^2d^2+e^2}}\right)+\left(\sec^{-1}(cx)+2\arcsin\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\right)\log\left(1+\frac{\left(e-\sqrt{-\right.}}{}\right.}{}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*((4*I)*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]*ArcTan[((-
(c*d) + e)*Tan[ArcSec[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcSec[c*x] + 2*A
rcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]])*Log[1 + ((e - Sqrt[-(c^2*d^2) + e^2])*E^(
I*ArcSec[c*x]))/(c*d)] + (ArcSec[c*x] - 2*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]
)*Log[1 + ((e + Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcSec[c*x]))/(c*d)] - ArcSec[
c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - I*(PolyLog[2, ((-e + Sqrt[-(c^2*d^2)
+ e^2])*E^(I*ArcSec[c*x]))/(c*d)] + PolyLog[2, -(((e + Sqrt[-(c^2*d^2) + e^
2])*E^(I*ArcSec[c*x]))/(c*d))]) + (I/2)*PolyLog[2, -E^((2*I)*ArcSec[c*x])])
)/e
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.83

method	result
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\,\mathrm{arcsec}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{b\,\mathrm{arcsec}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2}}{e+\sqrt{-c^2d^2+e^2}}\right.}{e}$
derivativedivides	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\mathrm{arcsec}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{\mathrm{arcsec}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}+e}{e+\sqrt{-c^2d^2+e^2}}\right.}{e}\right.}{}$
default	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\mathrm{arcsec}(cx)\ln\left(\frac{-cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}-e}{-e+\sqrt{-c^2d^2+e^2}}\right)}{e}+\frac{\mathrm{arcsec}(cx)\ln\left(\frac{cd\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)+\sqrt{-c^2d^2+e^2}+e}{e+\sqrt{-c^2d^2+e^2}}\right.}{e}\right.}{}$

```
[In] int((a+b*arcsec(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(e*x+d)/e+b/e*arcsec(c*x)*ln((-c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))+(-c^2*
d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2)))+b/e*arcsec(c*x)*ln((c*d*(1/c/x
```

```
+I*(1-1/c^2/x^2)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(e+(-c^2*d^2+e^2)^(1/2)))-I
*b/e*dilog((c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(e+(-
c^2*d^2+e^2)^(1/2)))-I*b/e*dilog((-c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))+(-c^2*
d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2)))-b/e*arcsec(c*x)*ln(1+I*(1/c/x+
I*(1-1/c^2/x^2)^(1/2)))-b/e*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)
))+I*b/e*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I*b/e*dilog(1-I*(1/c/x+I*
(1-1/c^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d),x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{d+ex}\,dx$$

```
[In] integrate((a+b*asec(c*x))/(e*x+d),x)
[Out] Integral((a + b*asec(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d),x, algorithm="maxima")
[Out] b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x + d), x) + a*log(e*x +
 d)/e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{d + ex}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{d + ex}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{d + e\,x}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x), x)
```

3.61 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2} \, dx = -\frac{b\csc^{-1}(cx)}{de} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)} - \frac{b\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2-e^2}}$$

```
[Out] -b*arccsc(c*x)/d/e+(-a-b*arcsec(c*x))/e/(e*x+d)-b*arctanh((c^2*d+e/x)/c/(c^
2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d/(c^2*d^2-e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5334, 1582, 1489, 858, 222, 739, 212}

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2} \, dx = -\frac{a+b\sec^{-1}(cx)}{e(d+ex)} - \frac{b\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}} - \frac{b\csc^{-1}(cx)}{de}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x)^2,x]
```

```
[Out] -((b*ArcCsc[c*x])/(d*e)) - (a + b*ArcSec[c*x])/(e*(d + e*x)) - (b*ArcTanh[(
c^2*d + e/x)/(c*Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])/(d*Sqrt[c^2*d^
2 - e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\sec^{-1}(cx)}{e(d+ex)}+\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)}\,dx}{ce}$$

$$= -\frac{a+b\sec^{-1}(cx)}{e(d+ex)} + \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)x^3}dx}{ce}$$

$$= -\frac{a+b\sec^{-1}(cx)}{e(d+ex)} - \frac{b\text{Subst}\left(\int \frac{x}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{ce}$$

$$= -\frac{a+b\sec^{-1}(cx)}{e(d+ex)} + \frac{b\text{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{cd} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{cde}$$

$$= -\frac{b\csc^{-1}(cx)}{de} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)} - \frac{b\text{Subst}\left(\int \frac{1}{d^2-\frac{e^2}{c^2}-x^2}dx, x, \frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{cd}$$

$$= -\frac{b\csc^{-1}(cx)}{de} - \frac{a+b\sec^{-1}(cx)}{e(d+ex)} - \frac{b\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2-e^2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.37

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}dx = -\frac{a}{e(d+ex)} - \frac{b\sec^{-1}(cx)}{e(d+ex)} - \frac{b\arcsin\left(\frac{1}{cx}\right)}{de} - \frac{b\log(d+ex)}{d\sqrt{c^2d^2-e^2}} + \frac{b\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d\sqrt{c^2d^2-e^2}}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x)^2,x]
```

```
[Out] -(a/(e*(d + e*x))) - (b*ArcSec[c*x])/(e*(d + e*x)) - (b*ArcSin[1/(c*x)])/(d
*e) - (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 - e^2]) + (b*Log[e + c*(c*d - Sqrt[c
^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 - e^2])
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.86

method	result
parts	$-\frac{a}{(ex+d)e}+\frac{b\left(-\frac{c^2\,\mathrm{arcsec}(cx)}{(cex+cd)e}-\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)}{c}$
derivativedivides	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\mathrm{arcsec}(cx)}{(cex+cd)e}-\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)}{c}$
default	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\mathrm{arcsec}(cx)}{(cex+cd)e}-\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)}{c}$

```
[In] int((a+b*arcsec(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(e*x+d)/e+b/c*(-c^2/(c*e*x+c*d)/e*arcsec(c*x)-1/e*(c^2*x^2-1)^(1/2)*(arc
tan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)-ln(2*(((c^2*d^2-e^2)/e^2
)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d)))/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x/d/((c^2*d^2-e^2)/e^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(98) = 196$.

Time = 0.31 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.59

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx$$

$$=\left[-\frac{ac^2d^3-ade^2-\sqrt{c^2d^2-e^2}(be^2x+bde)\log\left(\frac{c^3d^2x+cde-\sqrt{c^2d^2-e^2}(c^2dx+e)+\left(c^2d^2-\sqrt{c^2d^2-e^2}cd-e^2\right)\sqrt{c^2x^2-1}}{ex+d}\right)+(b}{c^2d^4e-d^2e^3+(c^2d^3e^2}\right.$$

$$-\frac{ac^2d^3-ade^2-2\sqrt{-c^2d^2+e^2}(be^2x+bde)\arctan\left(-\frac{\sqrt{-c^2d^2+e^2}\sqrt{c^2x^2-1}e-\sqrt{-c^2d^2+e^2}(cex+cd)}{c^2d^2-e^2}\right)+(bc^2d^3-bd}{c^2d^4e-d^2e^3+(c^2d^3e^2-de^4}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-(a*c^2*d^3 - a*d*e^2 - sqrt(c^2*d^2 - e^2)*(b*e^2*x + b*d*e)*log((c^3*d^2
*x + c*d*e - sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 - sqrt(c^2*d^2 -
```

```
e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + (b*c^2*d^3 - b*d*e^2)*arcse
c(c*x) - 2*(b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x + sq
rt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x), -(a*c^2*
d^3 - a*d*e^2 - 2*sqrt(-c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*arctan(-(sqrt(-c^2
*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))/(c^2*
d^2 - e^2)) + (b*c^2*d^3 - b*d*e^2)*arcsec(c*x) - 2*(b*c^2*d^3 - b*d*e^2 +
(b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2
*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

Sympy [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a + b\operatorname{asec}(cx)}{(d+ex)^2}\,dx$$

```
[In] integrate((a+b*asec(c*x))/(e*x+d)**2,x)
[Out] Integral((a + b*asec(c*x))/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{b\operatorname{arcsec}(cx) + a}{(ex+d)^2}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^2,x, algorithm="maxima")
[Out] ((c^2*e^2*x + c^2*d*e)*integrate(x*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))/
(c^2*e^2*x^3 + c^2*d*e*x^2 - e^2*x - d*e + (c^2*e^2*x^3 + c^2*d*e*x^2 - e^2
*x - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) - arctan(sqrt(c*x + 1)*sqrt(
c*x - 1))*b/(e^2*x + d*e) - a/(e^2*x + d*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{(d+ex)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^2,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^2}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{\left(d+e\,x\right)^2}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x)^2,x)
[Out] int((a + b*acos(1/(c*x)))/(d + e*x)^2, x)
```

3.62 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 172

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{b\csc^{-1}(cx)}{2d^2e} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} - \frac{b(2c^2d^2-e^2)\,\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}}$$

```
[Out] -1/2*b*arccsc(c*x)/d^2/e+1/2*(-a-b*arcsec(c*x))/e/(e*x+d)^2-1/2*b*(2*c^2*d^
2-e^2)*arctanh((c^2*d+e/x)/c/(c^2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^2/(
c^2*d^2-e^2)^(3/2)+1/2*b*c*e*(1-1/c^2/x^2)^(1/2)/d/(c^2*d^2-e^2)/(e+d/x)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5334, 1582, 1489, 1665, 858, 222, 739, 212}

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} - \frac{b(2c^2d^2-e^2)\,\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(\frac{d}{x}+e\right)} - \frac{b\csc^{-1}(cx)}{2d^2e}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x)^3,x]
```

```
[Out] (b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(2*d*(c^2*d^2 - e^2)*(e + d/x)) - (b*ArcCsc[c
*x])/(2*d^2*e) - (a + b*ArcSec[c*x])/(2*e*(d + e*x)^2) - (b*(2*c^2*d^2 - e^
2)*ArcTanh[(c^2*d + e/x)/(c*Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])/(2
*d^2*(c^2*d^2 - e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} + \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^2}\,dx}{2ce}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} + \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)^2x^4}\,dx}{2ce}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} - \frac{b\text{Subst}\left(\int \frac{x^2}{(e+dx)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce}$$

$$= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} + \frac{(bc)\text{Subst}\left(\int \frac{e-\left(d-\frac{e^2}{c^2d}\right)x}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2e\left(c^2d^2-e^2\right)}$$

$$= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2cd^2e} + \frac{\left(bc\left(2-\frac{e^2}{c^2d^2}\right)\right)\text{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2\left(c^2d^2-e^2\right)}$$

$$= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{b\csc^{-1}(cx)}{2d^2e} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} - \frac{\left(bc\left(2-\frac{e^2}{c^2d^2}\right)\right)\text{Subst}\left(\int \frac{1}{d^2-\frac{e^2}{c^2}-x^2}\,dx, x, \frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2\left(c^2d^2-e^2\right)}$$

$$= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{b\csc^{-1}(cx)}{2d^2e} - \frac{a+b\sec^{-1}(cx)}{2e(d+ex)^2} - \frac{b(2c^2d^2-e^2)\operatorname{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \frac{1}{2}\left(-\frac{a}{e(d+ex)^2} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x}{d\left(c^2d^2-e^2\right)(d+ex)} - \frac{b\sec^{-1}(cx)}{e(d+ex)^2} - \frac{b\arcsin\left(\frac{1}{cx}\right)}{d^2e} + \frac{b(-2c^2d^2+e^2)\log(d+ex)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}} + \frac{b(2c^2d^2-e^2)\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}\right)$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d^2 - e^2)*
(d + e*x)) - (b*ArcSec[c*x])/(e*(d + e*x)^2) - (b*ArcSin[1/(c*x)])/(d^2*e)
+ (b*(-2*c^2*d^2 + e^2)*Log[d + e*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2
 - e^2]) + (b*(2*c^2*d^2 - e^2)*Log[e + c*(c*d - Sqrt[c^2*d^2 - e^2]*Sqrt[1
 - 1/(c^2*x^2)])*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2 - e^2]))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(159) = 318$.

Time = 2.07 (sec) , antiderivative size = 574, normalized size of antiderivative = 3.34

method	result
parts	$-\frac{a}{2(ex+d)^2 e}+\frac{b\left(-\frac{c^3 \operatorname{arcsec}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{c^2x^2-1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^2ex-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d \cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\frac{a c^3}{2(cex+cd)^2 e}+b c^3\left(-\frac{\operatorname{arcsec}(cx)}{2(cex+cd)^2 e}-\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^3+\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^2ex \cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{a c^3}{2(cex+cd)^2 e}+b c^3\left(-\frac{\operatorname{arcsec}(cx)}{2(cex+cd)^2 e}-\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^3+\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^2ex \cdots\right.}{\cdots}\right.}{\cdots}$

```
[In] int((a+b*arcsec(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(e*x+d)^2/e+b/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arcsec(c*x)+1/2/e*(c^2*x^2
-1)^(1/2)*(-arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c^3*d^2*e
*x-arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c^3*d^3+2*ln(2*(((
c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*c^3*d^2
*e*x+2*ln(2*(((c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*
x+c*d))*c^3*d^3+(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*c*d*e^2+arctan(
1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*e^3*c*x+arctan(1/(c^2*x^2-1)
^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c*d*e^2-ln(2*(((c^2*d^2-e^2)/e^2)^(1/2)*(
c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*e^3*c*x-ln(2*(((c^2*d^2-e^2)/e^2
)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*c*d*e^2)/((c^2*x^2-1)/c
^2/x^2)^(1/2)/x/d^2/(c^2*d^2-e^2)/(c*e*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(156) = 312$.

Time = 0.55 (sec) , antiderivative size = 1117, normalized size of antiderivative = 6.49

$$\int \frac{a + b\sec^{-1}(cx)}{(d+ex)^3}\,dx$$
$$= \Bigg[-\frac{ac^4d^6 - bc^3d^5e - 2\,ac^2d^4e^2 + bcd^3e^3 + ad^2e^4 - (bc^3d^3e^3 - bcde^5)x^2 - (2\,bc^2d^4e - bd^2e^3 + (2\,bc^2d^2e^3 - b}{}$$
$$-\frac{ac^4d^6 - bc^3d^5e - 2\,ac^2d^4e^2 + bcd^3e^3 + ad^2e^4 - (bc^3d^3e^3 - bcde^5)x^2 - 2\,(2\,bc^2d^4e - bd^2e^3 + (2\,bc^2d^2e^3 -}{}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*c^4*d^6 - b*c^3*d^5*e - 2*a*c^2*d^4*e^2 + b*c*d^3*e^3 + a*d^2*e^4
- (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d
^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(c^2*d^2 - e^2)*
log((c^3*d^2*x + c*d*e - sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 - sqr
t(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) - 2*(b*c^3*d^4*e^
2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arcsec(c*x)
- 2*(b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2
*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*arctan(-
c*x + sqrt(c^2*x^2 - 1)) - (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e^3 - b*
d*e^5)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^
6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3
*e^6)*x), -1/2*(a*c^4*d^6 - b*c^3*d^5*e - 2*a*c^2*d^4*e^2 + b*c*d^3*e^3 + a
*d^2*e^4 - (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - 2*(2*b*c^2*d^4*e - b*d^2*e^3 +
 (2*b*c^2*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(-c^2
*d^2 + e^2)*arctan(-(sqrt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d
^2 + e^2)*(c*e*x + c*d))/(c^2*d^2 - e^2)) - 2*(b*c^3*d^4*e^2 - b*c*d^2*e^4)
*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arcsec(c*x) - 2*(b*c^4*d^6 -
 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b*e^6)*x^
2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*arctan(-c*x + sqrt(c^2*x
^2 - 1)) - (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e^3 - b*d*e^5)*x)*sqrt(c
^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 - 2*c^2*d^
4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3*e^6)*x)]
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{a+b\,\mathrm{asec}\,(cx)}{(d+ex)^3}\,dx$$

```
[In] integrate((a+b*asec(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*asec(c*x))/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2*e)*integrate(1/2*x*e^(1/2*log
(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x -
d^2*e + (c^2*d^2*e - e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x
- d^2*e + (c^2*d^2*e - e^3)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) - arc
tan(sqrt(c*x + 1)*sqrt(c*x - 1))*b/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/2*a/(
e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^3}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{\left(d+e\,x\right)^3}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x)^3,x)
[Out] int((a + b*acos(1/(c*x)))/(d + e*x)^3, x)
```

3.63 $\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 372

$$\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} + \frac{28bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] 2/5*(e*x+d)^(5/2)*(a+b*arcsec(c*x))/e+4/15*b*e*(-c^2*x^2+1)*(e*x+d)^(1/2)/c
^3/x/(1-1/c^2/x^2)^(1/2)+28/15*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(
1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(1-1/c^2/x^2
)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*(-c*
x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c
^2*x^2+1)^(1/2)/c^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*d^3*EllipticP
i(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e
))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5334, 1588, 972, 733, 430, 947, 174, 552, 551, 858, 435, 945, 1598}

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}+\frac{4bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}+\frac{28bd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b\sqrt{1-c^2x^2}(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}+\frac{4be(1-c^2x^2)\sqrt{d+ex}}{15c^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

```
[In] Int[(d + e*x)^(3/2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] (4*b*e*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*(
d + e*x)^(5/2)*(a + b*ArcSec[c*x]))/(5*e) + (28*b*d*Sqrt[d + e*x]*Sqrt[1 -
c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2
*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*(2*c^2*d^2 +
 e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) + (4*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipt
icPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*c*e*Sqrt[1 - 1/
(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
```

```
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} - \frac{(2b)\int \frac{(d+ex)^{5/2}}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{5ce}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{5/2}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{d^3}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{3de^2x}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{e^3x^2}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\right)dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} - \frac{\left(6bd^2\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(2bd^3\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(6bde\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(2be^2\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{x^2}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} - \frac{\left(6bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(6bd^2\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(2be\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{-\frac{ex}{c^2}+2dx^2}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(2bd^3\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(12bd^2\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= \frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}$$
$$+ \frac{12bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{\left(2be\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{-\frac{e}{c^2}+2dx}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(4bd^3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(12bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$
$$- \frac{\left(12bd^2\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}$$
$$+ \frac{12bd\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{\left(4bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(2b\left(-2d^2-\frac{e^2}{c^2}\right)\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}$$
$$+ \frac{12bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{\left(8bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$
$$- \frac{\left(4b\left(-2d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}$$
$$+ \frac{28bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{4b(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.35 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.90

$$\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \frac{1}{15}\left(-\frac{4be\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c} + \frac{6a(d+ex)^{5/2}}{e} + \frac{6b(d+ex)^{5/2}\sec^{-1}(cx)}{e} + \frac{4ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-7cd(cd-e)E\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(9c^2d^2-7cde+e^2)\,\text{EllipticF}\left(i\ldots\right.\right.}{c^3e\sqrt{-\frac{c}{cd+e}}\sqrt{1\ldots}}\right.$$

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] ((-4*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (6*a*(d + e*x)^(5/2))/e
 + (6*b*(d + e*x)^(5/2)*ArcSec[c*x])/e + ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d
) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-7*c*d*(c*d - e)*EllipticE[I*ArcSinh[S
qrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (9*c^2*d^2 - 7*c
*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d +
 e)/(c*d - e)] - 3*c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d
+ e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^3*e*Sqrt[-(c/(c*d + e))]*S
qrt[1 - 1/(c^2*x^2)]*x))/15
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. 2(335) = 670.

Time = 9.08 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.15

method	result
derivativedivides	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsec}(cx)}{5}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}+9d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{cd\cdots}\right.\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsec}(cx)}{5}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}+9d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{cd\cdots}\right.\right.}{\cdots}\right.}{\cdots}$
parts	$\frac{2a(ex+d)^{\frac{5}{2}}}{5e}+\frac{2b\left(\frac{(ex+d)^{\frac{5}{2}}\operatorname{arcsec}(cx)}{5}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}-2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}+9d^2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-c\cdots}{cd+e}}\right.}{\cdots}\right.}{\cdots}$

```
[In] int((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/5*a*(e*x+d)^(5/2)+b*(1/5*(e*x+d)^(5/2)*arcsec(c*x)-2/15/c^3*((c/(c*d
-e))^(1/2)*c^2*(e*x+d)^(5/2)+9*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*
(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c
*d-e)/(c*d+e))^(1/2))*c^2-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)
+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(
c*d+e))^(1/2))*c^2*d^2-3*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d
)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d
-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2-2*(c/(c*d-e))^(1/2)*c^2*d*(e
*x+d)^(3/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e
))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))
*c*d*e-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1
/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*
e+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2
)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*(e*x+d)^(1/2))/(c/(c
*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1
/2)))
```

Fricas [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Timed out}$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int (a+b\operatorname{asec}(cx))\,(d+ex)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x+d)**(3/2)*(a+b*asec(c*x)),x)
[Out] Integral((a + b*asec(c*x))*(d + e*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int (ex+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)\,dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^(3/2)*(b*arcsec(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)^{3/2}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))*(d + e*x)^(3/2),x)
[Out] int((a + b*acos(1/(c*x)))*(d + e*x)^(3/2), x)
```

3.64 $\int \sqrt{d+ex}(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 315

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} + \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{4bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arcsec(c*x))/e+4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^
(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(1-
1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/3*b*d*EllipticF(1/2*(-c*x+1)^(
1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2
+1)^(1/2)/c^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*d^2*EllipticPi(1/2*
(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/
2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5334, 1588, 972, 733, 430, 947, 174, 552, 551, 858, 435}

$$\begin{aligned}
&\int \sqrt{d+ex}(a+b\sec^{-1}(cx))\,dx \\
&= \frac{2(d+ex)^{3/2}\,(a+b\sec^{-1}(cx))}{3e} \\
&\quad + \frac{4bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&\quad + \frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&\quad + \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}
\end{aligned}$$

```
[In] Int[Sqrt[d + e*x]*(a + b*ArcSec[c*x]),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcSec[c*x]))/(3*e) + (4*b*Sqrt[d + e*x]*Sqrt[1 -
 c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2
*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*d*Sqrt[(c*(d
 + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2
]], (2*e)/(c*d + e)])/(3*c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*
d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{(2b)\int \frac{(d+ex)^{3/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{3ce} \\
&= \frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{3/2}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= \frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} \\
&\quad - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(\frac{2de}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{d^2}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{e^2x}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\right)dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= \frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{\left(4bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(2bd^2\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2be\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$=\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}-\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$+\frac{\left(2bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{\left(2bd^2\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$+\frac{\left(8bd\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$=\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}$$

$$+\frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+\frac{\left(4bd^2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$+\frac{\left(4b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$

$$-\frac{\left(4bd\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$=\frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}+\frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$

$$+\frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+\frac{\left(4bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} + \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$

$$+ \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+ \frac{4bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.34 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.88

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= \frac{2\left(a(d+ex)^{3/2}+b(d+ex)^{3/2}\sec^{-1}(cx)+\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left((-cd+e)E\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(2cd-e)\,\text{Elli}\ldots}{c^2\ldots}\right.}{3e}$$

```
[In] Integrate[Sqrt[d + e*x]*(a + b*ArcSec[c*x]),x]
```

```
[Out] (2*(a*(d + e*x)^(3/2) + b*(d + e*x)^(3/2)*ArcSec[c*x] + ((2*I)*b*Sqrt[(e*(1
 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*((-(c*d) + e)*EllipticE[
I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (2*c*
d - e)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(
c*d - e)] - c*d*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt
[d + e*x]], (c*d + e)/(c*d - e)]))/(c^2*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^
2*x^2)]*x)))/(3*e)
```

Maple [A] (verified)

Time = 7.74 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsec}(cx)}{3}-\frac{2\left(2d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\mathrm{EllipticE}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\,\mathrm{Ellip}\right.}{}\right.}{}$
default	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsec}(cx)}{3}-\frac{2\left(2d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\mathrm{EllipticE}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\,\mathrm{Ellip}\right.}{}\right.}{}$
parts	$\frac{2a(ex+d)^{\frac{3}{2}}}{3e}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arcsec}(cx)}{3}-\frac{2\left(2d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\mathrm{EllipticE}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\,\mathrm{E}\right.}{}\right.}{}$

```
[In] int((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arcsec(c*x)-2/3/c^2*(2*d*Elli
pticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c-EllipticE(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-d*EllipticPi((
e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^
(1/2))*c+EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))
*e-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e)*((
-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)/(c/(c*d
-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2
)))
```

Fricas [F]

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}\,(b\operatorname{arcsec}(cx)+a)\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arcsec(c*x) + a), x)
```

Sympy [F]

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx = \int (a+b\operatorname{asec}(cx))\sqrt{d+ex}\,dx$$

```
[In] integrate((e*x+d)**(1/2)*(a+b*asec(c*x)),x)
[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}(b\operatorname{arcsec}(cx)+a)\,dx$$

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arcsec(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\sqrt{d+e\,x}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int((a + b*acos(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.65 $\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 212

$$
\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx &= \frac{2\sqrt{d+ex}\,(a+b\sec^{-1}(cx))}{e} \\
&+ \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\
&+ \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}
\end{aligned}
$$

```
[Out] 2*(a+b*arcsec(c*x))*(e*x+d)^(1/2)/e+4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2
),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c
^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*
2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)
^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{5334, 1588, 958, 733, 430, 947, 174, 552, 551\}$

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}(a+b\sec^{-1}(cx))}{e} + \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcSec[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcSec[c*x]))/e + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)
]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e
)])/(c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*d*Sqrt[(c*(d + e*x))
/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{d+ex}(a+b\sec^{-1}(cx))}{e} - \frac{(2b)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{ce}$$

$$= \frac{2\sqrt{d+ex}(a+b\sec^{-1}(cx))}{e} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2\sqrt{d+ex}(a+b\sec^{-1}(cx))}{e} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{c\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2\sqrt{d+ex}(a+b\sec^{-1}(cx))}{e} - \frac{\left(2bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}(a+b\sec^{-1}(cx))}{e} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{\left(4bd\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2\sqrt{d+ex}(a+b\sec^{-1}(cx))}{e} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{\left(4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \sqrt{1-cx}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}(a+b\sec^{-1}(cx))}{e} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\left(a\sqrt{d+ex}+b\sqrt{d+ex}\sec^{-1}(cx)+\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right),\frac{cd+e}{cd-e}\right)-\text{EllipticPi}\left(1+\frac{e}{cd},i\right.\right.}{c\sqrt{-\frac{c}{cd+e}}\sqrt{1-\frac{1}{c^2x^2}}x}\right.}{e}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*Sqrt[d + e*x] + b*Sqrt[d + e*x]*ArcSec[c*x] + ((2*I)*b*Sqrt[(e*(1 + c
*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(EllipticF[I*ArcSinh[Sqrt[-(
c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - EllipticPi[1 + e/(c*d)
, I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c*
Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^2)]*x)))/e
```

Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arcsec}(cx)-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticPi}\left(\sqrt{\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}\right.}{e}$
default	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arcsec}(cx)-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticPi}\left(\sqrt{\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}\right.}{e}$
parts	$\frac{2a\sqrt{ex+d}}{e}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arcsec}(cx)-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticPi}\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}\right.}{e}$

```
[In] int((a+b*arcsec(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*((e*x+d)^(1/2)*a+b*((e*x+d)^(1/2)*arcsec(c*x)-2/c*((-c*(e*x+d)+c*d-e)/(
c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)*(
c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(c/(c*d-
e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*x+d)
^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx=\int \frac{b\,\operatorname{arcsec}(cx)+a}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsec(c*x) + a)/sqrt(e*x + d), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex}}\,dx=\int \frac{a+b\,\operatorname{asec}(cx)}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*asec(c*x))/sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{\sqrt{d + ex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a + b\sec^{-1}(cx)}{\sqrt{d + ex}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{\sqrt{ex + d}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{\sqrt{d + ex}}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{\sqrt{d + e\,x}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x)^(1/2), x)
```

3.66 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2(a+b\sec^{-1}(cx))}{e\sqrt{d+ex}} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] -2*(a+b*arcsec(c*x))/e/(e*x+d)^(1/2)-4*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1
/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/
2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5334, 1588, 947, 174, 552, 551}

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2(a+b\sec^{-1}(cx))}{e\sqrt{d+ex}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSec[c*x]))/(e*Sqrt[d + e*x]) - (4*b*Sqrt[(c*(d + e*x))/(c*d +
 e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(
c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\sec^{-1}(cx))}{e\sqrt{d+ex}} + \frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{ce}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{e\sqrt{d+ex}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{e\sqrt{d+ex}} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{e\sqrt{d+ex}} - \frac{\left(4b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{e\sqrt{d+ex}} - \frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \sqrt{1-cx}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{e\sqrt{d+ex}} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2, \arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2\left((-1+c^2x^2)\,(a+b\sec^{-1}(cx)) + 2bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2, \arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)\right)}{e\sqrt{d+ex}\,(-1+c^2x^2)}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*((-1 + c^2*x^2)*(a + b*ArcSec[c*x]) + 2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqr
t[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 -
c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(e*Sqrt[d + e*x]*(-1 + c^2*x^2))
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.81

method	result	s
derivativedivides	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	2
default	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	2
parts	$-\frac{2a}{\sqrt{ex+d}\,e}+\frac{2b\left(-\frac{\operatorname{arcsec}(cx)}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	2

```
[In] int((a+b*arcsec(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arcsec(c*x)-2/c/((c^2*(e*x+d)^2-2
*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/d/(c/(c*d-e))^(1/2)*((-c*(
e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))
^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{b\operatorname{arcsec}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arcsec(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*asec(c*x))/(d + e*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x)^(3/2), x)
```

3.67 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 298

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{4be(1-c^2x^2)}{3cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] -2/3*(a+b*arcsec(c*x))/e/(e*x+d)^(3/2)-4/3*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e^
2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d
^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*EllipticPi(1/
2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(
1/2)*(-c^2*x^2+1)^(1/2)/c/d/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5334, 1588, 972, 759, 21, 733, 435, 947, 174, 552, 551}

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3dx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cdex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4be(1-c^2x^2)}{3cdx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*e*(1 - c^2*x^2))/(3*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) - (2*(a + b*ArcSec[c*x]))/(3*e*(d + e*x)^(3/2)) + (4*b*Sqrt[d + e
*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
 e)])/(3*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d
+ e)]) - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2,
 ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*d*e*Sqrt[1 - 1/(c^2*
x^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
```

```
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\right)dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3cd\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3cde\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be(1-c^2x^2)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{\left(4b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3cd\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{3cde\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be(1-c^2x^2)}{3cd\,(c^2d^2-e^2)\,\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3cd\,\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(4b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be(1-c^2x^2)}{3cd\,(c^2d^2-e^2)\,\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \sqrt{1-cx}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{\left(4b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2d\,\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$

$$= -\frac{4be(1-c^2x^2)}{3cd\,(c^2d^2-e^2)\,\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\sec^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\,(c^2d^2-e^2)\,\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.38 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.09

$$\int\frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \frac{2\left(-\frac{a}{(d+ex)^{3/2}}+\frac{2bce^2\sqrt{1-\frac{1}{c^2x^2}}x}{(c^2d^3-de^2)\sqrt{d+ex}}-\frac{b\sec^{-1}(cx)}{(d+ex)^{3/2}}-\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-cdE\left(i\,\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\right.\right.\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (2*(-(a/(d + e*x)^(3/2)) + (2*b*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d^3 -
d*e^2)*Sqrt[d + e*x]) - (b*ArcSec[c*x])/(d + e*x)^(3/2) - ((2*I)*b*Sqrt[(e*
(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-(c*d*EllipticE[I*Arc
Sinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]) + c*d*Ellip
ticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] +
(c*d + e)*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e
*x]], (c*d + e)/(c*d - e)]))/(d^2*(-(c/(c*d + e)))^(3/2)*(c*d + e)^2*Sqrt[1
 - 1/(c^2*x^2)]*x)))/(3*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(270) = 540$.

Time = 7.33 (sec) , antiderivative size = 875, normalized size of antiderivative = 2.94

method	result
derivativedivides	$\frac{-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2\sqrt{ex+d}-\sqrt{\frac{-c(\cdots}{}}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsec}(cx)}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2\sqrt{ex+d}-\sqrt{\frac{-c(\cdots}{}}\cdots\right.}{\cdots}\right.}{\cdots}$
parts	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}e}+\frac{2b\left(-\frac{\operatorname{arcsec}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^2}{3}-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\cdots}\right.}{3}\cdots\right.}{\cdots}$

```
[In] int((a+b*arcsec(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arcsec(c*x)-2/3/c*(((-c*(e*
x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)
-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elli
pticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*(e*x
+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(
1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(
1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d*(e*x+
d)^2+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*
EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e*(e
*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))
^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c
```

```
*d*e*(e*x+d)^(1/2)+2*(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)-((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*e^2*(e
*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d^3+(c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(c/(
c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/d^2/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c
^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arcsec(c*x) + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e
*x + d^3), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{(d+ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Integral((a + b*asec(c*x))/(d + e*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^{5/2}}\, dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{(ex + d)^{\frac{5}{2}}}\, dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex)^{5/2}}\, dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{5/2}}\, dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x)^(5/2),x)
[Out] int((a + b*acos(1/(c*x)))/(d + e*x)^(5/2), x)
```

3.68 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 540

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx &= -\frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} \\ &\quad -\frac{16bce(1-c^2x^2)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}-\frac{4be(1-c^2x^2)}{5cd^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\ &\quad -\frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}}+\frac{4b(7c^2d^2-3e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15\left(c^2d^3-de^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} \\ &\quad -\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\ &\quad -\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}\end{aligned}$$

```
[Out] -2/5*(a+b*arcsec(c*x))/e/(e*x+d)^(5/2)-4/15*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e
^2)/x/(e*x+d)^(3/2)/(1-1/c^2/x^2)^(1/2)-16/15*b*c*e*(-c^2*x^2+1)/(c^2*d^2-e
^2)^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*e*(-c^2*x^2+1)/c/d^2/(c^2*d
^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/15*b*(7*c^2*d^2-3*e^2)*Ellipt
icE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c
^2*x^2+1)^(1/2)/(c^2*d^3-d*e^2)^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)-4/15*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2
))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/x/(1-1/c^2/
x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1
```

```
/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d^2/e
/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.18, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5334, 1588, 972, 759, 849, 858, 733, 435, 430, 21, 947, 174, 552, 551}

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15dx\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{d+ex}} + \frac{16bc^2\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)^2\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{16bce(1-c^2x^2)}{15x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)^2\sqrt{d+ex}} - \frac{4be(1-c^2x^2)}{5cd^2x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{d+ex}} - \frac{4be(1-c^2x^2)}{15cdx\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)(d+ex)^{3/2}}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (-4*b*e*(1 - c^2*x^2))/(15*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
 e*x)^(3/2)) - (16*b*c*e*(1 - c^2*x^2))/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*e*(1 - c^2*x^2))/(5*c*d^2*(c^2*d^2 - e^2)*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*(a + b*ArcSec[c*x]))/(5*e*(d +
e*x)^(5/2)) + (16*b*c^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2
*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*(c^
2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*
Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c
*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]
```

```
*x*Sqrt[d + e*x]) - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*El
lipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*c*d^2*e*Sqr
t[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
```

```
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5334

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSec[c*x])/(e*(m + 1))), x] - Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{5/2}}\,dx}{5ce}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{5/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{5/2}\sqrt{-\frac{1}{c^2}+x^2}} - \frac{e}{d^2(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{1}{d^2x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\right)dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{5/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be(1-c^2x^2)}{15cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{4be(1-c^2x^2)}{5cd^2\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{\left(4b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd^2\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(4b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{-\frac{3d}{2}+\frac{ex}{2}}{(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15cd\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be(1-c^2x^2)}{15cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce(1-c^2x^2)}{15\,(c^2d^2-e^2)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{4be(1-c^2x^2)}{5cd^2\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}}$$
$$- \frac{\left(8b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\frac{1}{4}\left(3d^2+\frac{e^2}{c^2}\right)+dex}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15cd\left(d^2-\frac{e^2}{c^2}\right)^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd^2\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(4b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}}$$

$$-\frac{16bce(1-c^2x^2)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4be(1-c^2x^2)}{5cd^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$-\frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{\left(8b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15c\left(d^2-\frac{e^2}{c^2}\right)^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$-\frac{\left(2b\left(-d^2+\frac{e^2}{c^2}\right)\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15cd\left(d^2-\frac{e^2}{c^2}\right)^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$-\frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+\frac{\left(4b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2d^2\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$

$$= -\frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce(1-c^2x^2)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{4be(1-c^2x^2)}{5cd^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}}$$
$$+\frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$-\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+\frac{\left(16b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2\left(d^2-\frac{e^2}{c^2}\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$
$$+\frac{\left(4b\left(-d^2+\frac{e^2}{c^2}\right)\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2d\left(d^2-\frac{e^2}{c^2}\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce(1-c^2x^2)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{4be(1-c^2x^2)}{5cd^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\sec^{-1}(cx))}{5e(d+ex)^{5/2}}$$
$$+\frac{16bc^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+\frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$-\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$-\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.62 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.75

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \frac{2\left(-\frac{3a}{(d+ex)^{5/2}}+\frac{2bce^2\sqrt{1-\frac{1}{c^2x^2}}\,x\left(-e^2(4d+3ex)+c^2d^2(8d+7ex)\right)}{(c^2d^3-de^2)^2(d+ex)^{3/2}}-\frac{3b\sec^{-1}(cx)}{(d+ex)^{5/2}}+\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (2*((-3*a)/(d + e*x)^(5/2) + (2*b*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*(-(e^2*(4*d
 + 3*e*x)) + c^2*d^2*(8*d + 7*e*x)))/((c^2*d^3 - d*e^2)^2*(d + e*x)^(3/2))
- (3*b*ArcSec[c*x])/(d + e*x)^(5/2) + ((2*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) +
 e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(c*d*(7*c^2*d^2 - 3*e^2)*EllipticE[I*ArcSi
nh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - c*d*(6*c^2*d
^2 - c*d*e - 3*e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]]
, (c*d + e)/(c*d - e)] - 3*(c*d - e)*(c*d + e)^2*EllipticPi[1 + e/(c*d), I*
ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(d^3*(c
*d - e)*(-(c/(c*d + e)))^(3/2)*(c*d + e)^3*Sqrt[1 - 1/(c^2*x^2)]*x)))/(15*e
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. 2(491) = 982.

Time = 8.78 (sec) , antiderivative size = 1618, normalized size of antiderivative = 3.00

method	result	size
derivativedivides	Expression too large to display	1618
default	Expression too large to display	1618
parts	Expression too large to display	1642

```
[In] int((a+b*arcsec(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arcsec(c*x)+2/15/c*(7*(c/(c
*d-e))^(1/2)*c^4*d^3*(e*x+d)^3-6*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e
*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d
-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1
/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^
(1/2),((c*d-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^4*d^
4*(e*x+d)^(3/2)-13*(c/(c*d-e))^(1/2)*c^4*d^4*(e*x+d)^2-7*((-c*(e*x+d)+c*d-e
```

)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3*e*(e*x+d)^(3/2)+7*((-c*
(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3*e*(e*x+d)^
(3/2)+5*(c/(c*d-e))^(1/2)*c^4*d^5*(e*x+d)-3*(c/(c*d-e))^(1/2)*c^2*d*e^2*(e*
x+d)^3+2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1
/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*
d^2*e^2*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d
+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)+6*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*(
(-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2
),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3
/2)+(c/(c*d-e))^(1/2)*c^4*d^6+5*(c/(c*d-e))^(1/2)*c^2*d^2*e^2*(e*x+d)^2+3*(
(-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ellipt
icF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^3*(e*x+d
)^(3/2)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(
1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d
e^3(e*x+d)^(3/2)-8*(c/(c*d-e))^(1/2)*c^2*d^3*e^2*(e*x+d)-3*((-c*(e*x+d)+c
*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^
(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*
e^4*(e*x+d)^(3/2)-2*(c/(c*d-e))^(1/2)*c^2*d^4*e^2+3*(c/(c*d-e))^(1/2)*d*e^4
*(e*x+d)+(c/(c*d-e))^(1/2)*d^2*e^4)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(3/2)
/(c*d+e)/(c^2*d^2-e^2)/d^3/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c
^2/e^2/x^2)^(1/2)))

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x + d)*(b*arcsec(c*x) + a)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e
^2*x^2 + 4*d^3*e*x + d^4), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asec(c*x))/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{7/2}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x)^(7/2),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x)^(7/2), x)
```

3.69 $\int x^4(d+ex^2)(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 206

$$\begin{aligned}\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx =& -\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} \\ & -\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}-\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} \\ & +\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right) \\ & -\frac{b(42c^2d+25e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{560c^6\sqrt{c^2x^2}}\end{aligned}$$

```
[Out] 1/5*d*x^5*(a+b*arcsec(c*x))+1/7*e*x^7*(a+b*arcsec(c*x))-1/560*b*(42*c^2*d+2
5*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)-1/560*b*(42*c^2*d
+25*e)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)-1/840*b*(42*c^2*d+25*e)*x^
4*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)-1/42*b*e*x^6*(c^2*x^2-1)^(1/2)/c/(c
^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used

$= \{14, 5346, 12, 470, 327, 223, 212\}$

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right) - \frac{bx\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(42c^2d+25e)}{560c^6\sqrt{c^2x^2}} - \frac{bex^6\sqrt{c^2x^2-1}}{42c\sqrt{c^2x^2}} - \frac{bx^2\sqrt{c^2x^2-1}(42c^2d+25e)}{560c^5\sqrt{c^2x^2}} - \frac{bx^4\sqrt{c^2x^2-1}(42c^2d+25e)}{840c^3\sqrt{c^2x^2}}$$

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/560*(b*(42*c^2*d + 25*e)*x^2*Sqrt[-1 + c^2*x^2])/(c^5*Sqrt[c^2*x^2]) - (
b*(42*c^2*d + 25*e)*x^4*Sqrt[-1 + c^2*x^2])/(840*c^3*Sqrt[c^2*x^2]) - (b*e*
x^6*Sqrt[-1 + c^2*x^2])/(42*c*Sqrt[c^2*x^2]) + (d*x^5*(a + b*ArcSec[c*x]))/
5 + (e*x^7*(a + b*ArcSec[c*x]))/7 - (b*(42*c^2*d + 25*e)*x*ArcTanh[(c*x)/Sq
rt[-1 + c^2*x^2]])/(560*c^6*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^4(7d+5ex^2)}{35\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$=\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^4(7d+5ex^2)}{\sqrt{-1+c^2x^2}}\,dx}{35\sqrt{c^2x^2}}$$

$$=-\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)+\frac{\left(bc\left(-42d-\frac{25e}{c^2}\right)x\right)\int\frac{x^4}{\sqrt{-1+c^2x^2}}\,dx}{210\sqrt{c^2x^2}}$$

$$=-\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}-\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)+\frac{\left(b\left(-42d-\frac{25e}{c^2}\right)x\right)\int\frac{x^2}{\sqrt{-1+c^2x^2}}\,dx}{280c\sqrt{c^2x^2}}$$

$$=-\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}-\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}-\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right)+\frac{\left(b\left(-42d-\frac{25e}{c^2}\right)x\right)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{560c^3\sqrt{c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} - \frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} \\
&\quad - \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right) \\
&\quad + \frac{\left(b\left(-42d-\frac{25e}{c^2}\right)x\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, \frac{x}{\sqrt{-1+c^2x^2}}\right)}{560c^3\sqrt{c^2x^2}} \\
&= -\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} - \frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} \\
&\quad + \frac{1}{5}dx^5\left(a+b\sec^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\sec^{-1}(cx)\right) - \frac{b(42c^2d+25e)\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{560c^6\sqrt{c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.68

$$
\int x^4\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx
$$

$$
= \frac{48ac^7x^5(7d+5ex^2) - bc^2\sqrt{1-\frac{1}{c^2x^2}}x^2(75e+2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4)) + 48bc^7x^5(7d+5ex}{1680c^7}
$$

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] (48*a*c^7*x^5*(7*d + 5*e*x^2) - b*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*(75*e + 2*c
^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5*e
*x^2)*ArcSec[c*x] - 3*b*(42*c^2*d + 25*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x
])/(1680*c^7)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.59

method	result
parts	$a\left(\frac{1}{7}e\,x^7+\frac{1}{5}d\,x^5\right)+\frac{b\,\mathrm{arcsec}(cx)e\,x^7}{7}+\frac{b\,\mathrm{arcsec}(cx)x^5d}{5}-\frac{b(c^2x^2-1)x^4e}{42c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b(c^2x^2-1)x^2d}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b(c^2x^2-1)}{168c^5\sqrt{\frac{c^2x^2}{c^2}}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\,\mathrm{arcsec}(cx)d\,c^5x^5}{5}+\frac{b\,c^5\,\mathrm{arcsec}(cx)e\,x^7}{7}-\frac{b\left(c^2x^2-1\right)c^2x^2d}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\,c^2\left(c^2x^2-1\right)x^4e}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)d}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2\right.}{168\sqrt{c}}}{c^5}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\,\mathrm{arcsec}(cx)d\,c^5x^5}{5}+\frac{b\,c^5\,\mathrm{arcsec}(cx)e\,x^7}{7}-\frac{b\left(c^2x^2-1\right)c^2x^2d}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\,c^2\left(c^2x^2-1\right)x^4e}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b\left(c^2x^2-1\right)d}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{5b\left(c^2x^2\right.}{168\sqrt{c}}}{c^5}$

```
[In] int(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*e*x^7+1/5*d*x^5)+1/7*b*arcsec(c*x)*e*x^7+1/5*b*arcsec(c*x)*x^5*d-1/4
2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4*e-1/20*b/c^3*(c^2*x^2-1
)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*d-5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^
2/x^2)^(1/2)*x^2*e-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d-5/1
12*b/c^7*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e-3/40*b/c^6*(c^2*x^2-1)^(
1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))-5/112*b/c^8*
(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.93

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{240\,ac^7ex^7+336\,ac^7dx^5+48\,(5\,bc^7ex^7+7\,bc^7dx^5-7\,bc^7d-5\,bc^7e)\,\mathrm{arcsec}\,(cx)+96\,(7\,bc^7d+5\,bc^7e)\,\mathrm{arcta}}{}$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 + 48*(5*b*c^7*e*x^7 + 7*b*c^7*d*x
^5 - 7*b*c^7*d - 5*b*c^7*e)*arcsec(c*x) + 96*(7*b*c^7*d + 5*b*c^7*e)*arctan
(-c*x + sqrt(c^2*x^2 - 1)) + 3*(42*b*c^2*d + 25*b*e)*log(-c*x + sqrt(c^2*x^
2 - 1)) - (40*b*c^5*e*x^5 + 2*(42*b*c^5*d + 25*b*c^3*e)*x^3 + 3*(42*b*c^3*d
 + 25*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.98

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{adx^5}{5}+\frac{aex^7}{7}+\frac{bdx^5\,\mathrm{asec}\,(cx)}{5}+\frac{bex^7\,\mathrm{asec}\,(cx)}{7} - \frac{bd\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\mathrm{acosh}\,(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\mathrm{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c} - \frac{be\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\,\mathrm{acosh}\,(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\,\mathrm{asin}\,(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

```
[In] integrate(x**4*(e*x**2+d)*(a+b*asec(c*x)),x)
```

```
[Out] a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*asec(c*x)/5 + b*e*x**7*asec(c*x)/7 - b*d
*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1))
 - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2)
 > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 +
1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/
(5*c) - b*e*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sqrt(c**
2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**
2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x**7/(6*s
qrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x**3/(48*c*
*3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(
c*x)/(16*c**6), True))/(7*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.44

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{7}aex^7+\frac{1}{5}adx^5 + \frac{1}{80}\left(16\,x^5\operatorname{arcsec}(cx)+\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bd + \frac{1}{672}\left(96\,x^7\operatorname{arcsec}(cx)-\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\log\left(\sqrt{-\frac{1}{c^2}}\right.}{c^6}}{c}\right.$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/80*(16*x^5*arcsec(c*x) + (2*(3*(-1/(c^2*x^2)
+ 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/
(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(
-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arcsec(c*x) - (2*(15*(-1
/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2)
+ 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*
x^2) - 1) + c^6) + 15*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/
(c^2*x^2) + 1) - 1)/c^6)/c)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17474 vs. 2(178) = 356.

Time = 2.76 (sec) , antiderivative size = 17474, normalized size of antiderivative = 84.83

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/1680*(336*b*c^2*d*arccos(1/(c*x))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^
8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) - 126*b*c^2*d*log(abs(
sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21
*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c
*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 126*b*c^2*d*log(a
bs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1
/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 336*a*c^2*d/(c
^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^
2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/
(c*x) + 1)^14) - 1008*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^8 + 7*c
^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) -
 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8
*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
1)^14)*(1/(c*x) + 1)^2) - 882*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^
2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c
^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^1
2 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^2) + 882*b*c^2*
d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 +
7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2
) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*
c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x)
```

+ 1)^14)*(1/(c*x) + 1)^2) - 420*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c
^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8
*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
1)^14)*(1/(c*x) + 1)) - 1008*a*c^2*d*(1/(c^2*x^2) - 1)/((c^8 + 7*c^8*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/
(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*
x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1
/(c*x) + 1)^2) + 240*b*e*arccos(1/(c*x))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*
x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/
(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) + 336*b*c^2*d*(1/
(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*
(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) +
1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^4) - 75*b*
e*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^8 + 7*c^8*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) -
1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14) - 2646*b*
c^2*d*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((
c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/
(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c
^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^1
0 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1
/(c*x) + 1)^14)*(1/(c*x) + 1)^4) + 75*b*e*log(abs(sqrt(-1/(c^2*x^2) + 1) -
1/(c*x) - 1))/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
+ 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/
(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2
*x^2) - 1)^7/(1/(c*x) + 1)^14) + 2646*b*c^2*d*(1/(c^2*x^2) - 1)^2*log(abs(s
qrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21
c^8(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c
x) + 1)^12 + c^8(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^4) +
1008*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21
c^8(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c

```
*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^3) +
240*a*e/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2
) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*
c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*
x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2)
- 1)^7/(1/(c*x) + 1)^14) + 336*a*c^2*d*(1/(c^2*x^2) - 1)^2/((c^8 + 7*c^8*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^
4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(
c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14
)*(1/(c*x) + 1)^4) - 1680*b*e*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^8 + 7*c
^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) -
 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8
*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
1)^14)*(1/(c*x) + 1)^2) + 1680*b*c^2*d*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/
((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/
(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)
^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/
(1/(c*x) + 1)^14)*(1/(c*x) + 1)^6) - 525*b*e*(1/(c^2*x^2) - 1)*log(abs(sqrt
(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^
8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^2) - 441
0*b*c^2*d*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1)
)/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(
1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) +
1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^
7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^6) + 525*b*e*(1/(c^2*x^2) - 1)*log(abs(sq
rt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*
c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*
x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^2) + 4
410*b*c^2*d*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) -
1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8
*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x)
+ 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1
)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^6) - 330*b*e*sqrt(-1/(c^2*x^2) + 1)/((c
^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^
```

```
2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/
(c*x) + 1)^14)*(1/(c*x) + 1)) - 756*b*c^2*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^
2*x^2) + 1)/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
+ 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(
1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*
x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^5) - 1680*a*e*(1/(c^2*x^2) - 1)
/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1
)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7
/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^2) + 1680*a*c^2*d*(1/(c^2*x^2) - 1)^3/((c^
8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2
*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
+ 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(
c*x) + 1)^14)*(1/(c*x) + 1)^6) + 5040*b*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x
))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*
(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) +
 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)
^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^4) - 1680*b*c^2*d*(1/(c^2*x^2) - 1)^4*ar
ccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^
6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5
/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^
2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) - 1575*b*e*(1/(c^2*x^2) -
1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*
c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*
x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2
) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c
*x) + 1)^4) - 4410*b*c^2*d*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) +
1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) -
 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(
1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) + 1575*b*e*(1/(c^2*x^
2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(
c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
+ 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(
1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^
2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*
(1/(c*x) + 1)^4) + 4410*b*c^2*d*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^
```

2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) + 280*b*e*(-1/(c^2*x^2) + 1)^(3/2)/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^3) + 5040*a*e*(1/(c^2*x^2) - 1)^2/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^4) - 1680*a*c^2*d*(1/(c^2*x^2) - 1)^4/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) - 8400*b*e*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^6) - 336*b*c^2*d*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^10) - 2625*b*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^6) - 2646*b*c^2*d*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^10) + 2625*b*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*

c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^6) + 2646*b*c^2*d*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^10) - 850*b*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^5) + 756*b*c^2*d*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^9) - 8400*a*e*(1/(c^2*x^2) - 1)^3/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^6) - 336*a*c^2*d*(1/(c^2*x^2) - 1)^5/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^10) + 8400*b*e*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) + 1008*b*c^2*d*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^12) - 2625*b*e*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) - 882*b*c^2*d*(1/(c^2*x^2) - 1)^6*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2

/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(
c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^
10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(
1/(c*x) + 1)^14)*(1/(c*x) + 1)^12) + 2625*b*e*(1/(c^2*x^2) - 1)^4*log(abs(s
qrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21
c^8(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c
x) + 1)^12 + c^8(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) +
882*b*c^2*d*(1/(c^2*x^2) - 1)^6*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) -
1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8
*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x)
+ 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1
)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^12) + 1008*b*c^2*d*(1/(c^2*x^2) - 1)^5*
sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21
c^8(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^
2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c
^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^11) + 8400*a*e*(1/(c
^2*x^2) - 1)^4/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/
(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^
5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c
^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^8) + 1008*a*c^2*d*(1/(c^2*x^
2) - 1)^6/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/
(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^
2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^12) - 5040*b*e*(1/(c^2*x^2) - 1)^
5*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*
(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1
/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^10) - 336*b*c^2*d*(1/(c^2
*x^2) - 1)^7*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^
12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^14) - 1575*b*e
*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 +
 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7
c^8(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x

) + 1)^14)*(1/(c*x) + 1)^10) - 126*b*c^2*d*(1/(c^2*x^2) - 1)^7*log(abs(sqrt
(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^
8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^14) + 15
75*b*e*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(
(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(
c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^
10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(
1/(c*x) + 1)^14)*(1/(c*x) + 1)^10) + 126*b*c^2*d*(1/(c^2*x^2) - 1)^7*log(ab
s(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1
/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^14
) + 850*b*e*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/
(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*
x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1
/(c*x) + 1)^9) + 420*b*c^2*d*(1/(c^2*x^2) - 1)^6*sqrt(-1/(c^2*x^2) + 1)/((c
^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^
2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/
(c*x) + 1)^14)*(1/(c*x) + 1)^13) - 5040*a*e*(1/(c^2*x^2) - 1)^5/((c^8 + 7*c
^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) -
 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8
*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
1)^14)*(1/(c*x) + 1)^10) - 336*a*c^2*d*(1/(c^2*x^2) - 1)^7/((c^8 + 7*c^8*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^
4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(
c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14
)*(1/(c*x) + 1)^14) + 1680*b*e*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^8 +
7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2
) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*
c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x)
 + 1)^14)*(1/(c*x) + 1)^12) - 525*b*e*(1/(c^2*x^2) - 1)^6*log(abs(sqrt(-1/(
c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^

3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^12) + 525*b*e*(1/(c^2*x^2) - 1)^6*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^12) + 280*b*e*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^11) + 1680*a*e*(1/(c^2*x^2) - 1)^6/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^12) - 240*b*e*(1/(c^2*x^2) - 1)^7*arccos(1/(c*x))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^14) - 75*b*e*(1/(c^2*x^2) - 1)^7*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^14) + 75*b*e*(1/(c^2*x^2) - 1)^7*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^14) + 330*b*e*(1/(c^2*x^2) - 1)^6*sqrt(-1/(c^2*x^2) + 1)/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^13) - 240*a*e*(1/(c^2*x^2) - 1)^7/((c^8 + 7*c^8*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*(1/(c*x) + 1)^14))*c

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^4\left(e\,x^2+d\right)\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^4*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
[Out] int(x^4*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.70 $\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int x^2\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}-\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)-\frac{b(20c^2d+9e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}$$

```
[Out] 1/3*d*x^3*(a+b*arcsec(c*x))+1/5*e*x^5*(a+b*arcsec(c*x))-1/120*b*(20*c^2*d+9
*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^4/(c^2*x^2)^(1/2)-1/120*b*(20*c^2*d+
9*e)*x^2*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)-1/20*b*e*x^4*(c^2*x^2-1)^(1/
2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {14, 5346, 12, 470, 327, 223, 212}

$$\int x^2\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)-\frac{bx\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(20c^2d+9e)}{120c^4\sqrt{c^2x^2}}-\frac{bex^4\sqrt{c^2x^2-1}}{20c\sqrt{c^2x^2}}-\frac{bx^2\sqrt{c^2x^2-1}(20c^2d+9e)}{120c^3\sqrt{c^2x^2}}$$

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/120*(b*(20*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(c^3*Sqrt[c^2*x^2]) - (b
*e*x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + (d*x^3*(a + b*ArcSec[c*x]
))/3 + (e*x^5*(a + b*ArcSec[c*x]))/5 - (b*(20*c^2*d + 9*e)*x*ArcTanh[(c*x)/
Sqrt[-1 + c^2*x^2]])/(120*c^4*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2(5d+3ex^2)}{15\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2(5d+3ex^2)}{\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}}\\
&= -\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)+\frac{\left(bc\left(-20d-\frac{9e}{c^2}\right)x\right)\int\frac{x^2}{\sqrt{-1+c^2x^2}}\,dx}{60\sqrt{c^2x^2}}\\
&= -\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}-\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)+\frac{\left(b\left(-20d-\frac{9e}{c^2}\right)x\right)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{120c\sqrt{c^2x^2}}\\
&= -\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}-\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)+\frac{\left(b\left(-20d-\frac{9e}{c^2}\right)x\right)\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{120c\sqrt{c^2x^2}}\\
&= -\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}-\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\sec^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\sec^{-1}(cx)\right)-\frac{b(20c^2d+9e)\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.76

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{c^2x^2\left(8ac^3x(5d+3ex^2)-b\sqrt{1-\frac{1}{c^2x^2}}(9e+c^2(20d+6ex^2))\right)+8bc^5x^3(5d+3ex^2)\sec^{-1}(cx)-b(20c^2d+}{120c^5}$$

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] (c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) - b*Sqrt[1 - 1/(c^2*x^2)]*(9*e + c^2*(2
0*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcSec[c*x] - b*(20*c^2*d +
9*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(120*c^5)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.58

method	result
parts	$a\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b\,\operatorname{arcsec}(cx)e\,x^5}{5}+\frac{b\,\operatorname{arcsec}(cx)x^3d}{3}-\frac{b(c^2x^2-1)x^2e}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b(c^2x^2-1)d}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3b(c^2x^2-1)}{40c^5\sqrt{\frac{c^2x^2-}{c^2x^2}}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\,\operatorname{arcsec}(cx)d\,c^3x^3}{3}+\frac{b\,c^3\operatorname{arcsec}(cx)e\,x^5}{5}-\frac{b\left(c^2x^2-1\right)d}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right)x^2e}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\,\operatorname{arcsec}(cx)d\,c^3x^3}{3}+\frac{b\,c^3\operatorname{arcsec}(cx)e\,x^5}{5}-\frac{b\left(c^2x^2-1\right)d}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right)x^2e}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^3}$

```
[In] int(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*e*x^5+1/3*d*x^3)+1/5*b*arcsec(c*x)*e*x^5+1/3*b*arcsec(c*x)*x^3*d-1/2
0*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e-1/6*b/c^3*(c^2*x^2-1)
/((c^2*x^2-1)/c^2/x^2)^(1/2)*d-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)
^(1/2)*e-1/6*b/c^4*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x
+(c^2*x^2-1)^(1/2))-3/40*b/c^6*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2
)/x*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{24\,ac^5ex^5+40\,ac^5dx^3+8\,(3\,bc^5ex^5+5\,bc^5dx^3-5\,bc^5d-3\,bc^5e)\,\mathrm{arcsec}\,(cx)+16\,(5\,bc^5d+3\,bc^5e)\arctan\left(-\right.}{120\,c}$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3 -
 5*b*c^5*d - 3*b*c^5*e)*arcsec(c*x) + 16*(5*b*c^5*d + 3*b*c^5*e)*arctan(-c*
x + sqrt(c^2*x^2 - 1)) + (20*b*c^2*d + 9*b*e)*log(-c*x + sqrt(c^2*x^2 - 1))
 - (6*b*c^3*e*x^3 + (20*b*c^3*d + 9*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.83

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\,\mathrm{asec}\,(cx)}{3}+\frac{bex^5\,\mathrm{asec}\,(cx)}{5} - \frac{bd\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c} - \frac{be\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\mathrm{acosh}\,(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\mathrm{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

```
[In] integrate(x**2*(e*x**2+d)*(a+b*asec(c*x)),x)
```

```
[Out] a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*asec(c*x)/3 + b*e*x**5*asec(c*x)/5 - b*d
*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**
2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1
)) - I*asin(c*x)/(2*c**2), True))/(3*c) - b*e*Piecewise((c*x**5/(4*sqrt(c**
2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2
- 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**
2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2
*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.44

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{5}aex^5+\frac{1}{3}adx^3 + \frac{1}{12}\left(4x^3\operatorname{arcsec}(cx)-\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd + \frac{1}{80}\left(16x^5\operatorname{arcsec}(cx)+\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2)
+ 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 -
log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arcsec(c*x) + (2
*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2)
- 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)
/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9792 vs. 2(139) = 278.

Time = 1.93 (sec) , antiderivative size = 9792, normalized size of antiderivative = 60.82

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/120*(40*b*c^2*d*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) +
 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 20*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2) +
1) + 1/(c*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*
(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5
```

/(1/(c*x) + 1)^10) + 20*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) + 40*a*c^2*d/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 40*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 100*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) + 100*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 40*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)) - 40*a*c^2*d*(1/(c^2*x^2) - 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) + 24*b*e*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 80*b*c^2*d*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) - 9*b*e*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 200*b*c^2*d*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) + 9*b*e*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1

/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^1
0) + 200*b*c^2*d*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*
x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5
c^6(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x)
 + 1)^10)*(1/(c*x) + 1)^4) + 80*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)/((c^6 + 5*
c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) -
 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x)
+ 1)^3) + 24*a*e/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1
/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(
1/(c*x) + 1)^10) - 80*a*c^2*d*(1/(c^2*x^2) - 1)^2/((c^6 + 5*c^6*(1/(c^2*x^2
) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^
6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x)
+ 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) - 120*b
e(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*
(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) + 80*b*c^2*d*(1/(c^2
*x^2) - 1)^3*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*
x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) - 45*b*e*(1/(c^2*x^2) - 1)*l
og(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1
/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 200*b*c^2
d(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6
 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/
(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x
^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(
c*x) + 1)^6) + 45*b*e*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/
(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
+ 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c
x) + 1)^10)(1/(c*x) + 1)^2) + 200*b*c^2*d*(1/(c^2*x^2) - 1)^3*log(abs(sqr
t(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) - 30*b*e*sqrt(-1/(c^2
*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*
x) + 1)^10)*(1/(c*x) + 1)) - 120*a*e*(1/(c^2*x^2) - 1)/((c^6 + 5*c^6*(1/(c^

```
2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) +
80*a*c^2*d*(1/(c^2*x^2) - 1)^3/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^
2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) + 240*b*e*(1/(c^2*x^2) - 1
)^2*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^
6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x)
 + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)
^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) + 40*b*c^2*d*(1/(c^2*x^2) - 1)^4*arcc
os(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
+ 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c
*x) + 1)^10)*(1/(c*x) + 1)^8) - 90*b*e*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/
(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^
2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) - 100*b*c^2*d*(1/(c^2*x^2)
 - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) +
90*b*e*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(
(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c
^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*
(1/(c*x) + 1)^4) + 100*b*c^2*d*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2
) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10
*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) -
 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) + 80*b*c^2*d*(1/(c^2*x^2) - 1)^3*s
qrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*
c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*
x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^7) + 12*b*e*(-1/(c^2*x^2) + 1)^(3/2)/(
(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c
^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*
(1/(c*x) + 1)^3) + 240*a*e*(1/(c^2*x^2) - 1)^2/((c^6 + 5*c^6*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1
)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) + 40*a*c^2
*d*(1/(c^2*x^2) - 1)^4/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10
*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c
```

*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) -
 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) - 240*b*e*(1/(c^2*x^2) - 1)^3*arcc
os(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
+ 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c
x) + 1)^10)(1/(c*x) + 1)^6) - 40*b*c^2*d*(1/(c^2*x^2) - 1)^5*arccos(1/(c*
x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*
(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)
^10)*(1/(c*x) + 1)^10) - 90*b*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^
2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 1
0*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2)
- 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) - 20*b*c^2*d*(1/(c^2*x^2) - 1)^5*
log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1
)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) + 90*b*e*
(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 +
5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x
) + 1)^6) + 20*b*c^2*d*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) -
 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/
(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1
/(c*x) + 1)^10)*(1/(c*x) + 1)^10) + 40*b*c^2*d*(1/(c^2*x^2) - 1)^4*sqrt(-1/
(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/
(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1
/(c*x) + 1)^10)*(1/(c*x) + 1)^9) - 240*a*e*(1/(c^2*x^2) - 1)^3/((c^6 + 5*c^
6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) +
1)^6) - 40*a*c^2*d*(1/(c^2*x^2) - 1)^5/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(
c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^
6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) + 120*b*e*(1/(c^2
*x^2) - 1)^4*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*
x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) - 45*b*e*(1/(c^2*x^2) - 1)^4
*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) +

```
1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) + 45*b*e*
(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 +
5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x
) + 1)^8) + 12*b*e*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1
)^7) + 120*a*e*(1/(c^2*x^2) - 1)^4/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1
/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) - 24*b*e*(1/(c^2*x^2)
- 1)^5*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10
*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) -
 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) - 9*b*e*(1/(c^2*x^2) - 1)^5*log(a
bs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(
1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) + 9*b*e*(1/(c^
2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)
^10) + 30*b*e*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(
c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
+ 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^9)
- 24*a*e*(1/(c^2*x^2) - 1)^5/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*
x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.71 $\int (d+ex^2)(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 109

$$\int (d+ex^2)(a+b\sec^{-1}(cx))\,dx = -\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx(a+b\sec^{-1}(cx)) + \frac{1}{3}ex^3(a+b\sec^{-1}(cx)) - \frac{b(6c^2d+e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

```
[Out] d*x*(a+b*arcsec(c*x))+1/3*e*x^3*(a+b*arcsec(c*x))-1/6*b*(6*c^2*d+e)*x*arcta
nh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)^(1/2)-1/6*b*e*x^2*(c^2*x^2-1)^(1/2)
/c/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5336, 12, 396, 223, 212}

$$\int (d+ex^2)(a+b\sec^{-1}(cx))\,dx = dx(a+b\sec^{-1}(cx)) + \frac{1}{3}ex^3(a+b\sec^{-1}(cx)) - \frac{bx\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(6c^2d+e)}{6c^2\sqrt{c^2x^2}} - \frac{bex^2\sqrt{c^2x^2-1}}{6c\sqrt{c^2x^2}}$$

```
[In] Int[(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/6*(b*e*x^2*Sqrt[-1 + c^2*x^2])/(c*Sqrt[c^2*x^2]) + d*x*(a + b*ArcSec[c*x
]) + (e*x^3*(a + b*ArcSec[c*x]))/3 - (b*(6*c^2*d + e)*x*ArcTanh[(c*x)/Sqrt[
-1 + c^2*x^2]])/(6*c^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5336

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x]
- Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{3d+ex^2}{3\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= dx\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{3d+ex^2}{\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}}\\
&= -\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\sec^{-1}(cx)\right)\\
&\quad+\frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right)+\frac{\left(b\left(-6c^2d-e\right)x\right)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{6c\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx\left(a+b\sec^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right) + \frac{\left(b(-6c^2d-e)\,x\right)\operatorname{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, \frac{x}{\sqrt{-1+c^2x^2}}\right)}{6c\sqrt{c^2x^2}}$$

$$= -\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx\left(a+b\sec^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\sec^{-1}(cx)\right) - \frac{b(6c^2d+e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.38

$$\int \left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = adx + \frac{1}{3}aex^3 - \frac{bex^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c} + bdx\sec^{-1}(cx) + \frac{1}{3}bex^3\sec^{-1}(cx) - \frac{bd\sqrt{1-\frac{1}{c^2x^2}}\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}} - \frac{be\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 - (b*e*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d*
x*ArcSec[c*x] + (b*e*x^3*ArcSec[c*x])/3 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcT
anh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2] - (b*e*Log[x*(1 + Sqrt[(-
1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.21

method	result
parts	$a\left(\frac{1}{3}x^3e+dx\right)+\frac{b\left(\frac{c\,\mathrm{arcsec}(cx)x^3e}{3}+\mathrm{arcsec}(cx)xcd-\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-\right.\right.}{6c^3x\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c}$
derivativedivides	$\frac{\frac{a\left(c^3dx+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\mathrm{arcsec}(cx)d\,c^3x+\frac{\mathrm{arcsec}(cx)e\,c^3x^3}{3}-\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^2}}{c}$
default	$\frac{\frac{a\left(c^3dx+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\mathrm{arcsec}(cx)d\,c^3x+\frac{\mathrm{arcsec}(cx)e\,c^3x^3}{3}-\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^2}}{c}$

```
[In] int((e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*x^3*e+d*x)+b/c*(1/3*c*arcsec(c*x)*x^3*e+arcsec(c*x)*x*c*d-1/6/c^3*(c
^2*x^2-1)^(1/2)*(6*d*c^2*ln(c*x+(c^2*x^2-1)^(1/2))+e*c*x*(c^2*x^2-1)^(1/2)+
e*ln(c*x+(c^2*x^2-1)^(1/2)))/x/((c^2*x^2-1)/c^2/x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.29

$$\int\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3ex^3+6\,ac^3dx-\sqrt{c^2x^2-1}bcex+2\left(bc^3ex^3+3\,bc^3dx-3\,bc^3d-bc^3e\right)\mathrm{arcsec}\left(cx\right)+4\left(3\,bc^3d+bc^3e\right)\mathrm{a}}{6\,c^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e*x^3 + 6*a*c^3*d*x - sqrt(c^2*x^2 - 1)*b*c*e*x + 2*(b*c^3*e*x
^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*arcsec(c*x) + 4*(3*b*c^3*d + b*c^3*
e)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (6*b*c^2*d + b*e)*log(-c*x + sqrt(c^2
*x^2 - 1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= adx + \frac{aex^3}{3} + bdx\operatorname{asec}(cx) + \frac{bex^3\operatorname{asec}(cx)}{3} - \frac{bd\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} - \frac{be\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((e*x**2+d)*(a+b*asec(c*x)),x)
```

```
[Out] a*d*x + a*e*x**3/3 + b*d*x*asec(c*x) + b*e*x**3*asec(c*x)/3 - b*d*Piecewise
((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - b*e*Piecewise(
(x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-
I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin
(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.41

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{1}{3}aex^3 + \frac{1}{12}\left(4x^3\operatorname{arcsec}(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be + adx + \frac{\left(2cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2c}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/12*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(
c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c
^2*x^2) + 1) - 1)/c^2)/c)*b*e + a*d*x + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-
1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4051 vs. 2(95) = 190.

Time = 1.28 (sec) , antiderivative size = 4051, normalized size of antiderivative = 37.17

$$\int (d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] 1/6*(6*b*c^2*d*arccos(1/(c*x))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1
/(c*x) + 1)^6) - 6*b*c^2*d*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(
c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) + 6*b*c^2*d*log(a
bs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6) + 6*a*c^2*d/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6) + 6*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^4 +
 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*
x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) - 18*
b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((
c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2)
+ 18*b*c^2*d*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1
))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1
)^2) + 6*a*c^2*d*(1/(c^2*x^2) - 1)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) + 2*b*e*arccos(1/(c*x))/(c^4 + 3*c^4*(
1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^
4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 6*b*c^2*d*(1/(c^2*x^2) - 1)^
2*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(
1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^
6)*(1/(c*x) + 1)^4) - b*e*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c
^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6) - 18*b*c^2*d*(1/(c
^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 3*c^4
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + b*e*log(a
bs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6) + 18*b*c^2*d*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/
(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
```

)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) + 2*a*e/(c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6) - 6*a*c^2*d*(1/(c^2*x^2) - 1)^2/((c^4 + 3*c^4*(1/(
c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
 c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 6*b*e*(1/(c^2*
x^2) - 1)*arccos(1/(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*
x) + 1)^6)*(1/(c*x) + 1)^2) - 6*b*c^2*d*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))
/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^
6) - 3*b*e*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))
/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^
2) - 6*b*c^2*d*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x)
 + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x)
 + 1)^6) + 3*b*e*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x)
 - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x)
 + 1)^2) + 6*b*c^2*d*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1
/(c*x) - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1
/(c*x) + 1)^6) - 2*b*e*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 3*c^4*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)) - 6*a*e*(1/(c^2*x^2) - 1)/((c
^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1
/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^2) -
 6*a*c^2*d*(1/(c^2*x^2) - 1)^3/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6)*(1/(c*x) + 1)^6) + 6*b*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x)
)/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)
^4) - 3*b*e*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) +
1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) +
1)^4) + 3*b*e*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x)
- 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x)
+ 1)^4) + 6*a*e*(1/(c^2*x^2) - 1)^2/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^4) - 2*b*e*(1/(c^2*x^2) - 1)^3*arccos(1/
(c*x))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x)
 + 1)^6) - b*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x)

```
 + 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x)
 + 1)^6) + b*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x)
 - 1))/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x)
 + 1)^6) + 2*b*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 3*c^4*(
1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^
4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*(1/(c*x) + 1)^5) - 2*a*e*(1/(c
^2*x^2) - 1)^3/((c^4 + 3*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3*c^4*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6)*
(1/(c*x) + 1)^6))*c
```

Mupad [F(-1)]

Timed out.

$$\int (d + ex^2) (a + b \sec^{-1}(cx)) \, dx = \int (e\,x^2 + d) \left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((d + e*x^2)*(a + b*acos(1/(c*x))),x)
[Out] int((d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.72 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^2}\,dx = \frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{x} + ex(a+b\sec^{-1}(cx)) - \frac{bex\,\mathrm{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

```
[Out] -d*(a+b*arcsec(c*x))/x+e*x*(a+b*arcsec(c*x))-b*e*x*arctanh(c*x/(c^2*x^2-1)^
(1/2))/(c^2*x^2)^(1/2)+b*c*d*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5346, 462, 223, 212}

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^2}\,dx = -\frac{d(a+b\sec^{-1}(cx))}{x} + ex(a+b\sec^{-1}(cx)) - \frac{bex\,\mathrm{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}} + \frac{bcd\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^2,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*x^2] - (d*(a + b*ArcSec[c*x]))/x + e*x*
(a + b*ArcSec[c*x]) - (b*e*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[c^2*x^
2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(a+b\sec^{-1}(cx))}{x}+ex\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{-d+ex^2}{x^2\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{x}+ex\left(a+b\sec^{-1}(cx)\right)-\frac{(bcex)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{x}+ex\left(a+b\sec^{-1}(cx)\right)\\
&\quad -\frac{(bcex)\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}
\end{aligned}
$$

$$= \frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{x} + ex(a+b\sec^{-1}(cx)) - \frac{bex\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^2}\,dx = -\frac{ad}{x} + aex + bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{bd\sec^{-1}(cx)}{x} + bex\sec^{-1}(cx) - \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x + b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcSec[c*x
])/x + b*e*x*ArcSec[c*x] - (b*e*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[
-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

method	result	siz
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(\frac{\operatorname{arcsec}(cx)ex}{c}-\frac{\operatorname{arcsec}(cx)d}{xc}-\frac{\sqrt{c^2x^2-1}\left(-d\,c^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^4x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)$	11
derivativedivides	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arcsec}(cx)xe-\frac{\operatorname{arcsec}(cx)dc}{x}-\frac{\sqrt{c^2x^2-1}\left(-d\,c^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}\right)$	12
default	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arcsec}(cx)xe-\frac{\operatorname{arcsec}(cx)dc}{x}-\frac{\sqrt{c^2x^2-1}\left(-d\,c^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}\right)$	12

```
[In] int((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(e*x-d/x)+b*c*(1/c*arcsec(c*x)*e*x-arcsec(c*x)*d/x/c-1/c^4*(c^2*x^2-1)^(1
/2)*(-d*c^2*(c^2*x^2-1)^(1/2)+e*ln(c*x+(c^2*x^2-1)^(1/2))*c*x)/x^2/((c^2*x^
2-1)/c^2/x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = \frac{bc^2dx + acex^2 + bex\log\left(-cx+\sqrt{c^2x^2-1}\right) + \sqrt{c^2x^2-1}bcd - acd - 2\,(bcd-bce)x\arctan\left(-cx+\sqrt{c^2x^2\right.}}{cx}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (b*c^2*d*x + a*c*e*x^2 + b*e*x*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^2
 - 1)*b*c*d - a*c*d - 2*(b*c*d - b*c*e)*x*arctan(-c*x + sqrt(c^2*x^2 - 1))
+ (b*c*e*x^2 - b*c*d + (b*c*d - b*c*e)*x)*arcsec(c*x))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = -\frac{ad}{x} + aex + bcd\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd\operatorname{asec}(cx)}{x} + bex\operatorname{asec}(cx) - \frac{be\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$

```
[In] integrate((e*x**2+d)*(a+b*asec(c*x))/x**2,x)
```

```
[Out] -a*d/x + a*e*x + b*c*d*sqrt(1 - 1/(c**2*x**2)) - b*d*asec(c*x)/x + b*e*x*as
ec(c*x) - b*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), Tr
ue))/c
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = \left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsec}(cx)}{x}\right)bd + aex + \frac{\left(2\,cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be}{2\,c} - \frac{ad}{x}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] (c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*d + a*e*x + 1/2*(2*c*x*arcsec(
c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*
b*e/c - a*d/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(79) = 158$.

Time = 0.63 (sec) , antiderivative size = 1088, normalized size of antiderivative = 12.51

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

```
[Out] -(b*c^2*d*arccos(1/(c*x))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) +
 a*c^2*d/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + 2*b*c^2*d*(1/(c^
2*x^2) - 1)*arccos(1/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
)*(1/(c*x) + 1)^2) - 2*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^2 - c^2*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)) + 2*a*c^2*d*(1/(c^2*x^2) - 1)/((
c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) - b*e*arcco
s(1/(c*x))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) + b*c^2*d*(1/(c^
2*x^2) - 1)^2*arccos(1/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)
^4)*(1/(c*x) + 1)^4) + b*e*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(
c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) - b*e*log(abs(sqrt(-1/(c^2*x
^2) + 1) - 1/(c*x) - 1))/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4) +
2*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4)*(1/(c*x) + 1)^3) - a*e/(c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4) + a*c^2*d*(1/(c^2*x^2) - 1)^2/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4)*(1/(c*x) + 1)^4) + 2*b*e*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^2 -
 c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2) + 2*a*e*(1/(c^2*
x^2) - 1)/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c*x) + 1)^2)
 - b*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/
(1/(c*x) + 1)^4)*(1/(c*x) + 1)^4) - b*e*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1
/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4)*(1/(c*x) + 1)^4) + b*e*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2)
+ 1) - 1/(c*x) - 1))/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4)*(1/(c
*x) + 1)^4) - a*e*(1/(c^2*x^2) - 1)^2/((c^2 - c^2*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4)*(1/(c*x) + 1)^4))*c
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^2}\,dx = a\,e\,x - \frac{d\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)-b\,c\,x\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)}{x} - \frac{b\,e\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c} + b\,e\,x\,\mathrm{acos}\left(\frac{1}{c\,x}\right)$$

```
[In] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^2,x)
```

```
[Out] a*e*x - (d*(a + b*acos(1/(c*x)) - b*c*x*(1 - 1/(c^2*x^2))^(1/2)))/x - (b*e*
atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c + b*e*x*acos(1/(c*x))
```

3.73 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx = \frac{bc(2c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}+\frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{3x^3}-\frac{e(a+b\sec^{-1}(cx))}{x}$$

```
[Out] -1/3*d*(a+b*arcsec(c*x))/x^3-e*(a+b*arcsec(c*x))/x+1/9*b*c*(2*c^2*d+9*e)*(c
^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/9*b*c*d*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5346, 12, 464, 270}

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx = -\frac{d(a+b\sec^{-1}(cx))}{3x^3}-\frac{e(a+b\sec^{-1}(cx))}{x}+\frac{bc\sqrt{c^2x^2-1}(2c^2d+9e)}{9\sqrt{c^2x^2}}+\frac{bcd\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^4,x]
```

```
[Out] (b*c*(2*c^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/(9*Sqrt[c^2*x^2]) + (b*c*d*Sqrt[-1
 + c^2*x^2])/(9*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcSec[c*x]))/(3*x^3) - (e*(
a + b*ArcSec[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(a+b\sec^{-1}(cx))}{3x^3} - \frac{e(a+b\sec^{-1}(cx))}{x} - \frac{(bcx)\int \frac{-d-3ex^2}{3x^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d(a+b\sec^{-1}(cx))}{3x^3} - \frac{e(a+b\sec^{-1}(cx))}{x} - \frac{(bcx)\int \frac{-d-3ex^2}{x^4\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}} \\
&= \frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{3x^3} - \frac{e(a+b\sec^{-1}(cx))}{x} \\
&\quad - \frac{(bc(-2c^2d-9e)\,x)\int \frac{1}{x^2\sqrt{-1+c^2x^2}}\,dx}{9\sqrt{c^2x^2}}
\end{aligned}$$

$$=\frac{bc(2c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}+\frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{3x^3}-\frac{e(a+b\sec^{-1}(cx))}{x}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int\frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$=\frac{-3a(d+3ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+9ex^2)-3b(d+3ex^2)\sec^{-1}(cx)}{9x^3}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^4,x]
```

```
[Out] (-3*a*(d + 3*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 9*e*x^
2) - 3*b*(d + 3*e*x^2)*ArcSec[c*x])/(9*x^3)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03

method	result	size
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^3}\right)+b\,c^3\left(-\frac{\operatorname{arcsec}(cx)e}{c^3x}-\frac{\operatorname{arcsec}(cx)d}{3x^3c^3}+\frac{(c^2x^2-1)\left(2c^4d\,x^2+9c^2e\,x^2+c^2d\right)}{9c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^4}\right)$	108
derivativedivides	$c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{3c\,x^3}-\frac{\operatorname{arcsec}(cx)e}{cx}+\frac{\left(c^2x^2-1\right)\left(2c^4d\,x^2+9c^2e\,x^2+c^2d\right)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	121
default	$c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{3c\,x^3}-\frac{\operatorname{arcsec}(cx)e}{cx}+\frac{\left(c^2x^2-1\right)\left(2c^4d\,x^2+9c^2e\,x^2+c^2d\right)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	121

```
[In] int((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arcsec(c*x)*e/x-1/3*arcsec(c*x)*d/x^3/c^3+
1/9/c^6*(c^2*x^2-1)*(2*c^4*d*x^2+9*c^2*e*x^2+c^2*d)/((c^2*x^2-1)/c^2/x^2)^(
1/2)/x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx = -\frac{9\,aex^2+3\,ad+3\,(3\,bex^2+bd)\,\mathrm{arcsec}\,(cx)-\sqrt{c^2x^2-1}((2\,bc^2d+9\,be)x^2+bd)}{9\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*arcsec(c*x) - sqrt(c^2*x^2 -
1)*((2*b*c^2*d + 9*b*e)*x^2 + b*d))/x^3
```

Sympy [A] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx = -\frac{ad}{3x^3}-\frac{ae}{x}+bce\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd\,\mathrm{asec}\,(cx)}{3x^3}-\frac{be\,\mathrm{asec}\,(cx)}{x} + \frac{bd\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((e*x**2+d)*(a+b*asec(c*x))/x**4,x)
```

```
[Out] -a*d/(3*x**3) - a*e/x + b*c*e*sqrt(1 - 1/(c**2*x**2)) - b*d*asec(c*x)/(3*x*
*3) - b*e*asec(c*x)/x + b*d*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c
*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x*
*2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx = \left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\mathrm{arcsec}\,(cx)}{x}\right)be - \frac{1}{9}\,bd\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\,\mathrm{arcsec}\,(cx)}{x^3}\right)-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] (c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*e - 1/9*b*d*((c^4*(-1/(c^2*x^2
) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c*x)/x^3) - a*e/x
 - 1/3*a*d/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$=\frac{1}{9}\left(2\,bc^2d\sqrt{-\frac{1}{c^2x^2}+1}+9\,be\sqrt{-\frac{1}{c^2x^2}+1}-\frac{9\,be\arccos\left(\frac{1}{cx}\right)}{cx}+\frac{bd\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}-\frac{9\,ae}{cx}-\frac{3\,bd\arccos\left(\frac{1}{cx}\right)}{cx^3}\right.$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(2*b*c^2*d*sqrt(-1/(c^2*x^2) + 1) + 9*b*e*sqrt(-1/(c^2*x^2) + 1) - 9*b*
e*arccos(1/(c*x))/(c*x) + b*d*sqrt(-1/(c^2*x^2) + 1)/x^2 - 9*a*e/(c*x) - 3*
b*d*arccos(1/(c*x))/(c*x^3) - 3*a*d/(c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^4}\,dx=\int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^4, x)
```

3.74 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{2bc^3(12c^2d+25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{5x^5} - \frac{e(a+b\sec^{-1}(cx))}{3x^3}$$

```
[Out] -1/5*d*(a+b*arcsec(c*x))/x^5-1/3*e*(a+b*arcsec(c*x))/x^3+2/225*b*c^3*(12*c^
2*d+25*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/25*b*c*d*(c^2*x^2-1)^(1/2)/x^
4/(c^2*x^2)^(1/2)+1/225*b*c*(12*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {14, 5346, 12, 464, 277, 270}

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx = -\frac{d(a+b\sec^{-1}(cx))}{5x^5} - \frac{e(a+b\sec^{-1}(cx))}{3x^3} + \frac{bc\sqrt{c^2x^2-1}(12c^2d+25e)}{225x^2\sqrt{c^2x^2}} + \frac{bcd\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}} + \frac{2bc^3\sqrt{c^2x^2-1}(12c^2d+25e)}{225\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^6,x]
```

```
[Out] (2*b*c^3*(12*c^2*d + 25*e)*Sqrt[-1 + c^2*x^2])/(225*Sqrt[c^2*x^2]) + (b*c*d
*Sqrt[-1 + c^2*x^2])/(25*x^4*Sqrt[c^2*x^2]) + (b*c*(12*c^2*d + 25*e)*Sqrt[-
1 + c^2*x^2])/(225*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcSec[c*x]))/(5*x^5) - (
e*(a + b*ArcSec[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{d(a+b\sec^{-1}(cx))}{5x^5} - \frac{e(a+b\sec^{-1}(cx))}{3x^3} - \frac{(bcx)\int \frac{-3d-5ex^2}{15x^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{d(a+b\sec^{-1}(cx))}{5x^5} - \frac{e(a+b\sec^{-1}(cx))}{3x^3} - \frac{(bcx)\int \frac{-3d-5ex^2}{x^6\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}}$$

$$= \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{5x^5} - \frac{e(a+b\sec^{-1}(cx))}{3x^3} - \frac{(bc(-12c^2d-25e)\,x)\int \frac{1}{x^4\sqrt{-1+c^2x^2}}\,dx}{75\sqrt{c^2x^2}}$$

$$= \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{5x^5} - \frac{e(a+b\sec^{-1}(cx))}{3x^3} - \frac{(2bc^3(-12c^2d-25e)\,x)\int \frac{1}{x^2\sqrt{-1+c^2x^2}}\,dx}{225\sqrt{c^2x^2}}$$

$$= \frac{2bc^3(12c^2d+25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{5x^5} - \frac{e(a+b\sec^{-1}(cx))}{3x^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{-15a(3d+5ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(25ex^2(1+2c^2x^2)+3d(3+4c^2x^2+8c^4x^4))-15b(3d+5ex^2)\sec^{-1}(cx)}{225x^5}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^6,x]

[Out] (-15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(25*e*x^2*(1 + 2*c^2*x
^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*ArcSec[c*x])/
(225*x^5)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

method	result
parts	$a\left(-\frac{d}{5x^5}-\frac{e}{3x^3}\right)+b\,c^5\left(-\frac{\operatorname{arcsec}(cx)d}{5x^5c^5}-\frac{\operatorname{arcsec}(cx)e}{3c^5x^3}+\frac{(c^2x^2-1)\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25c^2e\,x^2+9\right.}{225c^8\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^6}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{5c^3x^5}-\frac{\operatorname{arcsec}(cx)e}{3c^3x^3}+\frac{(c^2x^2-1)\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25c^2e\,x^2+9c^2d\right)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$
default	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{5c^3x^5}-\frac{\operatorname{arcsec}(cx)e}{3c^3x^3}+\frac{(c^2x^2-1)\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25c^2e\,x^2+9c^2d\right)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$

```
[In] int((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/5*d/x^5-1/3*e/x^3)+b*c^5*(-1/5*arcsec(c*x)*d/x^5/c^5-1/3/c^5*arcsec(c
*x)*e/x^3+1/225/c^8*(c^2*x^2-1)*(24*c^6*d*x^4+50*c^4*e*x^4+12*c^4*d*x^2+25*
c^2*e*x^2+9*c^2*d)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{75\,aex^2+45\,ad+15\,(5\,bex^2+3\,bd)\operatorname{arcsec}(cx)-(2\,(12\,bc^4d+25\,bc^2e)x^4+(12\,bc^2d+25\,be)x^2+9\,bd)}{225\,x^5}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*arcsec(c*x) - (2*(12*b
*c^4*d + 25*b*c^2*e)*x^4 + (12*b*c^2*d + 25*b*e)*x^2 + 9*b*d)*sqrt(c^2*x^2
- 1))/x^5
```

Sympy [A] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= -\frac{ad}{5x^5} - \frac{ae}{3x^3} - \frac{bd\operatorname{asec}(cx)}{5x^5} - \frac{be\operatorname{asec}(cx)}{3x^3} + \frac{bd\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2| > 1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c} + \frac{be\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2| > 1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((e*x**2+d)*(a+b*asec(c*x))/x**6,x)
```

```
[Out] -a*d/(5*x**5) - a*e/(3*x**3) - b*d*asec(c*x)/(5*x**5) - b*e*asec(c*x)/(3*x*
*3) + b*d*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x
**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (
8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x*
*3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) + b*e*Piecewise((2*c*
*3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**
2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3
*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \frac{1}{75}\,bd\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{15\,\operatorname{arcsec}(cx)}{x^5}\right) - \frac{1}{9}\,be\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{3\,\operatorname{arcsec}(cx)}{x^3}\right) - \frac{ae}{3\,x^3} - \frac{ad}{5\,x^5}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/75*b*d*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2)
 + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcsec(c*x)/x^5) - 1/9*b*e*((c^4*(
-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c*x)/x
^3) - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$=\frac{1}{225}\left(24\,bc^4d\sqrt{-\frac{1}{c^2x^2}+1}+50\,bc^2e\sqrt{-\frac{1}{c^2x^2}+1}+\frac{12\,bc^2d\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}+\frac{25\,be\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}-\frac{75\,be\,\mathrm{arcc}}{cx^3}\right.$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 1/225*(24*b*c^4*d*sqrt(-1/(c^2*x^2) + 1) + 50*b*c^2*e*sqrt(-1/(c^2*x^2) + 1
) + 12*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/x^2 + 25*b*e*sqrt(-1/(c^2*x^2) + 1)/x
^2 - 75*b*e*arccos(1/(c*x))/(c*x^3) + 9*b*d*sqrt(-1/(c^2*x^2) + 1)/x^4 - 75
*a*e/(c*x^3) - 45*b*d*arccos(1/(c*x))/(c*x^5) - 45*a*d/(c*x^5))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^6}\,dx=\int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^6, x)
```

3.75 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 197

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{8bc^5(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}} + \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} + \frac{4bc^3(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}} - \frac{d(a+b\sec^{-1}(cx))}{7x^7} - \frac{e(a+b\sec^{-1}(cx))}{5x^5}$$

```
[Out] -1/7*d*(a+b*arcsec(c*x))/x^7-1/5*e*(a+b*arcsec(c*x))/x^5+8/3675*b*c^5*(30*c
^2*d+49*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/49*b*c*d*(c^2*x^2-1)^(1/2)/x
^6/(c^2*x^2)^(1/2)+1/1225*b*c*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^
2)^(1/2)+4/3675*b*c^3*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

= {14, 5346, 12, 464, 277, 270}

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx = -\frac{d(a+b\sec^{-1}(cx))}{7x^7} - \frac{e(a+b\sec^{-1}(cx))}{5x^5} + \frac{bc\sqrt{c^2x^2-1}(30c^2d+49e)}{1225x^4\sqrt{c^2x^2}} + \frac{bcd\sqrt{c^2x^2-1}}{49x^6\sqrt{c^2x^2}} + \frac{8bc^5\sqrt{c^2x^2-1}(30c^2d+49e)}{3675\sqrt{c^2x^2}} + \frac{4bc^3\sqrt{c^2x^2-1}(30c^2d+49e)}{3675x^2\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^8,x]
```

```
[Out] (8*b*c^5*(30*c^2*d + 49*e)*Sqrt[-1 + c^2*x^2])/(3675*Sqrt[c^2*x^2]) + (b*c*
d*Sqrt[-1 + c^2*x^2])/(49*x^6*Sqrt[c^2*x^2]) + (b*c*(30*c^2*d + 49*e)*Sqrt[
-1 + c^2*x^2])/(1225*x^4*Sqrt[c^2*x^2]) + (4*b*c^3*(30*c^2*d + 49*e)*Sqrt[-
1 + c^2*x^2])/(3675*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcSec[c*x]))/(7*x^7) -
(e*(a + b*ArcSec[c*x]))/(5*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}-\frac{(bcx)\int\frac{-5d-7ex^2}{35x^8\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}-\frac{(bcx)\int\frac{-5d-7ex^2}{x^8\sqrt{-1+c^2x^2}}\,dx}{35\sqrt{c^2x^2}}\\
&= \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}\\
&\quad-\frac{(bc(-30c^2d-49e)\,x)\int\frac{1}{x^6\sqrt{-1+c^2x^2}}\,dx}{245\sqrt{c^2x^2}}\\
&= \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}+\frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{7x^7}\\
&\quad-\frac{e(a+b\sec^{-1}(cx))}{5x^5}-\frac{(4bc^3(-30c^2d-49e)\,x)\int\frac{1}{x^4\sqrt{-1+c^2x^2}}\,dx}{1225\sqrt{c^2x^2}}\\
&= \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}+\frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}+\frac{4bc^3(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}}\\
&\quad-\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}-\frac{(8bc^5(-30c^2d-49e)\,x)\int\frac{1}{x^2\sqrt{-1+c^2x^2}}\,dx}{3675\sqrt{c^2x^2}}\\
&= \frac{8bc^5(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}}+\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}+\frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}\\
&\quad+\frac{4bc^3(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}}-\frac{d(a+b\sec^{-1}(cx))}{7x^7}-\frac{e(a+b\sec^{-1}(cx))}{5x^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$=\frac{-105a(5d+7ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(49ex^2(3+4c^2x^2+8c^4x^4)+15d(5+6c^2x^2+8c^4x^4+16c^6x^6))-105b}{3675x^7}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^8,x]
```

```
[Out] (-105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(49*e*x^2*(3 + 4*c^2*
x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(
5*d + 7*e*x^2)*ArcSec[c*x])/(3675*x^7)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.74

method	result
parts	$a\left(-\frac{e}{5x^5}-\frac{d}{7x^7}\right)+b\,c^7\left(-\frac{\operatorname{arcsec}(cx)e}{5c^7x^5}-\frac{\operatorname{arcsec}(cx)d}{7x^7c^7}+\frac{(c^2x^2-1)(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e}{3675c^{10}\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^8}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{7c^5x^7}-\frac{\operatorname{arcsec}(cx)e}{5c^5x^5}+\frac{(c^2x^2-1)(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4d\,x}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$
default	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)d}{7c^5x^7}-\frac{\operatorname{arcsec}(cx)e}{5c^5x^5}+\frac{(c^2x^2-1)(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4d\,x}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$

```
[In] int((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/5*e/x^5-1/7*d/x^7)+b*c^7*(-1/5/c^7*arcsec(c*x)*e/x^5-1/7*arcsec(c*x)*
d/x^7/c^7+1/3675/c^10*(c^2*x^2-1)*(240*c^8*d*x^6+392*c^6*e*x^6+120*c^6*d*x^
4+196*c^4*e*x^4+90*c^4*d*x^2+147*c^2*e*x^2+75*c^2*d)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x^8)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx = -\frac{735\,aex^2+525\,ad+105\,(7\,bex^2+5\,bd)\,\mathrm{arcsec}\,(cx)-(8\,(30\,bc^6d+49\,bc^4e)x^6+4\,(30\,bc^4d+49\,bc^2e)x^4+}{3675\,x^7}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*arcsec(c*x) - (8*(
30*b*c^6*d + 49*b*c^4*e)*x^6 + 4*(30*b*c^4*d + 49*b*c^2*e)*x^4 + 3*(30*b*c^
2*d + 49*b*e)*x^2 + 75*b*d)*sqrt(c^2*x^2 - 1))/x^7
```

Sympy [A] (verification not implemented)

Time = 29.28 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx = -\frac{ad}{7x^7}-\frac{ae}{5x^5}-\frac{bd\,\mathrm{asec}\,(cx)}{7x^7}-\frac{be\,\mathrm{asec}\,(cx)}{5x^5} + \frac{bd\left(\begin{cases}\frac{16c^7\sqrt{c^2x^2-1}}{35x}+\frac{8c^5\sqrt{c^2x^2-1}}{35x^3}+\frac{6c^3\sqrt{c^2x^2-1}}{35x^5}+\frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2|>1\\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x}+\frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3}+\frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5}+\frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise}\end{cases}\right)}{7c} + \frac{be\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$

```
[In] integrate((e*x**2+d)*(a+b*asec(c*x))/x**8,x)
```

```
[Out] -a*d/(7*x**7) - a*e/(5*x**5) - b*d*asec(c*x)/(7*x**7) - b*e*asec(c*x)/(5*x*
*5) + b*d*Piecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*
x**2 - 1)/(35*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x*
*2 - 1)/(7*x**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*
x) + 8*I*c**5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1
)/(35*x**5) + I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) + b*e*Piecewi
se((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3
) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c*
*2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c
**2*x**2 + 1)/(5*x**5), True))/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx = -\frac{1}{245}\,bd\left(\frac{5\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{35\,\mathrm{arcsec}\,(c}{x^7}\right. + \frac{1}{75}\,be\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\mathrm{arcsec}\,(cx)}{x^5}\right) - \frac{ae}{5\,x^5}-\frac{ad}{7\,x^7}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] -1/245*b*d*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(5/
2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c + 3
5*arcsec(c*x)/x^7) + 1/75*b*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1
/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcsec(c*x)/x
^5) - 1/5*a*e/x^5 - 1/7*a*d/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{1}{3675}\left(240\,bc^6d\sqrt{-\frac{1}{c^2x^2}+1}+392\,bc^4e\sqrt{-\frac{1}{c^2x^2}+1}+\frac{120\,bc^4d\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}+\frac{196\,bc^2e\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}+\frac{90}{}\right.$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^8,x, algorithm="giac")
```

```
[Out] 1/3675*(240*b*c^6*d*sqrt(-1/(c^2*x^2) + 1) + 392*b*c^4*e*sqrt(-1/(c^2*x^2)
+ 1) + 120*b*c^4*d*sqrt(-1/(c^2*x^2) + 1)/x^2 + 196*b*c^2*e*sqrt(-1/(c^2*x^
2) + 1)/x^2 + 90*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/x^4 + 147*b*e*sqrt(-1/(c^2*
x^2) + 1)/x^4 - 735*b*e*arccos(1/(c*x))/(c*x^5) + 75*b*d*sqrt(-1/(c^2*x^2)
+ 1)/x^6 - 735*a*e/(c*x^5) - 525*b*d*arccos(1/(c*x))/(c*x^7) - 525*a*d/(c*x
^7))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^8, x)
```

3.76 $\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 196

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b(4c^2d+3e)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}-\frac{b(8c^2d+9e)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}-\frac{b(4c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}-\frac{bex(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)$$

```
[Out] 1/6*d*x^6*(a+b*arcsec(c*x))+1/8*e*x^8*(a+b*arcsec(c*x))-1/72*b*(8*c^2*d+9*e
)*x*(c^2*x^2-1)^(3/2)/c^7/(c^2*x^2)^(1/2)-1/120*b*(4*c^2*d+9*e)*x*(c^2*x^2-
1)^(5/2)/c^7/(c^2*x^2)^(1/2)-1/56*b*e*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/
2)-1/24*b*(4*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

= {14, 5346, 12, 457, 78}

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)-\frac{bx(c^2x^2-1)^{5/2}\left(4c^2d+9e\right)}{120c^7\sqrt{c^2x^2}}-\frac{bx(c^2x^2-1)^{3/2}\left(8c^2d+9e\right)}{72c^7\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}(4c^2d+3e)}{24c^7\sqrt{c^2x^2}}-\frac{bex(c^2x^2-1)^{7/2}}{56c^7\sqrt{c^2x^2}}$$

```
[In] Int[x^5*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/24*(b*(4*c^2*d + 3*e)*x*Sqrt[-1 + c^2*x^2])/(c^7*Sqrt[c^2*x^2]) - (b*(8*
c^2*d + 9*e)*x*(-1 + c^2*x^2)^(3/2))/(72*c^7*Sqrt[c^2*x^2]) - (b*(4*c^2*d +
 9*e)*x*(-1 + c^2*x^2)^(5/2))/(120*c^7*Sqrt[c^2*x^2]) - (b*e*x*(-1 + c^2*x^
2)^(7/2))/(56*c^7*Sqrt[c^2*x^2]) + (d*x^6*(a + b*ArcSec[c*x]))/6 + (e*x^8*(
a + b*ArcSec[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^5(4d+3ex^2)}{24\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^5(4d+3ex^2)}{\sqrt{-1+c^2x^2}}\,dx}{24\sqrt{c^2x^2}}\\
&= \frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\text{Subst}\left(\int\frac{x^2(4d+3ex)}{\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{48\sqrt{c^2x^2}}\\
&= \frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)\\
&\quad-\frac{(bcx)\text{Subst}\left(\int\left(\frac{4c^2d+3e}{c^6\sqrt{-1+c^2x}}+\frac{(8c^2d+9e)\sqrt{-1+c^2x}}{c^6}+\frac{(4c^2d+9e)(-1+c^2x)^{3/2}}{c^6}+\frac{3e(-1+c^2x)^{5/2}}{c^6}\right)dx,x,x^2\right)}{48\sqrt{c^2x^2}}\\
&= -\frac{b(4c^2d+3e)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}-\frac{b(8c^2d+9e)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}\\
&\quad-\frac{b(4c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}-\frac{bex(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}}\\
&\quad+\frac{1}{6}dx^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\sec^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.60

$$\begin{aligned}
&\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx\\
&= \frac{1}{24}ax^6\left(4d+3ex^2\right)\\
&\quad-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x(144e+8c^2(28d+9ex^2)+2c^4(56dx^2+27ex^4)+c^6(84dx^4+45ex^6))}{2520c^7}\\
&\quad+\frac{1}{24}bx^6\left(4d+3ex^2\right)\sec^{-1}(cx)
\end{aligned}$$

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] (a*x^6*(4*d + 3*e*x^2))/24 - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(144*e + 8*c^2*(28*
d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6)))/(2
520*c^7) + (b*x^6*(4*d + 3*e*x^2)*ArcSec[c*x])/24
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

method	result
parts	$a\left(\frac{1}{8}e\,x^8+\frac{1}{6}d\,x^6\right)+\frac{b\left(\frac{c^6\,\mathrm{arcsec}(cx)e\,x^8}{8}+\frac{\mathrm{arcsec}(cx)d\,x^6c^6}{6}-\frac{(c^2x^2-1)(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2+72c^2e\,x\cdots}{2520c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}\cdots\right)}{c^6}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{6}c^8d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\left(\frac{\mathrm{arcsec}(cx)d\,c^8x^6}{6}+\frac{\mathrm{arcsec}(cx)e\,c^8x^8}{8}-\frac{(c^2x^2-1)(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2+72c^2e\,x^2+224\cdots}{2520\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\cdots\right)}{c^2}}{c^6}$
default	$\frac{\frac{a\left(\frac{1}{6}c^8d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\left(\frac{\mathrm{arcsec}(cx)d\,c^8x^6}{6}+\frac{\mathrm{arcsec}(cx)e\,c^8x^8}{8}-\frac{(c^2x^2-1)(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2+72c^2e\,x^2+224\cdots}{2520\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\cdots\right)}{c^2}}{c^6}$

```
[In] int(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/8*e*x^8+1/6*d*x^6)+b/c^6*(1/8*c^6*arcsec(c*x)*e*x^8+1/6*arcsec(c*x)*d*
x^6*c^6-1/2520/c^3*(c^2*x^2-1)*(45*c^6*e*x^6+84*c^6*d*x^4+54*c^4*e*x^4+112*
c^4*d*x^2+72*c^2*e*x^2+224*c^2*d+144*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.65

$$\int x^5\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{315\,ac^8ex^8+420\,ac^8dx^6+105\left(3\,bc^8ex^8+4\,bc^8dx^6\right)\mathrm{arcsec}\left(cx\right)-\left(45\,bc^6ex^6+6\left(14\,bc^6d+9\,bc^4e\right)x^4+224\cdots\right.}{2520\,c^8}$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^8*e*x^8 + 420*a*c^8*d*x^6 + 105*(3*b*c^8*e*x^8 + 4*b*c^8*d*
x^6)*arcsec(c*x) - (45*b*c^6*e*x^6 + 6*(14*b*c^6*d + 9*b*c^4*e)*x^4 + 224*b
*c^2*d + 8*(14*b*c^4*d + 9*b*c^2*e)*x^2 + 144*b*e)*sqrt(c^2*x^2 - 1))/c^8
```

Sympy [A] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.86

$$\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{adx^6}{6}+\frac{aex^8}{8}+\frac{bdx^6\operatorname{asec}(cx)}{6}+\frac{bex^8\operatorname{asec}(cx)}{8} - \frac{bd\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c} - \frac{be\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c}+\frac{6x^4\sqrt{c^2x^2-1}}{35c^3}+\frac{8x^2\sqrt{c^2x^2-1}}{35c^5}+\frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c}+\frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3}+\frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5}+\frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

```
[In] integrate(x**5*(e*x**2+d)*(a+b*asec(c*x)),x)
```

```
[Out] a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*asec(c*x)/6 + b*e*x**8*asec(c*x)/8 - b*d
*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15
*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt
(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt
(-c**2*x**2 + 1)/(15*c**5), True))/(6*c) - b*e*Piecewise((x**6*sqrt(c**2*x*
*2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3) + 8*x**2*sqrt(c**2*x**
2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7), Abs(c**2*x**2) > 1), (
I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c**2*x**2 + 1)/(35*c**3)
 + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sqrt(-c**2*x**2 + 1)/(35*
c**7), True))/(8*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94

$$\int x^5(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{8}aex^8+\frac{1}{6}adx^6 + \frac{1}{90}\left(15\,x^6\operatorname{arcsec}(cx) - \frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)bd + \frac{1}{280}\left(35\,x^8\operatorname{arcsec}(cx) - \frac{5\,c^6x^7\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}+21\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+35\,x\sqrt{-}}{c^7}\right.$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arcsec(c*x) - (3*c^4*x^5*(-1/(c^2*
x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x
^2) + 1))/c^5)*b*d + 1/280*(35*x^8*arcsec(c*x) - (5*c^6*x^7*(-1/(c^2*x^2) +
 1)^(7/2) + 21*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2)
+ 1)^(3/2) + 35*x*sqrt(-1/(c^2*x^2) + 1))/c^7)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13018 vs. $2(168) = 336$.

Time = 0.47 (sec) , antiderivative size = 13018, normalized size of antiderivative = 66.42

$$\int x^5 (d + ex^2) (a + b\sec^{-1}(cx)) \, dx = \text{Too large to display}$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/2520*(420*b*c^2*d*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^
9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x
) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) -
 1)^8/(1/(c*x) + 1)^16) + 420*a*c^2*d/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*
c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c
*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2)
 - 1)^8/(1/(c*x) + 1)^16) - 1680*b*c^2*d*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/
((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/
(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)
^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)
^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) +
1)^2) - 840*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c
^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8
 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6
/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^
2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)) - 1680*a*c^2*d*(1/(c^2*x^2)
- 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^
9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x)
 + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2)
 - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*
```

x) + 1)^2) + 315*b*e*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c
^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*
x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2)
- 1)^8/(1/(c*x) + 1)^16) + 1680*b*c^2*d*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))
/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1
)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1
)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) +
 1)^4) + 3640*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)/((c^9 + 8*c^9*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) -
1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1
/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^3) + 315*a*e/(c^9 + 8*c^9
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*
(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16) + 1680*a*c^2*d*(1/(c^2*x
^2) - 1)^2/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1
/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^
2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*
(1/(c*x) + 1)^4) - 2520*b*e*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^9 + 8*c^9
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*
(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^2) + 1680*
b*c^2*d*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^
8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^
6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c
^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^6) - 630*b*e*sqrt(-1/(c^2*x^
2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^
2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70
c^9(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c
*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x
^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/
(c*x) + 1)) - 9128*b*c^2*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^9

+ 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^5) - 2520*a*e*(1/(c^2*x^2) - 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^2) + 1680*a*c^2*d*(1/(c^2*x^2) - 1)^3/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^6) + 8820*b*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^4) - 4200*b*c^2*d*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^8) - 15064*b*c^2*d*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^7) + 1890*b*e*(-1/(c^2*x^2) + 1)^(3/2)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^3) + 8820*a*e*(1/(c^2*x^2) - 1)^2/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^4) - 4200*a*c^2*d*(1/(c^2*x^2) - 1)^4/((c^9 + 8

c^9(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*
c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*
x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^8) - 1
7640*b*e*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)
^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(
c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^6) + 1680*b*c^2*d*(1/(c^2*x
^2) - 1)^5*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^
2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^1
2 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1
/(c*x) + 1)^16)*(1/(c*x) + 1)^10) - 6678*b*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c
^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/
(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(
c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16
)*(1/(c*x) + 1)^5) - 15064*b*c^2*d*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) +
1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*
(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) +
 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) -
 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x)
 + 1)^9) - 17640*a*e*(1/(c^2*x^2) - 1)^3/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(
1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*
x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^6) + 1680*a*c^2*d*(1/(c^2*x^2)
- 1)^5/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2
) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*
c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*
x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^
2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(
c*x) + 1)^10) + 22050*b*e*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^9 + 8*c^9
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*
(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^8) + 1680*

b*c^2*d*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^
8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^
6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c
^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^12) - 9234*b*e*(1/(c^2*x^2)
- 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/
(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1
)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8
/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^7) - 9128*b*c^2*d*(1/(c^2*x^2) - 1)^5*sqrt
(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^
9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) +
1)^16)*(1/(c*x) + 1)^11) + 22050*a*e*(1/(c^2*x^2) - 1)^4/((c^9 + 8*c^9*(1/
(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
+ 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/
(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(
c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^
14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^8) + 1680*a*c^
2*d*(1/(c^2*x^2) - 1)^6/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2
8*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x
^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 +
8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c
x) + 1)^16)(1/(c*x) + 1)^12) - 17640*b*e*(1/(c^2*x^2) - 1)^5*arccos(1/(c*
x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9
*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x)
+ 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2)
- 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x
) + 1)^10) - 1680*b*c^2*d*(1/(c^2*x^2) - 1)^7*arccos(1/(c*x))/((c^9 + 8*c^9
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*
(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^14) - 9234
*b*e*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) -
1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(

1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^9) - 3640*b*c^2*d*(1/(c^
2*x^2) - 1)^6*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*
c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c
*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2)
 - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^13) - 17640*a*e*(1/(c^2*x^2) - 1)^5
/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1
)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1
)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) +
 1)^10) - 1680*a*c^2*d*(1/(c^2*x^2) - 1)^7/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c
^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8
 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6
/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^
2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^14) + 8820*b*e*(1/(c^2*x^2) -
 1)^6*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*
c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*
x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2
) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8
c^9(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x
) + 1)^16)*(1/(c*x) + 1)^12) + 420*b*c^2*d*(1/(c^2*x^2) - 1)^8*arccos(1/(c*
x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9
*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x)
+ 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2)
- 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x
) + 1)^16) - 6678*b*e*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*
c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c
^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x
) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^11) - 8
40*b*c^2*d*(1/(c^2*x^2) - 1)^7*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 5
6*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*
x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 +
 c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^15) + 8820*a*e*(1/
(c^2*x^2) - 1)^6/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(
1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1
)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*

(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1
)^16)*(1/(c*x) + 1)^12) + 420*a*c^2*d*(1/(c^2*x^2) - 1)^8/((c^9 + 8*c^9*(1/
(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/
(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(
c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^
14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^16) - 2520*b*e
*(1/(c^2*x^2) - 1)^7*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*
c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c
*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2)
 - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^14) - 1890*b*e*(1/(c^2*x^2) - 1)^6*
sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28
c^9(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^
2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 +
8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*
x) + 1)^16)*(1/(c*x) + 1)^13) - 2520*a*e*(1/(c^2*x^2) - 1)^7/((c^9 + 8*c^9*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(
1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^14) + 315*b
e(1/(c^2*x^2) - 1)^8*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(
c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 5
6*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/
(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^
2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^16) - 630*b*e*(1/(c^2*x^2) - 1)^7
*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2
8*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x
^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 +
 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c
x) + 1)^16)(1/(c*x) + 1)^15) + 315*a*e*(1/(c^2*x^2) - 1)^8/((c^9 + 8*c^9*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(
1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) +
1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^16))*c

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right) \, dx = \int x^5 \left(e\,x^2+d\right) \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x^5*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
[Out] int(x^5*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.77 $\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int x^3\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b(3c^2d+2e)\,x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}}-\frac{b(3c^2d+4e)\,x(-1+c^2x^2)^{3/2}}{36c^5\sqrt{c^2x^2}}-\frac{bex(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)$$

```
[Out] 1/4*d*x^4*(a+b*arcsec(c*x))+1/6*e*x^6*(a+b*arcsec(c*x))-1/36*b*(3*c^2*d+4*e
)*x*(c^2*x^2-1)^(3/2)/c^5/(c^2*x^2)^(1/2)-1/30*b*e*x*(c^2*x^2-1)^(5/2)/c^5/
(c^2*x^2)^(1/2)-1/12*b*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5346, 12, 457, 78}

$$\int x^3\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{4}dx^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\sec^{-1}(cx)\right)-\frac{bx(c^2x^2-1)^{3/2}\left(3c^2d+4e\right)}{36c^5\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}(3c^2d+2e)}{12c^5\sqrt{c^2x^2}}-\frac{bex(c^2x^2-1)^{5/2}}{30c^5\sqrt{c^2x^2}}$$

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/12*(b*(3*c^2*d + 2*e)*x*Sqrt[-1 + c^2*x^2])/(c^5*Sqrt[c^2*x^2]) - (b*(3*
c^2*d + 4*e)*x*(-1 + c^2*x^2)^(3/2))/(36*c^5*Sqrt[c^2*x^2]) - (b*e*x*(-1 +
c^2*x^2)^(5/2))/(30*c^5*Sqrt[c^2*x^2]) + (d*x^4*(a + b*ArcSec[c*x]))/4 + (e
*x^6*(a + b*ArcSec[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{4}dx^4\big(a+b\sec^{-1}(cx)\big)+\frac{1}{6}ex^6\big(a+b\sec^{-1}(cx)\big)-\frac{(bcx)\int\frac{x^3(3d+2ex^2)}{12\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{1}{4}dx^4\left(a + b\sec^{-1}(cx)\right) + \frac{1}{6}ex^6\left(a + b\sec^{-1}(cx)\right) - \frac{(bcx)\int \frac{x^3(3d+2ex^2)}{\sqrt{-1+c^2x^2}}\,dx}{12\sqrt{c^2x^2}}$$

$$= \frac{1}{4}dx^4\left(a + b\sec^{-1}(cx)\right) + \frac{1}{6}ex^6\left(a + b\sec^{-1}(cx)\right) - \frac{(bcx)\mathrm{Subst}\left(\int \frac{x(3d+2ex)}{\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{24\sqrt{c^2x^2}}$$

$$= \frac{1}{4}dx^4\left(a + b\sec^{-1}(cx)\right) + \frac{1}{6}ex^6\left(a + b\sec^{-1}(cx)\right) - \frac{(bcx)\mathrm{Subst}\left(\int \left(\frac{3c^2d+2e}{c^4\sqrt{-1+c^2x}} + \frac{(3c^2d+4e)\sqrt{-1+c^2x}}{c^4} + \frac{2e(-1+c^2x)^{3/2}}{c^4}\right)\,dx, x, x^2\right)}{24\sqrt{c^2x^2}}$$

$$= -\frac{b(3c^2d+2e)\,x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}} - \frac{b(3c^2d+4e)\,x(-1+c^2x^2)^{3/2}}{36c^5\sqrt{c^2x^2}} - \frac{bex(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{1}{4}dx^4\left(a + b\sec^{-1}(cx)\right) + \frac{1}{6}ex^6\left(a + b\sec^{-1}(cx)\right)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int x^3\left(d + ex^2\right)\left(a + b\sec^{-1}(cx)\right)\,dx = \frac{1}{180}x\left(15ax^3\left(3d + 2ex^2\right) - \frac{b\sqrt{1-\frac{1}{c^2x^2}}(16e + c^2(30d + 8ex^2) + 3c^4(5dx^2 + 2ex^4))}{c^5} + 15bx^3\left(3d + 2ex^2\right)\sec^{-1}(cx)\right)$$

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] (x*(15*a*x^3*(3*d + 2*e*x^2) - (b*Sqrt[1 - 1/(c^2*x^2)]*(16*e + c^2*(30*d +
 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*ArcS
ec[c*x]))/180
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

method	result
parts	$a\left(\frac{1}{6}e\,x^{6}+\frac{1}{4}d\,x^{4}\right)+\frac{b\left(\frac{c^{4}\operatorname{arcsec}\left(cx\right)e\,x^{6}}{6}+\frac{\operatorname{arcsec}\left(cx\right)x^{4}c^{4}d}{4}-\frac{\left(c^{2}x^{2}-1\right)\left(6c^{4}e\,x^{4}+15c^{4}d\,x^{2}+8c^{2}e\,x^{2}+30c^{2}d+16e\right)}{180c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x}\right)}{c^{4}}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^{2}d\left(c^{2}e\,x^{2}+c^{2}d\right)^{2}}{2}-\frac{\left(c^{2}e\,x^{2}+c^{2}d\right)^{3}}{3}\right)}{2c^{2}e^{2}}-\frac{b\,c^{4}\operatorname{arcsec}\left(cx\right)d^{3}}{12e^{2}}+\frac{b\,\operatorname{arcsec}\left(cx\right)d\,c^{4}x^{4}}{4}+\frac{b\,c^{4}e\,\operatorname{arcsec}\left(cx\right)x^{6}}{6}-\frac{b\,c^{3}\sqrt{c^{2}x^{2}-1}\,d^{3}\arctan}{12e^{2}\sqrt{\frac{c^{2}x^{2}-}{c^{2}x^{2}}}}}{c^{4}}$
default	$\frac{-\frac{a\left(\frac{c^{2}d\left(c^{2}e\,x^{2}+c^{2}d\right)^{2}}{2}-\frac{\left(c^{2}e\,x^{2}+c^{2}d\right)^{3}}{3}\right)}{2c^{2}e^{2}}-\frac{b\,c^{4}\operatorname{arcsec}\left(cx\right)d^{3}}{12e^{2}}+\frac{b\,\operatorname{arcsec}\left(cx\right)d\,c^{4}x^{4}}{4}+\frac{b\,c^{4}e\,\operatorname{arcsec}\left(cx\right)x^{6}}{6}-\frac{b\,c^{3}\sqrt{c^{2}x^{2}-1}\,d^{3}\arctan}{12e^{2}\sqrt{\frac{c^{2}x^{2}-}{c^{2}x^{2}}}}}{c^{4}}$

```
[In] int(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arcsec(c*x)*e*x^6+1/4*arcsec(c*x)*x^
4*c^4*d-1/180/c^3*(c^2*x^2-1)*(6*c^4*e*x^4+15*c^4*d*x^2+8*c^2*e*x^2+30*c^2*
d+16*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.70

$$\int x^3\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{30\,ac^6ex^6+45\,ac^6dx^4+15\,(2\,bc^6ex^6+3\,bc^6dx^4)\,\mathrm{arcsec}\,(cx)-(6\,bc^4ex^4+30\,bc^2d+(15\,bc^4d+8\,bc^2e)x^2-}{180\,c^6}$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*e*x^6 + 45*a*c^6*d*x^4 + 15*(2*b*c^6*e*x^6 + 3*b*c^6*d*x^4)
*arcsec(c*x) - (6*b*c^4*e*x^4 + 30*b*c^2*d + (15*b*c^4*d + 8*b*c^2*e)*x^2 +
 16*b*e)*sqrt(c^2*x^2 - 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.78

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\operatorname{asec}(cx)}{4}+\frac{bex^6\operatorname{asec}(cx)}{6}-\frac{bd\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}-\frac{be\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

```
[In] integrate(x**3*(e*x**2+d)*(a+b*asec(c*x)),x)
```

```
[Out] a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*asec(c*x)/4 + b*e*x**6*asec(c*x)/6 - b*d
*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3)
, Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*
x**2 + 1)/(3*c**3), True))/(4*c) - b*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/
(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c*
*5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqr
t(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6
*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3\,x^4\operatorname{arcsec}(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd+\frac{1}{90}\left(15\,x^6\operatorname{arcsec}(cx)-\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*x^2
) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arcsec(c
*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(
3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7820 vs. $2(131) = 262$.

Time = 0.37 (sec) , antiderivative size = 7820, normalized size of antiderivative = 51.11

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/180*(45*b*c^2*d*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) +
 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)
^12) + 45*a*c^2*d/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(
1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)
^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 90*b*c^2*
d*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^
7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) +
 1)^12)*(1/(c*x) + 1)^2) - 90*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1
)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c
^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)) - 90*a*c^2*d*(1/(c^2*x^2) -
 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7
*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) +
 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2) + 30*b*
e*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(
1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)
^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 45*b*c^2*
d*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*
c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12)*(1/(c*x) + 1)^4) + 330*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)/((c^7 + 6
```

```
*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*
(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^3) + 30*a*e/(c^7 + 6*c^
7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/
(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 45*a*c^2*d*(1/(c^2*x^2) - 1)^2/((c^7 +
 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^
7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^4) - 180*b*e*(1/(c^2*
x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1
/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*
x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1
/(c*x) + 1)^2) + 180*b*c^2*d*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^7 + 6*
c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(
1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^6) - 60*b*e*sqrt(-1/(c^2
*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/
(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)) -
540*b*c^2*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/
(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2
) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^5) - 180*a*e*(1/(c^2*x^2) - 1)/((c
^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^
2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
+ c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2) + 180*a*c^2*d*
(1/(c^2*x^2) - 1)^3/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^
7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x)
 + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) -
 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x)
 + 1)^6) + 450*b*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/
(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2
) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^4) - 45*b*c^2*d*(1/(c^2*x^2) - 1)^
4*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*
(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
```

1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1
)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) +
1)^8) - 540*b*c^2*d*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c
^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1
/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^7) + 140*b*e*(-1/(c^2*x^2
) + 1)^(3/2)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
+ 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(
1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^3
) + 450*a*e*(1/(c^2*x^2) - 1)^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1
/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^
12)*(1/(c*x) + 1)^4) - 45*a*c^2*d*(1/(c^2*x^2) - 1)^4/((c^7 + 6*c^7*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 2
0*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2)
- 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^8) - 600*b*e*(1/(c^2*x^2) - 1)^3*ar
ccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^
6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/
(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^
6) - 90*b*c^2*d*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*
c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*
x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) -
1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^10) - 312*b*e*(1/(c^2*x^2) - 1)^2*sqr
t(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^
7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x)
+ 1)^5) - 330*b*c^2*d*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6
c^7(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*
(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^9) - 600*a*e*(1/(c^2*x^
2) - 1)^3/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(
c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^6) -
90*a*c^2*d*(1/(c^2*x^2) - 1)^5/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1

/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^
12)*(1/(c*x) + 1)^10) + 450*b*e*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^7 +
 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^
7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^8) + 45*b*c^2*d*(1/(c
^2*x^2) - 1)^6*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/
(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^1
2)*(1/(c*x) + 1)^12) - 312*b*e*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/(
(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(
c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^1
0 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^7) - 90*b*c^2*d
*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^
8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1
/(c*x) + 1)^12)*(1/(c*x) + 1)^11) + 450*a*e*(1/(c^2*x^2) - 1)^4/((c^7 + 6*c
^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) -
 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1
/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^8) + 45*a*c^2*d*(1/(c^2*x
^2) - 1)^6/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2
*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/
(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^12)
 - 180*b*e*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6
/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^10) - 140*b*e*(1/(c^2*x^2) - 1)^4*sqrt(-1/
(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/
(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5
/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)
^9) - 180*a*e*(1/(c^2*x^2) - 1)^5/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) -
 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*
(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1
)^12)*(1/(c*x) + 1)^10) + 30*b*e*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^7
+ 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(
c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x
^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c

```
^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^12) - 60*b*e*(1/(c^2
*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^
7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) +
 1)^12)*(1/(c*x) + 1)^11) + 30*a*e*(1/(c^2*x^2) - 1)^6/((c^7 + 6*c^7*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/
(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2
) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^12))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^3\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.78 $\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 138

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}}-\frac{bex(-1+c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}}+\frac{(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcd^2x\arctan\left(\sqrt{-1+c^2x^2}\right)}{4e\sqrt{c^2x^2}}$$

```
[Out] 1/4*(e*x^2+d)^2*(a+b*arcsec(c*x))/e-1/12*b*e*x*(c^2*x^2-1)^(3/2)/c^3/(c^2*x
^2)^(1/2)-1/4*b*c*d^2*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-1/4*b*(
2*c^2*d+e)*x*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5344, 457, 90, 65, 211}

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)}{4e}-\frac{bcd^2x\arctan\left(\sqrt{c^2x^2-1}\right)}{4e\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}(2c^2d+e)}{4c^3\sqrt{c^2x^2}}-\frac{bex(c^2x^2-1)^{3/2}}{12c^3\sqrt{c^2x^2}}$$

```
[In] Int[x*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/4*(b*(2*c^2*d + e)*x*Sqrt[-1 + c^2*x^2])/(c^3*Sqrt[c^2*x^2]) - (b*e*x*(-
1 + c^2*x^2)^(3/2))/(12*c^3*Sqrt[c^2*x^2]) + ((d + e*x^2)^2*(a + b*ArcSec[c
*x]))/(4*e) - (b*c*d^2*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(4*e*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5344

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{4e} - \frac{(bcx)\int \frac{(d+ex^2)^2}{x\sqrt{-1+c^2x^2}}\,dx}{4e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{4e} - \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^2}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{8e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{4e} - \frac{(bcx)\text{Subst}\left(\int \left(\frac{e(2c^2d+e)}{c^2\sqrt{-1+c^2x}} + \frac{d^2}{x\sqrt{-1+c^2x}} + \frac{e^2\sqrt{-1+c^2x}}{c^2}\right)dx, x, x^2\right)}{8e\sqrt{c^2x^2}}$$

$$= -\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}} - \frac{bex(-1+c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{4e} - \frac{(bcd^2x)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{8e\sqrt{c^2x^2}}$$

$$= -\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}} - \frac{bex(-1+c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{4e} - \frac{(bd^2x)\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{4ce\sqrt{c^2x^2}}$$

$$= -\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}} - \frac{bex(-1+c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{4e} - \frac{bcd^2x\arctan\left(\sqrt{-1+c^2x^2}\right)}{4e\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$\int x(d+ex^2)\,(a+b\sec^{-1}(cx))\,dx = \frac{x\left(3ac^3x(2d+ex^2) - b\sqrt{1-\frac{1}{c^2x^2}}(2e+c^2(6d+ex^2)) + 3bc^3x(2d+ex^2)\sec^{-1}(cx)\right)}{12c^3}$$

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
[Out] (x*(3*a*c^3*x*(2*d + e*x^2) - b*Sqrt[1 - 1/(c^2*x^2)]*(2*e + c^2*(6*d + e*x
^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcSec[c*x]))/(12*c^3)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.57

method	result
parts	$\frac{a(e\,x^2+d)^2}{4e}+\frac{b\,\mathrm{arcsec}(cx)e\,x^4}{4}+\frac{b\,\mathrm{arcsec}(cx)x^2d}{2}+\frac{b\,\mathrm{arcsec}(cx)d^2}{4e}-\frac{b(c^2x^2-1)xe}{12c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,d^2\arctan\left(\cdots\right)}{4ce\sqrt{\frac{c^2x^2-1}{c^2x^2}}\cdots}\cdots$
derivativedivides	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^2}{4c^2e}+\frac{b\,c^2\,\mathrm{arcsec}(cx)d^2}{4e}+\frac{b\,\mathrm{arcsec}(cx)d\,c^2x^2}{2}+\frac{b\,c^2e\,\mathrm{arcsec}(cx)x^4}{4}+\frac{bc\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{b\left(c^2x^2-1\right)\cdots}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\cdots}\cdots}{c^2}$
default	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^2}{4c^2e}+\frac{b\,c^2\,\mathrm{arcsec}(cx)d^2}{4e}+\frac{b\,\mathrm{arcsec}(cx)d\,c^2x^2}{2}+\frac{b\,c^2e\,\mathrm{arcsec}(cx)x^4}{4}+\frac{bc\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{b\left(c^2x^2-1\right)\cdots}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\cdots}\cdots}{c^2}$

```
[In] int(x*(e*x^2+d)*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*(e*x^2+d)^2/e+1/4*b*arcsec(c*x)*e*x^4+1/2*b*arcsec(c*x)*x^2*d+1/4*b/e
*arcsec(c*x)*d^2-1/12*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x*e+1/4
*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*arctan(1/(c^2*x^
2-1)^(1/2))-1/2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d-1/6*b/c^5
*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.62

$$\int x\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{3\,ac^4ex^4+6\,ac^4dx^2+3\,(bc^4ex^4+2\,bc^4dx^2)\,\mathrm{arcsec}\,(cx)-(bc^2ex^2+6\,bc^2d+2\,be)\sqrt{c^2x^2-1}}{12\,c^4}$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*e*x^4 + 6*a*c^4*d*x^2 + 3*(b*c^4*e*x^4 + 2*b*c^4*d*x^2)*arcse
c(c*x) - (b*c^2*e*x^2 + 6*b*c^2*d + 2*b*e)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.28

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\operatorname{asec}(cx)}{2}+\frac{bex^4\operatorname{asec}(cx)}{4} - \frac{bd\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c} - \frac{be\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

```
[In] integrate(x*(e*x**2+d)*(a+b*asec(c*x)),x)
```

```
[Out] a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*asec(c*x)/2 + b*e*x**4*asec(c*x)/4 - b*d
*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2
+ 1)/c, True))/(2*c) - b*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sq
rt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 +
1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.72

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{4}\,aex^4+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\operatorname{arcsec}(cx)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd + \frac{1}{12}\left(3\,x^4\operatorname{arcsec}(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arcsec(c*x) - x*sqrt(-1/(c^2*x^2) + 1)
/c)*b*d + 1/12*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x
*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3346 vs. 2(118) = 236.

Time = 0.33 (sec) , antiderivative size = 3346, normalized size of antiderivative = 24.25

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/12*(6*b*c^2*d*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 6*a*c^2*d/(c^
5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/
(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)
^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8)*(1/(c*x) + 1)) + 3*b*e*arccos(1/(c*x))/(c^5 + 4*c^5*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c
^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
 1)^8) - 12*b*c^2*d*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8)*(1/(c*x) + 1)^4) + 36*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)/((c^5 + 4*
c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 3*a*e/(c^5 + 4*c^5*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 1
2*a*c^2*d*(1/(c^2*x^2) - 1)^2/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4)
 - 12*b*e*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1
/(c*x) + 1)^2) - 6*b*e*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*
(1/(c*x) + 1)) - 36*b*c^2*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^
5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/
(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) - 12*a*e*(1/(c^2*x^2) - 1)/((c^5
+ 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) -
```

```
1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 18*b*e*(1/(c^2*x^2) - 1)^2*arccos
(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^
5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 6*b*c^2*d*(1/(c^2
*x^2) - 1)^4*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8)
 - 12*b*c^2*d*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8)*(1/(c*x) + 1)^7) + 10*b*e*(-1/(c^2*x^2) + 1)^(3/2)/((c^5 + 4*c^5*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)
^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 18*a*e*(1/(c^2*x^2) - 1)^2/((c^5 + 4*c^5*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)
^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 6*a*c^2*d*(1/(c^2*x^2) - 1)^4/((c^5 + 4*c
^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 12*b*e*(1/(c^2*x^2) - 1)^3*arccos(1/(c
*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/
(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 10*b*e*(1/(c^2*x^2) -
1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1
/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) -
 12*a*e*(1/(c^2*x^2) - 1)^3/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1
/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) +
 3*b*e*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^5 + 4*c^5*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 4*c^5*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/
(c*x) + 1)^8) - 6*b*e*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^5 + 4*
c^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) + 3*a*e*(1/(c^2*x^2) - 1)^4/((c^5 + 4*c
^5*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 6*c^5*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
 1)^4 + 4*c^5*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + c^5*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8))*c
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)\left(a+b\sec^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
[Out] int(x*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.79 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx = -\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} - \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2(a+b\sec^{-1}(cx)) + bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - \frac{1}{2}ibd\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

```
[Out] -1/2*I*b*d*arccsc(c*x)^2+1/2*e*x^2*(a+b*arcsec(c*x))+b*d*arccsc(c*x)*ln(1-(
I/c/x+(1-1/c^2/x^2)^(1/2))^2)-b*d*arccsc(c*x)*ln(1/x)-d*(a+b*arcsec(c*x))*l
n(1/x)-1/2*I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-1/2*b*e*x*(1-1/c^
2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules

used = {5348, 14, 4816, 6874, 270, 2363, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)\left(a+b\sec^{-1}(cx)\right)}{x}\,dx = -d\log\left(\frac{1}{x}\right)\left(a+b\sec^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\sec^{-1}(cx)\right) - \frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c}-\frac{1}{2}ibd\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right) - \frac{1}{2}ibd\csc^{-1}(cx)^2+bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - bd\log\left(\frac{1}{x}\right)\csc^{-1}(cx)$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x,x]
```

```
[Out] -1/2*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c - (I/2)*b*d*ArcCsc[c*x]^2 + (e*x^2*(a
+ b*ArcSec[c*x]))/2 + b*d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - b*d*
ArcCsc[c*x]*Log[x^(-1)] - d*(a + b*ArcSec[c*x])*Log[x^(-1)] - (I/2)*b*d*Pol
yLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4816

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCos[c*x], u, x] + Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{1}{2}ex^2\left(a+b\sec^{-1}(cx)\right)-d\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-\frac{b\text{Subst}\left(\int\frac{-\frac{e}{2x^2}+d\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{1}{2}ex^2\left(a+b\sec^{-1}(cx)\right)-d\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)$$
$$-\frac{b\text{Subst}\left(\int\left(-\frac{e}{2x^2\sqrt{1-\frac{x^2}{c^2}}}+\frac{d\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\right)dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{1}{2}ex^2\left(a+b\sec^{-1}(cx)\right)-d\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)$$
$$-\frac{(bd)\text{Subst}\left(\int\frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c}+\frac{(be)\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2c}$$

$$=-\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}ex^2\left(a+b\sec^{-1}(cx)\right)-bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$-d\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+(bd)\text{Subst}\left(\int\frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}ex^2\left(a+b\sec^{-1}(cx)\right)-bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$-d\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+(bd)\text{Subst}\left(\int x\cot(x)\,dx,x,\csc^{-1}(cx)\right)$$

$$=-\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}-\frac{1}{2}ibd\csc^{-1}(cx)^2+\frac{1}{2}ex^2\left(a+b\sec^{-1}(cx)\right)-bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$-d\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-(2ibd)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx,x,\csc^{-1}(cx)\right)$$

$$=-\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}-\frac{1}{2}ibd\csc^{-1}(cx)^2+\frac{1}{2}ex^2\left(a+b\sec^{-1}(cx)\right)$$
$$+bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)-bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$-d\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-(bd)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx,x,\csc^{-1}(cx)\right)$$

$$= -\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} - \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2(a+b\sec^{-1}(cx)) + bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{1}{2}(ibd)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)$$

$$= -\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} - \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2(a+b\sec^{-1}(cx)) + bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - \frac{1}{2}ibd\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int\frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x}\,dx = \frac{1}{2}aex^2 - \frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c} + \frac{1}{2}bex^2\sec^{-1}(cx) + \frac{1}{2}ibd\sec^{-1}(cx)^2 - bd\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) + ad\log(x) + \frac{1}{2}ibd\,\text{PolyLog}\left(2, -e^{2i\sec^{-1}(cx)}\right)$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x,x]
```

```
[Out] (a*e*x^2)/2 - (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*e*x^2*ArcSe
c[c*x])/2 + (I/2)*b*d*ArcSec[c*x]^2 - b*d*ArcSec[c*x]*Log[1 + E^((2*I)*ArcS
ec[c*x])] + a*d*Log[x] + (I/2)*b*d*PolyLog[2, -E^((2*I)*ArcSec[c*x])]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

method	result
parts	$\frac{ae\,x^2}{2}+ad\ln(x)+b\left(\frac{i\,\mathrm{arcsec}(cx)^2 d}{2}+\frac{e\left(c^2x^2\,\mathrm{arcsec}(cx)-xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2c^2}-d\,\mathrm{arcsec}(cx)\ln\left(1+\right.\right.$
derivativedivides	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{ic^2d\,\mathrm{arcsec}(cx)^2}{2}+\frac{e\left(c^2x^2\,\mathrm{arcsec}(cx)-xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2}-\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)c^2d\,\mathrm{arcs}\right.}{c^2}$
default	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{ic^2d\,\mathrm{arcsec}(cx)^2}{2}+\frac{e\left(c^2x^2\,\mathrm{arcsec}(cx)-xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2}-\ln\left(1+\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2\right)c^2d\,\mathrm{arcs}\right.}{c^2}$

```
[In] int((e*x^2+d)*(a+b*arcsec(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*e*x^2+a*d*ln(x)+b*(1/2*I*arcsec(c*x)^2*d+1/2*e*(c^2*x^2*arcsec(c*x)-x
*c*((c^2*x^2-1)/c^2/x^2)^(1/2)-I)/c^2-d*arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/
x^2)^(1/2))^2)+1/2*I*d*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x}\,dx=\int \frac{(ex^2+d)(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x}\,dx=\int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)}{x}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*asec(c*x))/x,x)
```

```
[Out] Integral((a + b*asec(c*x))*(d + e*x**2)/x, x)
```

Maxima [F]

$$\int \frac{(d + ex^2)(a + b\sec^{-1}(cx))}{x} \, dx = \int \frac{(ex^2 + d)(b\,\mathrm{arcsec}\,(cx) + a)}{x} \, dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + a*d*log(x) - 1/4*(-2*I*b*c^2*e*x^2*log(c) - 2*I*b*c^2*d*log(-
c*x + 1)*log(x) - 2*I*b*c^2*d*log(x)^2 - 2*I*b*c^2*d*dilog(c*x) - 2*I*b*c^2
*d*dilog(-c*x) + I*(b*e*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 8*b*d*integ
rate(1/2*log(x)/(c^2*x^3 - x), x))*c^2 + 4*c^2*integrate(1/2*(b*e*x^2 + 2*b
*d*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) - I*b*e*log(c*x -
1) - 2*(b*c^2*e*x^2 + 2*b*c^2*d*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
 + (I*b*c^2*e*x^2 + 2*I*b*c^2*d*log(x))*log(c^2*x^2) + (-2*I*b*c^2*d*log(x)
 - I*b*e)*log(c*x + 1) - 2*(I*b*c^2*e*x^2 + 2*I*b*c^2*d*log(c))*log(x))/c^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex^2)(a + b\sec^{-1}(cx))}{x} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expansi
on Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)(a + b\sec^{-1}(cx))}{x} \, dx = \int \frac{(e\,x^2 + d)\,\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x} \, dx$$

```
[In] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x, x)
```

3.80 $\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\begin{aligned}\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^3}\,dx &= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{1}{2}ibe\csc^{-1}(cx)^2 \\ &\quad - \frac{d(a+b\sec^{-1}(cx))}{2x^2} + be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) \\ &\quad - be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) \\ &\quad - \frac{1}{2}ibe\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

```
[Out] -1/4*b*c^2*d*arccsc(c*x)-1/2*I*b*e*arccsc(c*x)^2-1/2*d*(a+b*arcsec(c*x))/x^
2+b*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-b*e*arccsc(c*x)*ln(1/
x)-e*(a+b*arcsec(c*x))*ln(1/x)-1/2*I*b*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/
2))^2)+1/4*b*c*d*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules

used = {5348, 14, 4816, 12, 6874, 327, 222, 2363, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = -\frac{d(a+b\sec^{-1}(cx))}{2x^2} - e\log\left(\frac{1}{x}\right)(a+b\sec^{-1}(cx)) + \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{1}{2}ibe\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right) - \frac{1}{2}ibe\csc^{-1}(cx)^2 + be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - be\log\left(\frac{1}{x}\right)\csc^{-1}(cx)$$

```
[In] Int[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^3,x]
```

```
[Out] (b*c*d*Sqrt[1 - 1/(c^2*x^2)])/(4*x) - (b*c^2*d*ArcCsc[c*x])/4 - (I/2)*b*e*A
rcCsc[c*x]^2 - (d*(a + b*ArcSec[c*x]))/(2*x^2) + b*e*ArcCsc[c*x]*Log[1 - E^
((2*I)*ArcCsc[c*x])] - b*e*ArcCsc[c*x]*Log[x^(-1)] - e*(a + b*ArcSec[c*x])*
Log[x^(-1)] - (I/2)*b*e*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4816

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCos[c*x], u, x] + Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
```

```
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d(a+b\sec^{-1}(cx))}{2x^2} - e\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{b\text{Subst}\left(\int \frac{dx^2+2e\log(x)}{2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{d(a+b\sec^{-1}(cx))}{2x^2} - e\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{b\text{Subst}\left(\int \frac{dx^2+2e\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{d(a+b\sec^{-1}(cx))}{2x^2} - e\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{b\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{1-\frac{x^2}{c^2}}} + \frac{2e\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\right)dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{d(a+b\sec^{-1}(cx))}{2x^2} - e\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{(bd)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c} - \frac{(be)\text{Subst}\left(\int \frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{d(a+b\sec^{-1}(cx))}{2x^2} - be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \frac{1}{4}(bcd)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right) + (be)\text{Subst}\left(\int \frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{d(a+b\sec^{-1}(cx))}{2x^2} - be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + (be)\text{Subst}\left(\int x\cot(x)\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\sec^{-1}(cx))}{2x^2} - be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - (2ibe)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\sec^{-1}(cx))}{2x^2} + be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - (be)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\sec^{-1}(cx))}{2x^2} + be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{1}{2}(ibe)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\sec^{-1}(cx))}{2x^2} + be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - \frac{1}{2}ibe\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int\frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^3}\,dx = -\frac{ad}{2x^2} + \frac{bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x} - \frac{bd\sec^{-1}(cx)}{2x^2} + \frac{1}{2}ibe\sec^{-1}(cx)^2 - \frac{1}{4}bc^2d\arcsin\left(\frac{1}{cx}\right) - be\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right) + ae\log(x) + \frac{1}{2}ibe\,\text{PolyLog}\left(2, -e^{2i\sec^{-1}(cx)}\right)$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSec[c*x]))/x^3,x]
[Out] -1/2*(a*d)/x^2 + (b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*d*ArcSec
[c*x])/(2*x^2) + (I/2)*b*e*ArcSec[c*x]^2 - (b*c^2*d*ArcSin[1/(c*x)])/4 - b*
```

```
e*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] + a*e*Log[x] + (I/2)*b*e*PolyL
og[2, -E^((2*I)*ArcSec[c*x])]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

method	result
parts	$-\frac{ad}{2x^2}+ae\ln(x)+\frac{ib\,\mathrm{arcsec}(cx)^2e}{2}+\frac{b\,c^2d\,\mathrm{arcsec}(cx)}{4}+\frac{bcd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x}-\frac{bd\,\mathrm{arcsec}(cx)}{2x^2}-be\,\mathrm{arcsec}(cx)$
derivativedivides	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{ibe\,\mathrm{arcsec}(cx)^2}{2c^2}+\frac{bd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{bd\,\mathrm{arcsec}(cx)}{4}-\frac{b\,\mathrm{arcsec}(cx)d}{2c^2x^2}-\frac{be\,\mathrm{arcsec}(cx)\ln(}{}\right.$
default	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{ibe\,\mathrm{arcsec}(cx)^2}{2c^2}+\frac{bd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{bd\,\mathrm{arcsec}(cx)}{4}-\frac{b\,\mathrm{arcsec}(cx)d}{2c^2x^2}-\frac{be\,\mathrm{arcsec}(cx)\ln(}{}\right.$

```
[In] int((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*d/x^2+a*e*ln(x)+1/2*I*b*arcsec(c*x)^2*e+1/4*b*c^2*d*arcsec(c*x)+1/4*
b*c*d/x*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/2*b*d/x^2*arcsec(c*x)-b*e*arcsec(c*x)
*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+1/2*I*b*e*polylog(2,-(1/c/x+I*(1-1/c
^2/x^2)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)}{x^3}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*asec(c*x))/x**3,x)
```

```
[Out] Integral((a + b*asec(c*x))*(d + e*x**2)/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -(c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) -
arctan(sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b*e - 1/4*b*d*((c^4*x*sqrt(-1/(
c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(c^2
*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) + a*e*log(x) - 1/2*a*d/x^2
```

Giac [F]

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsec(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)*(a + b*acos(1/(c*x))))/x^3, x)
```

3.81 $\int x^2(d+ex^2)^2 (a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 252

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}-\frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}-\frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)-\frac{b(280c^4d^2+252c^2de+75e^2)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{1680c^6\sqrt{c^2x^2}}$$

```
[Out] 1/3*d^2*x^3*(a+b*arcsec(c*x))+2/5*d*e*x^5*(a+b*arcsec(c*x))+1/7*e^2*x^7*(a+
b*arcsec(c*x))-1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*x*arctanh(c*x/(c^2
*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)-1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2
)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)-1/840*b*e*(84*c^2*d+25*e)*x^4*(
c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)-1/42*b*e^2*x^6*(c^2*x^2-1)^(1/2)/c/(c^
2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {276, 5346, 12, 1281, 470, 327, 223, 212}

$$\int x^2 (d+ex^2)^2 (a+b\sec^{-1}(cx))\ dx = \frac{1}{3}d^2x^3(a+b\sec^{-1}(cx))+\frac{2}{5}dex^5(a+b\sec^{-1}(cx)) + \frac{1}{7}e^2x^7(a+b\sec^{-1}(cx)) - \frac{bx\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(280c^4d^2+252c^2de+75e^2)}{1680c^6\sqrt{c^2x^2}} - \frac{be^2x^6\sqrt{c^2x^2-1}}{42c\sqrt{c^2x^2}} - \frac{bex^4\sqrt{c^2x^2-1}(84c^2d+25e)}{840c^3\sqrt{c^2x^2}} - \frac{bx^2\sqrt{c^2x^2-1}(280c^4d^2+252c^2de+75e^2)}{1680c^5\sqrt{c^2x^2}}$$

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/1680*(b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x^2*Sqrt[-1 + c^2*x^2])/(c^
5*Sqrt[c^2*x^2]) - (b*e*(84*c^2*d + 25*e)*x^4*Sqrt[-1 + c^2*x^2])/(840*c^3*
Sqrt[c^2*x^2]) - (b*e^2*x^6*Sqrt[-1 + c^2*x^2])/(42*c*Sqrt[c^2*x^2]) + (d^2
*x^3*(a + b*ArcSec[c*x]))/3 + (2*d*e*x^5*(a + b*ArcSec[c*x]))/5 + (e^2*x^7*
(a + b*ArcSec[c*x]))/7 - (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x*ArcTanh[
(c*x)/Sqrt[-1 + c^2*x^2]])/(1680*c^6*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)\\&\quad+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2\left(35d^2+42dex^2+15e^2x^4\right)}{105\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\end{aligned}$$

$$= \frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2\left(35d^2+42dex^2+15e^2x^4\right)}{\sqrt{-1+c^2x^2}}\,dx}{105\sqrt{c^2x^2}}$$

$$= -\frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)-\frac{(bx)\int\frac{x^2\left(210c^2d^2+3e\left(84c^2d+25e\right)x^2\right)}{\sqrt{-1+c^2x^2}}\,dx}{630c\sqrt{c^2x^2}}$$

$$= -\frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}-\frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right) - -\frac{(b(-840c^4d^2-9e(84c^2d+25e))\,x)\int\frac{x^2}{\sqrt{-1+c^2x^2}}\,dx}{2520c^3\sqrt{c^2x^2}}$$

$$= -\frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}-\frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)- -\frac{(b(-840c^4d^2-9e(84c^2d+25e))\,x)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{5040c^5\sqrt{c^2x^2}}$$

$$= -\frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}-\frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right) + \frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)- -\frac{(b(-840c^4d^2-9e(84c^2d+25e))\,x)\,\mathrm{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{5040c^5\sqrt{c^2x^2}}$$

$$= -\frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}-\frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} - \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\sec^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\sec^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\sec^{-1}(cx)\right)-\frac{b(280c^4d^2+252c^2de+75e^2)\,x\,\mathrm{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{1680c^6\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.74

$$\int x^2\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$=\frac{c^2x^2\left(16ac^5x(35d^2+42dex^2+15e^2x^4)-b\sqrt{1-\frac{1}{c^2x^2}}(75e^2+2c^2e(126d+25ex^2)+8c^4(35d^2+21dex^2+5e\ldots\right.}{1\ldots}$$

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] (c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - b*Sqrt[1 - 1/(c^2
*x^2)]*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2 +
5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcSec[c*x]
- b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x]
)/(1680*c^7)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(222) = 444.

Time = 0.77 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.82

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}x^3d^2\right)+\frac{b\,\mathrm{arcsec}(cx)e^2x^7}{7}+\frac{2b\,\mathrm{arcsec}(cx)de\,x^5}{5}+\frac{b\,\mathrm{arcsec}(cx)d^2x^3}{3}-\frac{b(c^2x^2-1)x^4\ldots}{42c^3\sqrt{\frac{c^2x^2-\ldots}{c^2x^2}}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arcsec}(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\,\mathrm{arcsec}(cx)de\,x^5}{5}+\frac{b\,c^3\,\mathrm{arcsec}(cx)e^2x^7}{7}-\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right)\ldots}{10\sqrt{\frac{c^2x^2-\ldots}{c^2x^2}}}}{\ldots}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arcsec}(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\,\mathrm{arcsec}(cx)de\,x^5}{5}+\frac{b\,c^3\,\mathrm{arcsec}(cx)e^2x^7}{7}-\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\left(c^2x^2-1\right)\ldots}{10\sqrt{\frac{c^2x^2-\ldots}{c^2x^2}}}}{\ldots}$

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*x^3*d^2)+1/7*b*arcsec(c*x)*e^2*x^7+2/5*b*arc
sec(c*x)*d*e*x^5+1/3*b*arcsec(c*x)*d^2*x^3-1/42*b/c^3*(c^2*x^2-1)/((c^2*x^2
-1)/c^2/x^2)^(1/2)*x^4*e^2-1/10*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/
2)*x^2*d*e-5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e^2-1/6*
b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2-3/20*b/c^5*(c^2*x^2-1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)*d*e-1/6*b/c^4*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/
x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-5/112*b/c^7*(c^2*x^2-1)/((c^2*x^
2-1)/c^2/x^2)^(1/2)*e^2-3/20*b/c^6*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-5/112*b/c^8*(c^2*x^2-1)^(1/2)/((c^2*x
^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.08

$$\int x^2(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^7e^2x^7+672\,ac^7dex^5+560\,ac^7d^2x^3+16\left(15\,bc^7e^2x^7+42\,bc^7dex^5+35\,bc^7d^2x^3-35\,bc^7d^2-42\,bc^7de\right.}{}$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 + 16*(15*
b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b*c
^7*d*e - 15*b*c^7*e^2)*arcsec(c*x) + 32*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b
*c^7*e^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (280*b*c^4*d^2 + 252*b*c^2*d*e
 + 75*b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - (40*b*c^5*e^2*x^5 + 2*(84*b*c^
5*d*e + 25*b*c^3*e^2)*x^3 + (280*b*c^5*d^2 + 252*b*c^3*d*e + 75*b*c*e^2)*x)
*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.15

$$\int x^2(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{ad^2x^3}{3}+\frac{2adex^5}{5}+\frac{ae^2x^7}{7}+\frac{bd^2x^3\operatorname{asec}(cx)}{3}+\frac{2bdex^5\operatorname{asec}(cx)}{5}+\frac{be^2x^7\operatorname{asec}(cx)}{7}$$
$$-\frac{bd^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$
$$-\frac{2bde\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$
$$-\frac{be^2\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*asec(c*x)),x)
```

```
[Out] a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*asec(c*x)/3 +
2*b*d*e*x**5*asec(c*x)/5 + b*e**2*x**7*asec(c*x)/7 - b*d**2*Piecewise((x*sq
```

```
rt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x
**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)
/(2*c**2), True))/(3*c) - 2*b*d*e*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1))
 + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*ac
osh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
- 3*I*asin(c*x)/(8*c**4), True))/(5*c) - b*e**2*Piecewise((c*x**7/(6*sqrt(c
**2*x**2 - 1)) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**
2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6),
Abs(c**2*x**2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqr
t(-c**2*x**2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c*
*5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.61

$$\int x^2 \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{7}\,ae^2x^7 + \frac{2}{5}\,adex^5 + \frac{1}{3}\,ad^2x^3$$
$$+\frac{1}{12}\left(4\,x^3\,\mathrm{arcsec}\left(cx\right) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2$$
$$+\frac{1}{40}\left(16\,x^5\,\mathrm{arcsec}\left(cx\right) + \frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4} - \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bde$$
$$+\frac{1}{672}\left(96\,x^7\,\mathrm{arcsec}\left(cx\right) - \frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6} + \frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6} - \frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}\cdots}\right.}{c^6}}{c}\right.$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arcsec(c*x) - (
2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40*(1
6*x^5*arcsec(c*x) + (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) +
1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(
```

```
-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d*
e + 1/672*(96*x^7*arcsec(c*x) - (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c
^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) - 1)^3 +
3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*log(sqrt(-1
/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4760 vs. 2(222) = 444.

Time = 6.39 (sec) , antiderivative size = 4760, normalized size of antiderivative = 18.89

$$\int x^2(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/1680*(560*b*c^4*d^2*arccos(1/(c*x)) - 280*b*c^4*d^2*log(abs(sqrt(-1/(c^2*
x^2) + 1) + 1/(c*x) + 1)) + 280*b*c^4*d^2*log(abs(sqrt(-1/(c^2*x^2) + 1) -
1/(c*x) - 1)) + 560*a*c^4*d^2 + 560*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c
*x))/(1/(c*x) + 1)^2 - 1960*b*c^4*d^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^
2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^2 + 1960*b*c^4*d^2*(1/(c^2*x^2) -
 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^2 - 560*b*
c^4*d^2*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1) + 560*a*c^4*d^2*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 672*b*c^2*d*e*arccos(1/(c*x)) - 1680*b*c^4*d^2*(1/(c^
2*x^2) - 1)^2*arccos(1/(c*x))/(1/(c*x) + 1)^4 - 252*b*c^2*d*e*log(abs(sqrt(
-1/(c^2*x^2) + 1) + 1/(c*x) + 1)) - 5880*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*log(
abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^4 + 252*b*c^2*d*e*
log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1)) + 5880*b*c^4*d^2*(1/(c^2*x^2
) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^4 + 2
240*b*c^4*d^2*(-1/(c^2*x^2) + 1)^(3/2)/(1/(c*x) + 1)^3 + 672*a*c^2*d*e - 16
80*a*c^4*d^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 - 2016*b*c^2*d*e*(1/(c^2*x
^2) - 1)*arccos(1/(c*x))/(1/(c*x) + 1)^2 - 1680*b*c^4*d^2*(1/(c^2*x^2) - 1)
^3*arccos(1/(c*x))/(1/(c*x) + 1)^6 - 1764*b*c^2*d*e*(1/(c^2*x^2) - 1)*log(a
bs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^2 - 9800*b*c^4*d^2*
(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x)
 + 1)^6 + 1764*b*c^2*d*e*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) -
 1/(c*x) - 1))/(1/(c*x) + 1)^2 + 9800*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*log(abs
(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^6 - 840*b*c^2*d*e*sqr
t(-1/(c^2*x^2) + 1)/(1/(c*x) + 1) - 2800*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*sqrt
(-1/(c^2*x^2) + 1)/(1/(c*x) + 1)^5 - 2016*a*c^2*d*e*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 - 1680*a*c^4*d^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 240*b*e^2
*arccos(1/(c*x)) + 672*b*c^2*d*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/(1/(c*
x) + 1)^4 + 1680*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/(1/(c*x) + 1
)^8 - 75*b*e^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1)) - 5292*b*c^2*
d*e*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(
```

c*x) + 1)^4 - 9800*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^8 + 75*b*e^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1)) + 5292*b*c^2*d*e*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^4 + 9800*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^8 + 2016*b*c^2*d*e*(-1/(c^2*x^2) + 1)^(3/2)/(1/(c*x) + 1)^3 + 240*a*e^2 + 672*a*c^2*d*e*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 1680*a*c^4*d^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 - 1680*b*e^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/(1/(c*x) + 1)^2 + 3360*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/(1/(c*x) + 1)^6 + 1680*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/(1/(c*x) + 1)^10 - 525*b*e^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^2 - 8820*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^6 - 5880*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^10 + 525*b*e^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^2 + 8820*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^6 + 5880*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^10 - 330*b*e^2*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1) - 1512*b*c^2*d*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1)^5 + 2800*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1)^9 - 1680*a*e^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 3360*a*c^2*d*e*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 1680*a*c^4*d^2*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 5040*b*e^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/(1/(c*x) + 1)^4 - 3360*b*c^2*d*e*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/(1/(c*x) + 1)^8 - 560*b*c^4*d^2*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/(1/(c*x) + 1)^12 - 1575*b*e^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^4 - 8820*b*c^2*d*e*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^8 - 1960*b*c^4*d^2*(1/(c^2*x^2) - 1)^6*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^12 + 1575*b*e^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^4 + 8820*b*c^2*d*e*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^8 + 1960*b*c^4*d^2*(1/(c^2*x^2) - 1)^6*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^12 + 2240*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1)^11 + 280*b*e^2*(-1/(c^2*x^2) + 1)^(3/2)/(1/(c*x) + 1)^3 + 5040*a*e^2*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 - 3360*a*c^2*d*e*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 - 560*a*c^4*d^2*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 - 8400*b*e^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/(1/(c*x) + 1)^6 - 672*b*c^2*d*e*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/(1/(c*x) + 1)^10 - 560*b*c^4*d^2*(1/(c^2*x^2) - 1)^7*arccos(1/(c*x))/(1/(c*x) + 1)^14 - 2625*b*e^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^6 - 5292*b*c^2*d*e*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^10 - 280*b*c^4*d^2*(1/(c^2*x^2) - 1)^7*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^14 + 2625*b*e^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^6 + 5292*

b*c^2*d*e*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1)
)/(1/(c*x) + 1)^10 + 280*b*c^4*d^2*(1/(c^2*x^2) - 1)^7*log(abs(sqrt(-1/(c^2
*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^14 - 850*b*e^2*(1/(c^2*x^2) - 1)^2
*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1)^5 + 1512*b*c^2*d*e*(1/(c^2*x^2) - 1)^
4*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1)^9 + 560*b*c^4*d^2*(1/(c^2*x^2) - 1)^
6*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1)^13 - 8400*a*e^2*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6 - 672*a*c^2*d*e*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 - 560*
a*c^4*d^2*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + 8400*b*e^2*(1/(c^2*x^2) -
1)^4*arccos(1/(c*x))/(1/(c*x) + 1)^8 + 2016*b*c^2*d*e*(1/(c^2*x^2) - 1)^6*a
rccos(1/(c*x))/(1/(c*x) + 1)^12 - 2625*b*e^2*(1/(c^2*x^2) - 1)^4*log(abs(sq
rt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^8 - 1764*b*c^2*d*e*(1/(c
^2*x^2) - 1)^6*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)
^12 + 2625*b*e^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*
x) - 1))/(1/(c*x) + 1)^8 + 1764*b*c^2*d*e*(1/(c^2*x^2) - 1)^6*log(abs(sqrt(
-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^12 + 2016*b*c^2*d*e*(1/(c^2
*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1)^11 + 8400*a*e^2*(1/(c^2*x
^2) - 1)^4/(1/(c*x) + 1)^8 + 2016*a*c^2*d*e*(1/(c^2*x^2) - 1)^6/(1/(c*x) +
1)^12 - 5040*b*e^2*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/(1/(c*x) + 1)^10 - 6
72*b*c^2*d*e*(1/(c^2*x^2) - 1)^7*arccos(1/(c*x))/(1/(c*x) + 1)^14 - 1575*b*
e^2*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(
c*x) + 1)^10 - 252*b*c^2*d*e*(1/(c^2*x^2) - 1)^7*log(abs(sqrt(-1/(c^2*x^2)
+ 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^14 + 1575*b*e^2*(1/(c^2*x^2) - 1)^5*log(
abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^10 + 252*b*c^2*d*e
*(1/(c^2*x^2) - 1)^7*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x
) + 1)^14 + 850*b*e^2*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) +
 1)^9 + 840*b*c^2*d*e*(1/(c^2*x^2) - 1)^6*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) +
 1)^13 - 5040*a*e^2*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 - 672*a*c^2*d*e*(1
/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + 1680*b*e^2*(1/(c^2*x^2) - 1)^6*arccos(
1/(c*x))/(1/(c*x) + 1)^12 - 525*b*e^2*(1/(c^2*x^2) - 1)^6*log(abs(sqrt(-1/(
c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^12 + 525*b*e^2*(1/(c^2*x^2) - 1
)^6*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^12 + 280*b
e^2(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1)^11 + 1680*a*e
^2*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 - 240*b*e^2*(1/(c^2*x^2) - 1)^7*arc
cos(1/(c*x))/(1/(c*x) + 1)^14 - 75*b*e^2*(1/(c^2*x^2) - 1)^7*log(abs(sqrt(-
1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(1/(c*x) + 1)^14 + 75*b*e^2*(1/(c^2*x^2) -
 1)^7*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(1/(c*x) + 1)^14 + 330
*b*e^2*(1/(c^2*x^2) - 1)^6*sqrt(-1/(c^2*x^2) + 1)/(1/(c*x) + 1)^13 - 240*a*
e^2*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)*c/(c^8 + 7*c^8*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 + 21*c^8*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 35*c^8*(1/(c
^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 35*c^8*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8
 + 21*c^8*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 7*c^8*(1/(c^2*x^2) - 1)^6/
(1/(c*x) + 1)^12 + c^8*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14)

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2\,{\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

3.82 $\int (d+ex^2)^2 (a+b\sec^{-1}(cx))\, dx$

Optimal result

Integrand size = 18, antiderivative size = 191

$$\begin{aligned}\int (d+ex^2)^2 (a+b\sec^{-1}(cx))\, dx = &-\frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} \\ &-\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\big(a+b\sec^{-1}(cx)\big) \\ &+\frac{2}{3}dex^3\big(a+b\sec^{-1}(cx)\big)+\frac{1}{5}e^2x^5\big(a+b\sec^{-1}(cx)\big) \\ &-\frac{b(120c^4d^2+40c^2de+9e^2)\,x\,\text{arctanh}\Big(\frac{cx}{\sqrt{-1+c^2x^2}}\Big)}{120c^4\sqrt{c^2x^2}}\end{aligned}$$

```
[Out] d^2*x*(a+b*arcsec(c*x))+2/3*d*e*x^3*(a+b*arcsec(c*x))+1/5*e^2*x^5*(a+b*arcs
ec(c*x))-1/120*b*(120*c^4*d^2+40*c^2*d*e+9*e^2)*x*arctanh(c*x/(c^2*x^2-1)^(
1/2))/c^4/(c^2*x^2)^(1/2)-1/120*b*e*(40*c^2*d+9*e)*x^2*(c^2*x^2-1)^(1/2)/c^
3/(c^2*x^2)^(1/2)-1/20*b*e^2*x^4*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used

$= \{200, 5336, 12, 1173, 396, 223, 212\}$

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right) - \frac{bx\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(120c^4d^2+40c^2de+9e^2\right)}{120c^4\sqrt{c^2x^2}} - \frac{be^2x^4\sqrt{c^2x^2-1}}{20c\sqrt{c^2x^2}} - \frac{bex^2\sqrt{c^2x^2-1}(40c^2d+9e)}{120c^3\sqrt{c^2x^2}}$$

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/120*(b*e*(40*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(c^3*Sqrt[c^2*x^2]) -
(b*e^2*x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + d^2*x*(a + b*ArcSec[c
*x]) + (2*d*e*x^3*(a + b*ArcSec[c*x]))/3 + (e^2*x^5*(a + b*ArcSec[c*x]))/5
- (b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]]
)/(120*c^4*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 5336

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x]
- Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)\\
&\quad+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{15d^2+10dex^2+3e^2x^4}{15\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)\\
&\quad+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{15d^2+10dex^2+3e^2x^4}{\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}}\\
&= -\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)\\
&\quad+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)-\frac{(bx)\int\frac{60c^2d^2+e(40c^2d+9e)x^2}{\sqrt{-1+c^2x^2}}\,dx}{60c\sqrt{c^2x^2}}\\
&= -\frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}-\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}\\
&\quad+d^2x\left(a+b\sec^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right)\\
&\quad+\frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right)+\frac{(b(-120c^4d^2-e(40c^2d+9e))\,x)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{120c^3\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} - \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x\left(a+b\sec^{-1}(cx)\right) + \frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right) + \frac{\left(b(-120c^4d^2-e(40c^2d+9e))\,x\right)\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{120c^3\sqrt{c^2x^2}}$$

$$= -\frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} - \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x\left(a+b\sec^{-1}(cx)\right) + \frac{2}{3}dex^3\left(a+b\sec^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\sec^{-1}(cx)\right) - \frac{b(120c^4d^2+40c^2de+9e^2)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int \left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{c^2x\left(8ac^3(15d^2+10dex^2+3e^2x^4)-be\sqrt{1-\frac{1}{c^2x^2}}x(9e+c^2(40d+6ex^2))\right)+8bc^5x(15d^2+10dex^2+3e^2x^4}{120c^5}$$

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] (c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*e*Sqrt[1 - 1/(c^2*x^2
)]*x*(9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2
*x^4)*ArcSec[c*x] - b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt[1 -
1/(c^2*x^2)])*x])/(120*c^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(169) = 338.

Time = 0.42 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.78

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\,\text{arcsec}(cx)e^2x^5}{5}+\frac{2b\,\text{arcsec}(cx)de\,x^3}{3}+b\,\text{arcsec}\,(cx)\,d^2x-\frac{b(c^2x^2-1}{20c^3\sqrt{\frac{c^2}{}}}$
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\text{arcsec}(cx)d^2cx+\frac{2bc\,\text{arcsec}(cx)de\,x^3}{3}+\frac{bc\,\text{arcsec}(cx)e^2x^5}{5}-\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\text{arcsec}(cx)d^2cx+\frac{2bc\,\text{arcsec}(cx)de\,x^3}{3}+\frac{bc\,\text{arcsec}(cx)e^2x^5}{5}-\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c}$

```
[In] int((e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+1/5*b*arcsec(c*x)*e^2*x^5+2/3*b*arcsec(c*
x)*d*e*x^3+b*arcsec(c*x)*d^2*x-1/20*b/c^3*(c^2*x^2-1)*x^2/((c^2*x^2-1)/c^2/
x^2)^(1/2)*e^2-1/3*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e-b/c^2*
(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2
))-3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2-1/3*b/c^4*(c^2*x^
2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-3/40
*b/c^6*(c^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*ln(c*x+(c^2*x^2-
1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.24

$$\int \left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{24\,ac^5e^2x^5+80\,ac^5dex^3+120\,ac^5d^2x+8\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x-15\,bc^5d^2-10\,bc^5de-3\,bc\right.}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(3*b*c^5*e
^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e -
3*b*c^5*e^2)*arcsec(c*x) + 16*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*a
rctan(-c*x + sqrt(c^2*x^2 - 1)) + (120*b*c^4*d^2 + 40*b*c^2*d*e + 9*b*e^2)*
log(-c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^3*e^2*x^3 + (40*b*c^3*d*e + 9*b*c*e^
2)*x)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.86

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\,\mathrm{asec}\,(cx) + \frac{2bdex^3\,\mathrm{asec}\,(cx)}{3} + \frac{be^2x^5\,\mathrm{asec}\,(cx)}{5} - \frac{bd^2\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c}$$
$$- \frac{2bde\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$
$$- \frac{be^2\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\mathrm{acosh}\,(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\mathrm{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

```
[In] integrate((e*x**2+d)**2*(a+b*asec(c*x)),x)
```

```
[Out] a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*asec(c*x) + 2*b*d*e*x*
*3*asec(c*x)/3 + b*e**2*x**5*asec(c*x)/5 - b*d**2*Piecewise((acosh(c*x), Ab
s(c**2*x**2) > 1), (-I*asin(c*x), True))/c - 2*b*d*e*Piecewise((x*sqrt(c**2
*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*
sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**
2), True))/(3*c) - b*e**2*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/
(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)
/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**
3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*as
in(c*x)/(8*c**4), True))/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.55

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{5}\,ae^2x^5 + \frac{2}{3}\,adex^3 + \frac{1}{6}\left(4\,x^3\,\mathrm{arcsec}\,(cx) - \frac{\frac{2\,\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde + \frac{1}{80}\left(16\,x^5\,\mathrm{arcsec}\,(cx) + \frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\,\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4} - \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be^2 + ad^2x + \frac{\left(2\,cx\,\mathrm{arcsec}\,(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2\,c}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/6*(4*x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^
2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2
 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/80*(16*x^5*arcsec(c*x)
 + (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*
x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1)
 + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2 + a*d^2*x + 1/2
*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-sqrt(-1/(c^2*x
^2) + 1) + 1))*b*d^2/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14166 vs. 2(169) = 338.

Time = 4.60 (sec) , antiderivative size = 14166, normalized size of antiderivative = 74.17

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/120*(120*b*c^4*d^2*arccos(1/(c*x))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2)
```

```
 - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 120*b*c^4*d^2*log(abs(sqrt(-1/(c^2*x
^2) + 1) + 1/(c*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 1
0*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2)
 - 1)^5/(1/(c*x) + 1)^10) + 120*b*c^4*d^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1
/(c*x) - 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c
*x) + 1)^10) + 120*a*c^4*d^2/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x
^2) - 1)^5/(1/(c*x) + 1)^10) + 360*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*
x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*
(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)
^10)*(1/(c*x) + 1)^2) - 600*b*c^4*d^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^
2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x
^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) + 600*b*c^4*d^2*(1/(c^2*x^2)
 - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*
c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) + 360
*a*c^4*d^2*(1/(c^2*x^2) - 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*
x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) + 80*b*c^2*d*e*arccos(1/(c*x
))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^1
0) + 240*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) -
40*b*c^2*d*e*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^6 + 5*c^6*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^
4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/
(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 1200*b*c^4*d^
2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6
 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(
c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c
*x) + 1)^4) + 40*b*c^2*d*e*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(
```

c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/
(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^
2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) +
 1200*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x
) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^
2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*
c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x)
+ 1)^10)*(1/(c*x) + 1)^4) + 80*a*c^2*d*e/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*
x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c
^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) + 240*a*c^4*d^2*(1/(c^2*x^2) - 1)^
2/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1
/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^1
0)*(1/(c*x) + 1)^4) - 80*b*c^2*d*e*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^6
+ 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(
c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c
*x) + 1)^2) - 240*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^6 + 5*c
^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) +
 1)^6) - 200*b*c^2*d*e*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1
/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(
c*x) + 1)^10)*(1/(c*x) + 1)^2) - 1200*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*log(abs
(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*
x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c
^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) + 200*b*c^2*d*e*(
1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c
^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) -
1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) +
 1)^2) + 1200*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1)
- 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1
/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(
1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) - 80*b*c^2*d*e*sqrt(-1/(c^2*x^2) + 1)/((c
^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2
*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1
/(c*x) + 1)) - 80*a*c^2*d*e*(1/(c^2*x^2) - 1)/((c^6 + 5*c^6*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1

/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 240*a*c^4
d^2(1/(c^2*x^2) - 1)^3/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2)
 - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) + 24*b*e^2*arccos(1/(c*x))/(c^6
+ 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(
c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 160
*b*c^2*d*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1
)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) - 360*b*c^
4*d^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(
1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) - 9*b*e^2*log(a
bs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 400*b*c^2*d*e*(1/(c^2*x^2) - 1)
^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2
) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^
6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x)
+ 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) - 600*b
*c^4*d^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))
/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/
(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
)*(1/(c*x) + 1)^8) + 9*b*e^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))
/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(
c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)
 + 400*b*c^2*d*e*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*
x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5
c^6(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x)
 + 1)^10)*(1/(c*x) + 1)^4) + 600*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt
(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1
/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) + 160*b*c^2*d*e*(-1/(c
^2*x^2) + 1)^(3/2)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^

5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^3) + 24*a*e^2/(c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10) - 160*a*c^2*d*e*(1/(c^2*x^2) - 1)^2/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) - 360*a*c^4*d^2*(1/(c^2*x^2) - 1)^4/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) - 120*b*e^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) + 160*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) - 120*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) - 45*b*e^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) - 400*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) - 120*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) + 45*b*e^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^2) + 400*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) + 120*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*log(abs(sqr

t(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) - 30*b*e^2*sqrt(-1/(
c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^
6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/
(c*x) + 1)^10)*(1/(c*x) + 1)) - 120*a*e^2*(1/(c^2*x^2) - 1)/((c^6 + 5*c^6*(
1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)
^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^
2) + 160*a*c^2*d*e*(1/(c^2*x^2) - 1)^3/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(
c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^
6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) - 120*a*c^4*d^2*(1
/(c^2*x^2) - 1)^5/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*
(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5
/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) + 240*b*e^2*(1/(c^2*x^2) - 1)^2*arccos
(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x
) + 1)^10)*(1/(c*x) + 1)^4) + 80*b*c^2*d*e*(1/(c^2*x^2) - 1)^4*arccos(1/(c*
x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) -
 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*
(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)
^10)*(1/(c*x) + 1)^8) - 90*b*e^2*(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x
^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2)
 - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) - 200*b*c^2*d*e*(1/(c^2*x^2) - 1
)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^
2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c
^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x)
 + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) + 90*b
e^2(1/(c^2*x^2) - 1)^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c
^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2
*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1
/(c*x) + 1)^4) + 200*b*c^2*d*e*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2
) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10
c^6(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c
*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) -
 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) + 160*b*c^2*d*e*(1/(c^2*x^2) - 1)^
3*sqrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +

10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^7) + 12*b*e^2*(-1/(c^2*x^2) + 1)^(3/2)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^3) + 240*a*e^2*(1/(c^2*x^2) - 1)^2/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^4) + 80*a*c^2*d*e*(1/(c^2*x^2) - 1)^4/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) - 240*b*e^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) - 80*b*c^2*d*e*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) - 90*b*e^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) - 40*b*c^2*d*e*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) + 90*b*e^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^6) + 40*b*c^2*d*e*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) + 80*b*c^2*d*e*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^9) - 240*a*e^2*(1/(c^2*x^2) - 1)^3/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6

+ 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(
c*x) + 1)^10)*(1/(c*x) + 1)^6) - 80*a*c^2*d*e*(1/(c^2*x^2) - 1)^5/((c^6 + 5
c^6(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x)
+ 1)^10) + 120*b*e^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^6 + 5*c^6*(1/
(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
+ 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(
1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8)
- 45*b*e^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) +
1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) -
1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*
(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)
^10)*(1/(c*x) + 1)^8) + 45*b*e^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x
^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2)
- 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^8) + 12*b*e^2*(1/(c^2*x^2) - 1)^3*s
qrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*
c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*
x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^7) + 120*a*e^2*(1/(c^2*x^2) - 1)^4/((c
^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2
*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1
/(c*x) + 1)^8) - 24*b*e^2*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^6 + 5*c^6
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) +
1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1
)^10) - 9*b*e^2*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x
) + 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^
2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*
c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x)
+ 1)^10)*(1/(c*x) + 1)^10) + 9*b*e^2*(1/(c^2*x^2) - 1)^5*log(abs(sqrt(-1/(c
^2*x^2) + 1) - 1/(c*x) - 1))/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
2 + 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*
x^2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^10) + 30*b*e^2*(1/(c^2*x^2) - 1
)^4*sqrt(-1/(c^2*x^2) + 1)/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 10*c^6*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(
1/(c*x) + 1)^6 + 5*c^6*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^
2) - 1)^5/(1/(c*x) + 1)^10)*(1/(c*x) + 1)^9) - 24*a*e^2*(1/(c^2*x^2) - 1)^5
/((c^6 + 5*c^6*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 10*c^6*(1/(c^2*x^2) - 1)
^2/(1/(c*x) + 1)^4 + 10*c^6*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 5*c^6*(1/
(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + c^6*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10

```
)*(1/(c*x) + 1)^10))*c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(e\,x^2+d\right)^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
[Out] int((d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

3.83 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^2}\,dx = \frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{x} + 2dex(a+b\sec^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\sec^{-1}(cx)) - \frac{be(12c^2d+e)\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

```
[Out] -d^2*(a+b*arcsec(c*x))/x+2*d*e*x*(a+b*arcsec(c*x))+1/3*e^2*x^3*(a+b*arcsec(
c*x))-1/6*b*e*(12*c^2*d+e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)^(
1/2)+b*c*d^2*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/6*b*e^2*x^2*(c^2*x^2-1)^(1
/2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{276, 5346, 12, 1279, 396, 223, 212\}$

$$\int \frac{\left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{x} + 2dex\left(a+b\sec^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\sec^{-1}(cx)\right) - \frac{bex\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(12c^2d+e\right)}{6c^2\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}} - \frac{be^2x^2\sqrt{c^2x^2-1}}{6c\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^2,x]
```

```
[Out] (b*c*d^2*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*x^2] - (b*e^2*x^2*Sqrt[-1 + c^2*x^2])
/(6*c*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcSec[c*x]))/x + 2*d*e*x*(a + b*ArcSec[
c*x]) + (e^2*x^3*(a + b*ArcSec[c*x]))/3 - (b*e*(12*c^2*d + e)*x*ArcTanh[(c*
x)/Sqrt[-1 + c^2*x^2]])/(6*c^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\sec^{-1}(cx))}{x}+2dex(a+b\sec^{-1}(cx)) \\
&\quad +\frac{1}{3}e^2x^3(a+b\sec^{-1}(cx))-\frac{(bcx)\int\frac{-3d^2+6dex^2+e^2x^4}{3x^2\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d^2(a+b\sec^{-1}(cx))}{x}+2dex(a+b\sec^{-1}(cx)) \\
&\quad +\frac{1}{3}e^2x^3(a+b\sec^{-1}(cx))-\frac{(bcx)\int\frac{-3d^2+6dex^2+e^2x^4}{x^2\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}} \\
&= \frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{x}+2dex(a+b\sec^{-1}(cx)) \\
&\quad +\frac{1}{3}e^2x^3(a+b\sec^{-1}(cx))-\frac{(bcx)\int\frac{6de+e^2x^2}{\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}} \\
&= \frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}-\frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{x}+2dex(a+b\sec^{-1}(cx)) \\
&\quad +\frac{1}{3}e^2x^3(a+b\sec^{-1}(cx))--\frac{(b(-12c^2de-e^2)\,x)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{6c\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{x} + 2dex(a+b\sec^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\sec^{-1}(cx)) - -\frac{(b(-12c^2de-e^2)x)\,\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, \frac{x}{\sqrt{-1+c^2x^2}}\right)}{6c\sqrt{c^2x^2}}$$

$$= \frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{x} + 2dex(a+b\sec^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\sec^{-1}(cx)) - \frac{be(12c^2d+e)\,x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$
$$= \frac{c^2\left(b\sqrt{1-\frac{1}{c^2x^2}}x(6c^2d^2-e^2x^2)+2ac(-3d^2+6dex^2+e^2x^4)\right)+2bc^3(-3d^2+6dex^2+e^2x^4)\sec^{-1}(cx)-b}{6c^3x}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^2,x]
```

```
[Out] (c^2*(b*Sqrt[1 - 1/(c^2*x^2)]*x*(6*c^2*d^2 - e^2*x^2) + 2*a*c*(-3*d^2 + 6*d
*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcSec[c*x] - b
*e*(12*c^2*d + e)*x*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3*x)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.54

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+\frac{b\,e^2\,\text{arcsec}(cx)x^3}{3}+2be\,\text{arcsec}\,(cx)\,xd-\frac{b\,\text{arcsec}(cx)d^2}{x}-\frac{b(c^2x^2-1)e^2}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\,\text{arcsec}(cx)dex}{c}+\frac{b\,\text{arcsec}(cx)e^2x^3}{3c}-\frac{b\,\text{arcsec}(cx)d^2}{cx}+\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\,\text{arcsec}(cx)dex}{c}+\frac{b\,\text{arcsec}(cx)e^2x^3}{3c}-\frac{b\,\text{arcsec}(cx)d^2}{cx}+\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\right.$

```
[In] int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+1/3*b*e^2*arcsec(c*x)*x^3+2*b*e*arcsec(c*x)*x
*d-b*arcsec(c*x)*d^2/x-1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^
```

```
2+b/c*(c^2*x^2-1)/x^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2-2*b/c^2*(c^2*x^2-1)^(
1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-1/6*b/c^4*
(c^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*e
^2
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2}\,dx = \frac{2\,ac^3e^2x^4+6\,bc^4d^2x+12\,ac^3dex^2-6\,ac^3d^2-4\,(3\,bc^3d^2-6\,bc^3de-bc^3e^2)x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+(\cdots}{\cdots}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^4 + 6*b*c^4*d^2*x + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 4*(
3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) +
 (12*b*c^2*d*e + b*e^2)*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*e^2*x^4
+ 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x
)*arcsec(c*x) + (6*b*c^3*d^2 - b*c*e^2*x^2)*sqrt(c^2*x^2 - 1))/(c^3*x)
```

Sympy [A] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2}\,dx = -\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} + bcd^2\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\operatorname{asec}(cx)}{x} + 2bdex\operatorname{asec}(cx) + \frac{be^2x^3\operatorname{asec}(cx)}{3} - \frac{2bde\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} - \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**2,x)
```

```
[Out] -a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 + b*c*d**2*sqrt(1 - 1/(c**2*x**2)) -
b*d**2*asec(c*x)/x + 2*b*d*e*x*asec(c*x) + b*e**2*x**3*asec(c*x)/3 - 2*b*d*
```

```
e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c - b*e
**2*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*
x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2
+ 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2}\,dx = \frac{1}{3}ae^2x^3 + \left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsec}(cx)}{x}\right)bd^2 + \frac{1}{12}\left(4x^3\operatorname{arcsec}(cx) - \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 + 2\,adex + \frac{\left(2\,cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bde}{c} - \frac{ad^2}{x}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + (c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*d^2 + 1/12*(4*
x^3*arcsec(c*x) - (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) +
 log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)
/c)*b*e^2 + 2*a*d*e*x + (2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1
) + log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d*e/c - a*d^2/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6018 vs. $2(144) = 288$.

Time = 2.47 (sec) , antiderivative size = 6018, normalized size of antiderivative = 37.15

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
```

```
[Out] -1/6*(6*b*c^4*d^2*arccos(1/(c*x))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) +
 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4
```

/(1/(c*x) + 1)^8) + 6*a*c^4*d^2/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(
1/(c*x) + 1)^8) + 24*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^4 + 2*
c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 12*b*c
^4*d^2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8)*(1/(c*x) + 1)) + 24*a*c^4*d^2*(1/(c^2*x^2) - 1)/((c^4 + 2*c^
4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 12*b*c^2
*d*e*arccos(1/(c*x))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4
*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1
)^8) + 36*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^4 + 2*c^4*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 -
c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 12*b*c^2*d*e*lo
g(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*
x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^2*d*e*log(abs(sqrt(-1/(c^2*x^2) + 1)
- 1/(c*x) - 1))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)
+ 36*b*c^4*d^2*(-1/(c^2*x^2) + 1)^(3/2)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) - 12*a*c^2*d*e/(c^4 + 2*c^4*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 -
c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 36*a*c^4*d^2*(1/(c^2*x^2) - 1)^2
/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^
4) + 24*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^4 + 2*c^4*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^
4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 24*b*c^2*d*e*(1/(
c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 2*c^4*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 24*b*c^2*d
e(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 +
 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*
x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 36*
b*c^4*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 2*c^4*(1/(c^2*
x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4
*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) + 24*a*c^4*d^2*(1/(c
^2*x^2) - 1)^3/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*
(1/(c*x) + 1)^6) - 2*b*e^2*arccos(1/(c*x))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(
1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8) + 24*b*c^2*d*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x
))/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1

)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1
)^4) + 6*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^4 + 2*c^4*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c
^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + b*e^2*log(abs(sq
rt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1
)^4/(1/(c*x) + 1)^8) - b*e^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))
/(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 12*b*c^4*d^2*
(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x
^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) - 2*a*e^2/(c^4 + 2*c^4*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^
4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 24*a*c^2*d*e*(1/(c^2*x^2) - 1)^2/(
(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4)
 + 6*a*c^4*d^2*(1/(c^2*x^2) - 1)^4/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x)
 + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)
^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 8*b*e^2*(1/(c^2*x^2) - 1)*arccos(1/(
c*x))/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2)
- 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x)
+ 1)^2) + 2*b*e^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x
) + 1))/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x
) + 1)^2) - 24*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1)
 + 1/(c*x) + 1))/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1
/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8
)*(1/(c*x) + 1)^6) - 2*b*e^2*(1/(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) +
1) - 1/(c*x) - 1))/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*
(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8)*(1/(c*x) + 1)^2) + 24*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^
2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8)*(1/(c*x) + 1)^6) + 2*b*e^2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 2*c^
4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 8*a*e^2*(1
/(c^2*x^2) - 1)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)
*(1/(c*x) + 1)^2) - 12*b*e^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^4 + 2*
c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 12*b*c
^2*d*e*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x
^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 12*b*c^2*d*e*(1/(c^2*x^2) -
1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 2*c^4*(1/(c^2*x

```
^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*
(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 12*b*c^2*d*e*(1/(c^
2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 2*c^4*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 2*b*e^2*(-
1/(c^2*x^2) + 1)^(3/2)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*
c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x)
+ 1)^8)*(1/(c*x) + 1)^3) - 12*a*e^2*(1/(c^2*x^2) - 1)^2/((c^4 + 2*c^4*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 -
c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) - 12*a*c^2*d*e*(1
/(c^2*x^2) - 1)^4/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^
8)*(1/(c*x) + 1)^8) + 8*b*e^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^4 + 2
*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x)
 + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 2*b*e
^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4
 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 2
*b*e^2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(
(c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/
(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6)
 - 2*b*e^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 2*c^4*(1/(c^2
*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^
4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) + 8*a*e^2*(1/(c^2*x
^2) - 1)^3/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*
x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(
c*x) + 1)^6) - 2*b*e^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^4 + 2*c^4*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - b*e^2*(1/(c^
2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^4 + 2*c^4*
(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + b*e^2*(1/(
c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^4 + 2*c^
4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 2*b*e^2*
(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/
(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x
^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) - 2*a*e^2*(1/(c^2*x^2) - 1)^4/
((c^4 + 2*c^4*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 - 2*c^4*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 - c^4*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8
))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^2, x)
```

3.84 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^4}\,dx = \frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}+\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{3x^3}-\frac{2de(a+b\sec^{-1}(cx))}{x} + e^2x(a+b\sec^{-1}(cx))-\frac{be^2x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

```
[Out] -1/3*d^2*(a+b*arcsec(c*x))/x^3-2*d*e*(a+b*arcsec(c*x))/x+e^2*x*(a+b*arcsec(
c*x))-b*e^2*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/(c^2*x^2)^(1/2)+2/9*b*c*d*(c^2
*d+9*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/9*b*c*d^2*(c^2*x^2-1)^(1/2)/x^2
/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {276, 5346, 12, 1279, 462, 223, 212}

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^4}\,dx = -\frac{d^2(a+b\sec^{-1}(cx))}{3x^3}-\frac{2de(a+b\sec^{-1}(cx))}{x} + e^2x(a+b\sec^{-1}(cx))-\frac{be^2x\,\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}+\frac{2bcd\sqrt{c^2x^2-1}(c^2d+9e)}{9\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^4,x]
```

```
[Out] (2*b*c*d*(c^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/(9*Sqrt[c^2*x^2]) + (b*c*d^2*Sqr
t[-1 + c^2*x^2])/(9*x^2*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcSec[c*x]))/(3*x^3)
- (2*d*e*(a + b*ArcSec[c*x]))/x + e^2*x*(a + b*ArcSec[c*x]) - (b*e^2*x*ArcT
anh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[c^2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\sec^{-1}(cx))}{3x^3}-\frac{2de(a+b\sec^{-1}(cx))}{x}\\
&\quad+e^2x\big(a+b\sec^{-1}(cx)\big)-\frac{(bcx)\int\frac{-d^2-6dex^2+3e^2x^4}{3x^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{d^2(a+b\sec^{-1}(cx))}{3x^3}-\frac{2de(a+b\sec^{-1}(cx))}{x}\\
&\quad+e^2x\big(a+b\sec^{-1}(cx)\big)-\frac{(bcx)\int\frac{-d^2-6dex^2+3e^2x^4}{x^4\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}}\\
&= \frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{3x^3}-\frac{2de(a+b\sec^{-1}(cx))}{x}\\
&\quad+e^2x\big(a+b\sec^{-1}(cx)\big)-\frac{(bcx)\int\frac{-2d(c^2d+9e)+9e^2x^2}{x^2\sqrt{-1+c^2x^2}}\,dx}{9\sqrt{c^2x^2}}\\
&= \frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}+\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{3x^3}\\
&\quad-\frac{2de(a+b\sec^{-1}(cx))}{x}+e^2x\big(a+b\sec^{-1}(cx)\big)-\frac{(bce^2x)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}+\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}\\
&\quad-\frac{d^2(a+b\sec^{-1}(cx))}{3x^3}-\frac{2de(a+b\sec^{-1}(cx))}{x}\\
&\quad+e^2x\big(a+b\sec^{-1}(cx)\big)-\frac{(bce^2x)\,\text{Subst}\Big(\int\frac{1}{1-c^2x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\Big)}{\sqrt{c^2x^2}}\\
&= \frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}+\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{3x^3}\\
&\quad-\frac{2de(a+b\sec^{-1}(cx))}{x}+e^2x\big(a+b\sec^{-1}(cx)\big)-\frac{be^2x\,\text{arctanh}\Big(\frac{cx}{\sqrt{-1+c^2x^2}}\Big)}{\sqrt{c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^4} \, dx$$
$$= \frac{c\left(bcd\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+18ex^2)-3a(d^2+6dex^2-3e^2x^4)\right)-3bc(d^2+6dex^2-3e^2x^4)\sec^{-1}(cx)}{9cx^3}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^4,x]
[Out] (c*(b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 18*e*x^2) - 3*a*(d^2 +
 6*d*e*x^2 - 3*e^2*x^4)) - 3*b*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcSec[c*x]
- 9*b*e^2*x^3*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(9*c*x^3)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.46

method	result
parts	$a\left(e^2x-\frac{2de}{x}-\frac{d^2}{3x^3}\right)+b\ \mathrm{arcsec}\left(cx\right)e^2x-\frac{2b\ \mathrm{arcsec}(cx)de}{x}-\frac{b\ \mathrm{arcsec}(cx)d^2}{3x^3}+\frac{2bc(c^2x^2-1)d^2}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^2}+$
derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\ \mathrm{arcsec}(cx)e^2x}{c^3}-\frac{b\ \mathrm{arcsec}(cx)d^2}{3c^3x^3}-\frac{2b\ \mathrm{arcsec}(cx)de}{c^3x}+\frac{2b(c^2x^2-1)d^2}{9c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\ \mathrm{arcsec}(cx)e^2x}{c^3}-\frac{b\ \mathrm{arcsec}(cx)d^2}{3c^3x^3}-\frac{2b\ \mathrm{arcsec}(cx)de}{c^3x}+\frac{2b(c^2x^2-1)d^2}{9c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\right.$

```
[In] int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x,method=_RETURNVERBOSE)
[Out] a*(e^2*x-2*d*e/x-1/3*d^2/x^3)+b*arcsec(c*x)*e^2*x-2*b*arcsec(c*x)*d*e/x-1/3
*b*arcsec(c*x)*d^2/x^3+2/9*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^2*
d^2+2*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^2*d*e-b/c^2*(c^2*x^2-1)
^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))+1/9*b/c*
(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^4*d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$= \frac{9\,ace^2x^4 + 9\,be^2x^3\log\left(-cx+\sqrt{c^2x^2-1}\right) - 18\,acdex^2 - 6\,(bcd^2+6\,bcde-3\,bce^2)x^3\arctan\left(-cx+\sqrt{c^2x^2\right.}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*c*e^2*x^4 + 9*b*e^2*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - 18*a*c*d*e
*x^2 - 6*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*arctan(-c*x + sqrt(c^2*x^2 -
 1)) - 3*a*c*d^2 + 2*(b*c^4*d^2 + 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^2*x^4 - 6*b
*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*arcsec(c*x) +
 (b*c*d^2 + 2*(b*c^3*d^2 + 9*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

Sympy [A] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^4}\,dx = -\frac{ad^2}{3x^3} - \frac{2ade}{x} + ae^2x + 2bcde\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\operatorname{asec}(cx)}{3x^3} - \frac{2bde\operatorname{asec}(cx)}{x} + be^2x\operatorname{asec}(cx) + \frac{bd^2\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c} - \frac{be^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$

```
[In] integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**4,x)
```

```
[Out] -a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x + 2*b*c*d*e*sqrt(1 - 1/(c**2*x**2))
 - b*d**2*asec(c*x)/(3*x**3) - 2*b*d*e*asec(c*x)/x + b*e**2*x*asec(c*x) + b
*d**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(
3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sq
rt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c) - b*e**2*Piecewise((acosh(c*x), A
bs(c**2*x**2) > 1), (-I*asin(c*x), True))/c
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^4} dx$$
$$= 2\left(c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsec}(cx)}{x}\right)bde + ae^2x$$
$$- \frac{1}{9}bd^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{3\operatorname{arcsec}(cx)}{x^3}\right)$$
$$+ \frac{\left(2cx\operatorname{arcsec}(cx) - \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be^2}{2c}$$
$$- \frac{2ade}{x} - \frac{ad^2}{3x^3}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 2*(c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*d*e + a*e^2*x - 1/9*b*d^2*((
c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c
*x)/x^3) + 1/2*(2*c*x*arcsec(c*x) - log(sqrt(-1/(c^2*x^2) + 1) + 1) + log(-
sqrt(-1/(c^2*x^2) + 1) + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4968 vs. $2(140) = 280$.

Time = 99.04 (sec) , antiderivative size = 4968, normalized size of antiderivative = 31.44

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^4} dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
```

```
[Out] -1/9*(3*b*c^4*d^2*arccos(1/(c*x))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) +
 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4
/(1/(c*x) + 1)^8) + 3*a*c^4*d^2/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(
1/(c*x) + 1)^8) + 12*b*c^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^2 - 2*
c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 6*b*c^
4*d^2*sqrt(-1/(c^2*x^2) + 1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^
```

2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) + 12*a*c^4*d^2*(1/(c^2*x^2) - 1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 18*b*c^2*d*e*arccos(1/(c*x))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 18*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 2*b*c^4*d^2*(-1/(c^2*x^2) + 1)^(3/2)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) + 18*a*c^2*d*e/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) + 18*a*c^4*d^2*(1/(c^2*x^2) - 1)^2/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 12*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 36*b*c^2*d*e*sqrt(-1/(c^2*x^2) + 1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)) - 2*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) + 12*a*c^4*d^2*(1/(c^2*x^2) - 1)^3/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 9*b*e^2*arccos(1/(c*x))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 36*b*c^2*d*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 3*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8) + 9*b*e^2*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 9*b*e^2*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8) - 6*b*c^4*d^2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^7) - 36*b*c^2*d*e*(-1/(c^2*x^2) + 1)^(3/2)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^3) - 9*a*e^2/(c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2

+ 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(
c*x) + 1)^8) - 36*a*c^2*d*e*(1/(c^2*x^2) - 1)^2/((c^2 - 2*c^2*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(
c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 3*a*c^4*d^2*(1/(c^2*x^2
) - 1)^4/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*
x) + 1)^8) + 36*b*e^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^2 - 2*c^2*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 -
c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) - 18*b*e^2*(1/(c^
2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^2 - 2*c^2*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
- c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 18*b*e^2*(1/
(c^2*x^2) - 1)*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^2 - 2*c^2
*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^2) + 36*b*c^2*
d*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^2 - 2*c^2*(1/(c^2*x^2) -
1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c
^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^5) + 36*a*e^2*(1/(c^2*x^2) -
1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1
)^2) - 54*b*e^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^2 - 2*c^2*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*
(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 18*b*c^2*d*e*(1/(c^
2*x^2) - 1)^4*arccos(1/(c*x))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8)*(1/(c*x) + 1)^8) - 36*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*sqrt(-1/
(c^2*x^2) + 1)/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*
(1/(c*x) + 1)^7) - 54*a*e^2*(1/(c^2*x^2) - 1)^2/((c^2 - 2*c^2*(1/(c^2*x^2)
- 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(
c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^4) + 18*a*c^2*d*e*(1/(c^2*x^
2) - 1)^4/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c
*x) + 1)^8) + 36*b*e^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^2 - 2*c^2*(1
/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
- c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 18*b*e^2*(1/
(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) + 1))/((c^2 - 2*c
^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) +
1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) - 18*b*e^
2*(1/(c^2*x^2) - 1)^3*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x) - 1))/((c^2
- 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c
x) + 1)^6 - c^2(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^6) + 36
*a*e^2*(1/(c^2*x^2) - 1)^3/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2
+ 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c
x) + 1)^8)(1/(c*x) + 1)^6) - 9*b*e^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/

```
((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3
/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8
) - 9*b*e^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) + 1/(c*x) +
1))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) +
1)^8) + 9*b*e^2*(1/(c^2*x^2) - 1)^4*log(abs(sqrt(-1/(c^2*x^2) + 1) - 1/(c*x
) - 1))/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x
) + 1)^8) - 9*a*e^2*(1/(c^2*x^2) - 1)^4/((c^2 - 2*c^2*(1/(c^2*x^2) - 1)/(1/
(c*x) + 1)^2 + 2*c^2*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 - c^2*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8)*(1/(c*x) + 1)^8))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^4, x)
```

3.85 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 183

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{bc(24c^4d^2+100c^2de+225e^2)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{2bcd(6c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{5x^5} - \frac{2de(a+b\sec^{-1}(cx))}{3x^3} - \frac{e^2(a+b\sec^{-1}(cx))}{x}$$

```
[Out] -1/5*d^2*(a+b*arcsec(c*x))/x^5-2/3*d*e*(a+b*arcsec(c*x))/x^3-e^2*(a+b*arcse
c(c*x))/x+1/225*b*c*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(c^2*x^2-1)^(1/2)/(c^2
*x^2)^(1/2)+1/25*b*c*d^2*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)+2/225*b*c*d*
(6*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{276, 5346, 12, 1279, 464, 270\}$

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^6}\,dx = -\frac{d^2(a+b\sec^{-1}(cx))}{5x^5} - \frac{2de(a+b\sec^{-1}(cx))}{3x^3} - \frac{e^2(a+b\sec^{-1}(cx))}{x} + \frac{bcd^2\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}} + \frac{2bcd\sqrt{c^2x^2-1}(6c^2d+25e)}{225x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1}(24c^4d^2+100c^2de+225e^2)}{225\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^6,x]
```

```
[Out] (b*c*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[-1 + c^2*x^2])/(225*Sqrt[c^2
*x^2]) + (b*c*d^2*Sqrt[-1 + c^2*x^2])/(25*x^4*Sqrt[c^2*x^2]) + (2*b*c*d*(6*
c^2*d + 25*e)*Sqrt[-1 + c^2*x^2])/(225*x^2*Sqrt[c^2*x^2]) - (d^2*(a + b*Arc
Sec[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcSec[c*x]))/(3*x^3) - (e^2*(a + b*ArcS
ec[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\sec^{-1}(cx))}{5x^5}-\frac{2de(a+b\sec^{-1}(cx))}{3x^3} \\
&\quad -\frac{e^2(a+b\sec^{-1}(cx))}{x}-\frac{(bcx)\int\frac{-3d^2-10dex^2-15e^2x^4}{15x^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d^2(a+b\sec^{-1}(cx))}{5x^5}-\frac{2de(a+b\sec^{-1}(cx))}{3x^3} \\
&\quad -\frac{e^2(a+b\sec^{-1}(cx))}{x}-\frac{(bcx)\int\frac{-3d^2-10dex^2-15e^2x^4}{x^6\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}} \\
&= \frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{5x^5}-\frac{2de(a+b\sec^{-1}(cx))}{3x^3} \\
&\quad -\frac{e^2(a+b\sec^{-1}(cx))}{x}-\frac{(bcx)\int\frac{-2d(6c^2d+25e)-75e^2x^2}{x^4\sqrt{-1+c^2x^2}}\,dx}{75\sqrt{c^2x^2}} \\
&= \frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}+\frac{2bcd(6c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} \\
&\quad -\frac{d^2(a+b\sec^{-1}(cx))}{5x^5}-\frac{2de(a+b\sec^{-1}(cx))}{3x^3}-\frac{e^2(a+b\sec^{-1}(cx))}{x} \\
&\quad -\frac{(bc(-225e^2-4c^2d(6c^2d+25e))\,x)\int\frac{1}{x^2\sqrt{-1+c^2x^2}}\,dx}{225\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{bc(225e^2+4c^2d(6c^2d+25e))\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} + \frac{2bcd(6c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{5x^5} - \frac{2de(a+b\sec^{-1}(cx))}{3x^3} - \frac{e^2(a+b\sec^{-1}(cx))}{x}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^6}\,dx = \frac{-15a(3d^2+10dex^2+15e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(225e^2x^4+50dex^2(1+2c^2x^2)+3d^2(3+4c^2x^2+8c^4x^4))-}{225x^5}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(225
*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4))
- 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcSec[c*x])/(225*x^5)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96

method	result
parts	$a\left(-\frac{e^2}{x}-\frac{d^2}{5x^5}-\frac{2de}{3x^3}\right)+b\,c^5\left(-\frac{\operatorname{arcsec}(cx)e^2}{c^5x}-\frac{\operatorname{arcsec}(cx)d^2}{5x^5c^5}-\frac{2\operatorname{arcsec}(cx)de}{3c^5x^3}+\frac{(c^2x^2-1)(24c^8d^2x^4+100\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)e^2}{cx}-\frac{\operatorname{arcsec}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arcsec}(cx)de}{3c\,x^3}+\frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6d^2x}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)e^2}{cx}-\frac{\operatorname{arcsec}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arcsec}(cx)de}{3c\,x^3}+\frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6d^2x}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^4}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-e^2/x-1/5*d^2/x^5-2/3*d*e/x^3)+b*c^5*(-1/c^5*arcsec(c*x)*e^2/x-1/5*arcs
ec(c*x)*d^2/x^5/c^5-2/3/c^5*arcsec(c*x)*d*e/x^3+1/225/c^10*(c^2*x^2-1)*(24*
```

```
c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^2+9
*c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^6}\,dx = -\frac{225\,ae^2x^4+150\,adex^2+45\,ad^2+15\,(15\,be^2x^4+10\,bdex^2+3\,bd^2)\,\mathrm{arcsec}\,(cx)-((24\,bc^4d^2+100\,bc^2de}{225\,x^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b*
d*e*x^2 + 3*b*d^2)*arcsec(c*x) - ((24*b*c^4*d^2 + 100*b*c^2*d*e + 225*b*e^2
)*x^4 + 9*b*d^2 + 2*(6*b*c^2*d^2 + 25*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^5
```

Sympy [A] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.82

$$\begin{aligned}\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^6}\,dx &= -\frac{ad^2}{5x^5}-\frac{2ade}{3x^3}-\frac{ae^2}{x}+bce^2\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd^2\,\mathrm{asec}\,(cx)}{5x^5}-\frac{2bde\,\mathrm{asec}\,(cx)}{3x^3}\\ &\quad-\frac{be^2\,\mathrm{asec}\,(cx)}{x}+\frac{bd^2\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}\\ &\quad+\frac{2bde\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}\end{aligned}$$

```
[In] integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**6,x)
```

```
[Out] -a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x + b*c*e**2*sqrt(1 - 1/(c**2*
x**2)) - b*d**2*asec(c*x)/(5*x**5) - 2*b*d*e*asec(c*x)/(3*x**3) - b*e**2*as
ec(c*x)/x + b*d**2*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sq
rt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2
) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 +
1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) + 2*b*d*e*Pi
```

```
ecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3),
 Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2
*x**2 + 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \left(c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsec}(cx)}{x}\right)be^2 + \frac{1}{75}bd^2\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{15\operatorname{arcsec}(cx)}{x^5}\right) - \frac{2}{9}bde\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{3\operatorname{arcsec}(cx)}{x^3}\right)-\frac{ae^2}{x}-\frac{2ade}{3x^3}-\frac{ad^2}{5x^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] (c*sqrt(-1/(c^2*x^2) + 1) - arcsec(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(-1/(
c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2
*x^2) + 1))/c - 15*arcsec(c*x)/x^5) - 2/9*b*d*e*((c^4*(-1/(c^2*x^2) + 1)^(3
/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c + 3*arcsec(c*x)/x^3) - a*e^2/x - 2/3*
a*d*e/x^3 - 1/5*a*d^2/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \frac{1}{225}\left(24bc^4d^2\sqrt{-\frac{1}{c^2x^2}+1}+100bc^2de\sqrt{-\frac{1}{c^2x^2}+1}+225be^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{12bc^2d^2\sqrt{-\frac{1}{c^2x^2}+1}}{x^2}-\frac{225}{}\right.$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 1/225*(24*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1) + 100*b*c^2*d*e*sqrt(-1/(c^2*x^2
) + 1) + 225*b*e^2*sqrt(-1/(c^2*x^2) + 1) + 12*b*c^2*d^2*sqrt(-1/(c^2*x^2)
```

```
+ 1)/x^2 - 225*b*e^2*arccos(1/(c*x))/(c*x) + 50*b*d*e*sqrt(-1/(c^2*x^2) + 1
)/x^2 - 225*a*e^2/(c*x) - 150*b*d*e*arccos(1/(c*x))/(c*x^3) + 9*b*d^2*sqrt(
-1/(c^2*x^2) + 1)/x^4 - 150*a*d*e/(c*x^3) - 45*b*d^2*arccos(1/(c*x))/(c*x^5
) - 45*a*d^2/(c*x^5))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^6}\,dx=\int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^6,x)
[Out] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^6, x)
```

3.86 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^8}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 241

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{2bc^3(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} + \frac{bc(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{7x^7} - \frac{2de(a+b\sec^{-1}(cx))}{5x^5} - \frac{e^2(a+b\sec^{-1}(cx))}{3x^3}$$

```
[Out] -1/7*d^2*(a+b*arcsec(c*x))/x^7-2/5*d*e*(a+b*arcsec(c*x))/x^5-1/3*e^2*(a+b*a
rcsec(c*x))/x^3+2/11025*b*c^3*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-
1)^(1/2)/(c^2*x^2)^(1/2)+1/49*b*c*d^2*(c^2*x^2-1)^(1/2)/x^6/(c^2*x^2)^(1/2)
+2/1225*b*c*d*(15*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)+1/11025
*b*c*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 5346, 12, 1279, 464, 277, 270}

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8} dx = -\frac{d^2(a+b\sec^{-1}(cx))}{7x^7} - \frac{2de(a+b\sec^{-1}(cx))}{5x^5} - \frac{e^2(a+b\sec^{-1}(cx))}{3x^3} + \frac{bcd^2\sqrt{c^2x^2-1}}{49x^6\sqrt{c^2x^2}} + \frac{2bcd\sqrt{c^2x^2-1}(15c^2d+49e)}{1225x^4\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1}(360c^4d^2+1176c^2de+1225e^2)}{11025x^2\sqrt{c^2x^2}} + \frac{2bc^3\sqrt{c^2x^2-1}(360c^4d^2+1176c^2de+1225e^2)}{11025\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^8,x]
```

```
[Out] (2*b*c^3*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 + c^2*x^2])/(11025
*Sqrt[c^2*x^2]) + (b*c*d^2*Sqrt[-1 + c^2*x^2])/(49*x^6*Sqrt[c^2*x^2]) + (2*
b*c*d*(15*c^2*d + 49*e)*Sqrt[-1 + c^2*x^2])/(1225*x^4*Sqrt[c^2*x^2]) + (b*c
*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 + c^2*x^2])/(11025*x^2*Sqr
t[c^2*x^2]) - (d^2*(a + b*ArcSec[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcSec[c*x]
))/(5*x^5) - (e^2*(a + b*ArcSec[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\sec^{-1}(cx))}{7x^7}-\frac{2de(a+b\sec^{-1}(cx))}{5x^5} \\
&\quad -\frac{e^2(a+b\sec^{-1}(cx))}{3x^3}-\frac{(bcx)\int\frac{-15d^2-42dex^2-35e^2x^4}{105x^8\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d^2(a+b\sec^{-1}(cx))}{7x^7}-\frac{2de(a+b\sec^{-1}(cx))}{5x^5} \\
&\quad -\frac{e^2(a+b\sec^{-1}(cx))}{3x^3}-\frac{(bcx)\int\frac{-15d^2-42dex^2-35e^2x^4}{x^8\sqrt{-1+c^2x^2}}\,dx}{105\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{7x^7} - \frac{2de(a+b\sec^{-1}(cx))}{5x^5} - \frac{e^2(a+b\sec^{-1}(cx))}{3x^3} - \frac{(bcx)\int\frac{-6d(15c^2d+49e)-245e^2x^2}{x^6\sqrt{-1+c^2x^2}}\,dx}{735\sqrt{c^2x^2}}$$

$$= \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{7x^7} - \frac{2de(a+b\sec^{-1}(cx))}{5x^5} - \frac{e^2(a+b\sec^{-1}(cx))}{3x^3} - \frac{(bc(-1225e^2-24c^2d(15c^2d+49e))\,x)\int\frac{1}{x^4\sqrt{-1+c^2x^2}}\,dx}{3675\sqrt{c^2x^2}}$$

$$= \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} + \frac{bc(1225e^2+24c^2d(15c^2d+49e))\sqrt{-1+c^2x^2}}{11025x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{7x^7} - \frac{2de(a+b\sec^{-1}(cx))}{5x^5} - \frac{e^2(a+b\sec^{-1}(cx))}{3x^3} - \frac{(2bc^3(-1225e^2-24c^2d(15c^2d+49e))\,x)\int\frac{1}{x^2\sqrt{-1+c^2x^2}}\,dx}{11025\sqrt{c^2x^2}}$$

$$= \frac{2bc^3(1225e^2+24c^2d(15c^2d+49e))\sqrt{-1+c^2x^2}}{11025\sqrt{c^2x^2}} + \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} + \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} + \frac{bc(1225e^2+24c^2d(15c^2d+49e))\sqrt{-1+c^2x^2}}{11025x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{7x^7} - \frac{2de(a+b\sec^{-1}(cx))}{5x^5} - \frac{e^2(a+b\sec^{-1}(cx))}{3x^3}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int\frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$= \frac{-105a(15d^2+42dex^2+35e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(1225e^2x^4(1+2c^2x^2)+294dex^2(3+4c^2x^2+8c^4x^4)+4}{11025x^7}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^8,x]
```

```
[Out] (-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1
225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^4) + 45*
d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 42*d*e*x^2
+ 35*e^2*x^4)*ArcSec[c*x])/(11025*x^7)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86

method	result
parts	$a\left(-\frac{d^2}{7x^7}-\frac{2de}{5x^5}-\frac{e^2}{3x^3}\right)+b\,c^7\left(-\frac{\operatorname{arcsec}(cx)d^2}{7x^7c^7}-\frac{2\operatorname{arcsec}(cx)de}{5c^7x^5}-\frac{\operatorname{arcsec}(cx)e^2}{3c^7x^3}+\frac{(c^2x^2-1)(720c^{10}d^2x^6+\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^3x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)e^2}{3c^3x^3}-\frac{\operatorname{arcsec}(cx)d^2}{7c^3x^7}-\frac{2\operatorname{arcsec}(cx)de}{5c^3x^5}+\frac{(c^2x^2-1)(720c^{10}d^2x^6+2352c^8de\,x^6+\right.}{c^4}\right.$
default	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^3x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arcsec}(cx)e^2}{3c^3x^3}-\frac{\operatorname{arcsec}(cx)d^2}{7c^3x^7}-\frac{2\operatorname{arcsec}(cx)de}{5c^3x^5}+\frac{(c^2x^2-1)(720c^{10}d^2x^6+2352c^8de\,x^6+\right.}{c^4}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/7*d^2/x^7-2/5*d*e/x^5-1/3*e^2/x^3)+b*c^7*(-1/7*arcsec(c*x)*d^2/x^7/c^
7-2/5/c^7*arcsec(c*x)*d*e/x^5-1/3/c^7*arcsec(c*x)*e^2/x^3+1/11025/c^12*(c^2
*x^2-1)*(720*c^10*d^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+2450*c^6*e^2*x^6
+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*d*e*x^2+225*c^4*
d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^8)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\,(35\,be^2x^4+42\,bdex^2+15\,bd^2)\operatorname{arcsec}(cx)-(2\,(360\,bc^6d^2+1\cdots}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
+ 42*b*d*e*x^2 + 15*b*d^2)*arcsec(c*x) - (2*(360*b*c^6*d^2 + 1176*b*c^4*d*e
 + 1225*b*c^2*e^2)*x^6 + (360*b*c^4*d^2 + 1176*b*c^2*d*e + 1225*b*e^2)*x^4
+ 225*b*d^2 + 18*(15*b*c^2*d^2 + 49*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^7
```

Sympy [A] (verification not implemented)

Time = 30.11 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8}\,dx$$
$$= -\frac{ad^2}{7x^7} - \frac{2ade}{5x^5} - \frac{ae^2}{3x^3} - \frac{bd^2\operatorname{asec}(cx)}{7x^7} - \frac{2bde\operatorname{asec}(cx)}{5x^5} - \frac{be^2\operatorname{asec}(cx)}{3x^3}$$
$$+ \frac{bd^2\left(\begin{cases} \frac{16c^7\sqrt{c^2x^2-1}}{35x} + \frac{8c^5\sqrt{c^2x^2-1}}{35x^3} + \frac{6c^3\sqrt{c^2x^2-1}}{35x^5} + \frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2| > 1 \\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x} + \frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3} + \frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5} + \frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise} \end{cases}\right)}{7c}$$
$$+ \frac{2bde\left(\begin{cases} \frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2| > 1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise} \end{cases}\right)}{5c}$$
$$+ \frac{be^2\left(\begin{cases} \frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2| > 1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}\right)}{3c}$$

```
[In] integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**8,x)
```

```
[Out] -a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*d**2*asec(c*x)/(7
*x**7) - 2*b*d*e*asec(c*x)/(5*x**5) - b*e**2*asec(c*x)/(3*x**3) + b*d**2*Pi
ecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*x**2 - 1)/(3
5*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x**2 - 1)/(7*x
**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*x) + 8*I*c**
5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1)/(35*x**5)
+ I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) + 2*b*d*e*Piecewise((8*c*
*5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sq
rt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2
+ 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2
 + 1)/(5*x**5), True))/(5*c) + b*e**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)
/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqr
t(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8} \, dx =$$
$$-\frac{1}{245}\, bd^2 \left(\frac{5\, c^8 \left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} - 21\, c^8 \left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35\, c^8 \left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 35\, c^8 \sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{35 \text{ arcsec}\,(c}{x^7}\right.$$
$$+ \frac{2}{75}\, bde \left(\frac{3\, c^6 \left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\, c^6 \left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\, c^6 \sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{15 \text{ arcsec}\,(cx)}{x^5}\right)$$
$$- \frac{1}{9}\, be^2 \left(\frac{c^4 \left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\, c^4 \sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{3 \text{ arcsec}\,(cx)}{x^3}\right) - \frac{ae^2}{3\,x^3} - \frac{2\,ade}{5\,x^5} - \frac{ad^2}{7\,x^7}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] -1/245*b*d^2*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(
5/2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c +
 35*arcsec(c*x)/x^7) + 2/75*b*d*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6
*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c - 15*arcsec(c*
x)/x^5) - 1/9*b*e^2*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2
) + 1))/c + 3*arcsec(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/
x^7
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^8} \, dx$$
$$= \frac{1}{11025} \left(720\, bc^6 d^2 \sqrt{-\frac{1}{c^2x^2}+1} + 2352\, bc^4 de \sqrt{-\frac{1}{c^2x^2}+1} + 2450\, bc^2 e^2 \sqrt{-\frac{1}{c^2x^2}+1} + \frac{360\, bc^4 d^2 \sqrt{-\frac{1}{c^2x^2}+}}{x^2}\right.$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^8,x, algorithm="giac")
```

```
[Out] 1/11025*(720*b*c^6*d^2*sqrt(-1/(c^2*x^2) + 1) + 2352*b*c^4*d*e*sqrt(-1/(c^2
*x^2) + 1) + 2450*b*c^2*e^2*sqrt(-1/(c^2*x^2) + 1) + 360*b*c^4*d^2*sqrt(-1/
(c^2*x^2) + 1)/x^2 + 1176*b*c^2*d*e*sqrt(-1/(c^2*x^2) + 1)/x^2 + 270*b*c^2*
```

```
d^2*sqrt(-1/(c^2*x^2) + 1)/x^4 + 1225*b*e^2*sqrt(-1/(c^2*x^2) + 1)/x^2 - 36
75*b*e^2*arccos(1/(c*x))/(c*x^3) + 882*b*d*e*sqrt(-1/(c^2*x^2) + 1)/x^4 - 3
675*a*e^2/(c*x^3) - 4410*b*d*e*arccos(1/(c*x))/(c*x^5) + 225*b*d^2*sqrt(-1/
(c^2*x^2) + 1)/x^6 - 4410*a*d*e/(c*x^5) - 1575*b*d^2*arccos(1/(c*x))/(c*x^7
) - 1575*a*d^2/(c*x^7))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)}{x^8}\,dx=\int \frac{\left(e\,x^2+d\right)^2\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^8, x)
```

3.87 $\int x^3(d+ex^2)^2(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 242

$$\begin{aligned}
&\int x^3(d+ex^2)^2(a+b\sec^{-1}(cx))\,dx\\
&=-\frac{b(6c^4d^2+8c^2de+3e^2)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}-\frac{b(6c^4d^2+16c^2de+9e^2)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}\\
&\quad-\frac{be(8c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}-\frac{be^2x(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}}\\
&\quad+\frac{1}{4}d^2x^4(a+b\sec^{-1}(cx))+\frac{1}{3}dex^6(a+b\sec^{-1}(cx))+\frac{1}{8}e^2x^8(a+b\sec^{-1}(cx))
\end{aligned}$$

```
[Out] 1/4*d^2*x^4*(a+b*arcsec(c*x))+1/3*d*e*x^6*(a+b*arcsec(c*x))+1/8*e^2*x^8*(a+
b*arcsec(c*x))-1/72*b*(6*c^4*d^2+16*c^2*d*e+9*e^2)*x*(c^2*x^2-1)^(3/2)/c^7/
(c^2*x^2)^(1/2)-1/120*b*e*(8*c^2*d+9*e)*x*(c^2*x^2-1)^(5/2)/c^7/(c^2*x^2)^(
1/2)-1/56*b*e^2*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/2)-1/24*b*(6*c^4*d^2+8
*c^2*d*e+3*e^2)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

= {272, 45, 5346, 12, 1265, 785}

$$\int x^3\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)-\frac{bex(c^2x^2-1)^{5/2}\left(8c^2d+9e\right)}{120c^7\sqrt{c^2x^2}}-\frac{be^2x(c^2x^2-1)^{7/2}}{56c^7\sqrt{c^2x^2}}-\frac{bx(c^2x^2-1)^{3/2}\left(6c^4d^2+16c^2de+9e^2\right)}{72c^7\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}(6c^4d^2+8c^2de+3e^2)}{24c^7\sqrt{c^2x^2}}$$

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/24*(b*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*x*Sqrt[-1 + c^2*x^2])/(c^7*Sqrt[c^
2*x^2]) - (b*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*x*(-1 + c^2*x^2)^(3/2))/(72*c
^7*Sqrt[c^2*x^2]) - (b*e*(8*c^2*d + 9*e)*x*(-1 + c^2*x^2)^(5/2))/(120*c^7*S
qrt[c^2*x^2]) - (b*e^2*x*(-1 + c^2*x^2)^(7/2))/(56*c^7*Sqrt[c^2*x^2]) + (d^
2*x^4*(a + b*ArcSec[c*x]))/4 + (d*e*x^6*(a + b*ArcSec[c*x]))/3 + (e^2*x^8*(
a + b*ArcSec[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^3(6d^2+8dex^2+3e^2x^4)}{24\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^3(6d^2+8dex^2+3e^2x^4)}{\sqrt{-1+c^2x^2}}\,dx}{24\sqrt{c^2x^2}}\\
&= \frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)-\frac{(bcx)\text{Subst}\left(\int\frac{x(6d^2+8dex+3e^2x^2)}{\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{48\sqrt{c^2x^2}}\\
&= \frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)\\
&\quad-\frac{(bcx)\text{Subst}\left(\int\left(\frac{6c^4d^2+8c^2de+3e^2}{c^6\sqrt{-1+c^2x}}+\frac{(6c^4d^2+16c^2de+9e^2)\sqrt{-1+c^2x}}{c^6}+\frac{e(8c^2d+9e)(-1+c^2x)^{3/2}}{c^6}+\frac{3e^2(-1+c^2x)^{5/2}}{c^6}\right)dx,x,x^2\right)}{48\sqrt{c^2x^2}}\\
&= -\frac{b(6c^4d^2+8c^2de+3e^2)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}-\frac{b(6c^4d^2+16c^2de+9e^2)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}\\
&\quad-\frac{be(8c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}-\frac{be^2x(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}}\\
&\quad+\frac{1}{4}d^2x^4\left(a+b\sec^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\sec^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\sec^{-1}(cx)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.67

$$\int x^3\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{1}{24}ax^4\left(6d^2+8dex^2+3e^2x^4\right)$$
$$-\frac{b\sqrt{1-\frac{1}{c^2x^2}}x(144e^2+8c^2e(56d+9ex^2)+c^4(420d^2+224dex^2+54e^2x^4)+3c^6(70d^2x^2+56dex^4+15e}{2520c^7}$$
$$+\frac{1}{24}bx^4\left(6d^2+8dex^2+3e^2x^4\right)\sec^{-1}(cx)$$

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] (a*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4))/24 - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(14
4*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 54*e^2*x^4)
 + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/(2520*c^7) + (b*x^4*(6*d^
2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcSec[c*x])/24
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.82

method	result
parts	$a\left(\frac{1}{8}e^2x^8+\frac{1}{3}de\,x^6+\frac{1}{4}x^4d^2\right)+\frac{b\left(\frac{c^4\,\mathrm{arcsec}(cx)e^2x^8}{8}+\frac{c^4\,\mathrm{arcsec}(cx)de\,x^6}{3}+\frac{\mathrm{arcsec}(cx)d^2x^4c^4}{4}-\frac{\left(c^2x^2-1\right)\left(45c^6e^2x^6\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2d\left(c^2e\,x^2+c^2d\right)^3}{3}-\frac{\left(c^2e\,x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}-\frac{b\,c^4\,\mathrm{arcsec}(cx)d^4}{24e^2}+\frac{b\ \mathrm{arcsec}(cx)d^2c^4x^4}{4}+\frac{b\,c^4e\ \mathrm{arcsec}(cx)d\,x^6}{3}+\frac{b\,c^4e^2\,\mathrm{arcsec}(cx)x^8}{8}-}{}$
default	$\frac{-\frac{a\left(\frac{c^2d\left(c^2e\,x^2+c^2d\right)^3}{3}-\frac{\left(c^2e\,x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}-\frac{b\,c^4\,\mathrm{arcsec}(cx)d^4}{24e^2}+\frac{b\ \mathrm{arcsec}(cx)d^2c^4x^4}{4}+\frac{b\,c^4e\ \mathrm{arcsec}(cx)d\,x^6}{3}+\frac{b\,c^4e^2\,\mathrm{arcsec}(cx)x^8}{8}-}{}$

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*x^4*d^2)+b/c^4*(1/8*c^4*arcsec(c*x)*e^2*x^8+
1/3*c^4*arcsec(c*x)*d*e*x^6+1/4*arcsec(c*x)*d^2*x^4*c^4-1/2520/c^5*(c^2*x^2
-1)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+210*c^6*d^2*x^2+54*c^4*e^2*x^4+224*c^4*
d*e*x^2+420*c^4*d^2+72*c^2*e^2*x^2+448*c^2*d*e+144*e^2)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{315\,ac^8e^2x^8 + 840\,ac^8dex^6 + 630\,ac^8d^2x^4 + 105\left(3\,bc^8e^2x^8 + 8\,bc^8dex^6 + 6\,bc^8d^2x^4\right)\operatorname{arcsec}(cx) - \left(45\,bc^6e^2x\right.}{}$$

[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")

[Out] 1/2520*(315*a*c^8*e^2*x^8 + 840*a*c^8*d*e*x^6 + 630*a*c^8*d^2*x^4 + 105*(3*b*c^8*e^2*x^8 + 8*b*c^8*d*e*x^6 + 6*b*c^8*d^2*x^4)*arcsec(c*x) - (45*b*c^6*e^2*x^6 + 420*b*c^4*d^2 + 448*b*c^2*d*e + 6*(28*b*c^6*d*e + 9*b*c^4*e^2)*x^4 + 144*b*e^2 + 2*(105*b*c^6*d^2 + 112*b*c^4*d*e + 36*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^8

Sympy [A] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.04

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\operatorname{asec}(cx)}{4} + \frac{bdex^6\operatorname{asec}(cx)}{3} + \frac{be^2x^8\operatorname{asec}(cx)}{8} - \frac{bd^2\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c} + \frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c} + \frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c} - \frac{bde\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c} + \frac{4x^2\sqrt{c^2x^2-1}}{15c^3} + \frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c} + \frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3} + \frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{3c} - \frac{be^2\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c} + \frac{6x^4\sqrt{c^2x^2-1}}{35c^3} + \frac{8x^2\sqrt{c^2x^2-1}}{35c^5} + \frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c} + \frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3} + \frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5} + \frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

[In] integrate(x**3*(e*x**2+d)**2*(a+b*asec(c*x)),x)

[Out] a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*asec(c*x)/4 + b*d*e*x**6*asec(c*x)/3 + b*e**2*x**8*asec(c*x)/8 - b*d**2*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1

```
), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3),
True))/(4*c) - b*d*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqr
t(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2
) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/
(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(3*c) - b*e**2*Piece
wise((x**6*sqrt(c**2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3)
 + 8*x**2*sqrt(c**2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7),
 Abs(c**2*x**2) > 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c
**2*x**2 + 1)/(35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sq
rt(-c**2*x**2 + 1)/(35*c**7), True))/(8*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.06

$$\int x^3 (d+ex^2)^2 \left(a+b\sec^{-1}(cx)\right)\,dx = \frac{1}{8}\,ae^2x^8 + \frac{1}{3}\,adex^6 + \frac{1}{4}\,ad^2x^4 + \frac{1}{12}\left(3\,x^4\,\mathrm{arcsec}\,(cx) - \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd^2 + \frac{1}{45}\left(15\,x^6\,\mathrm{arcsec}\,(cx) - \frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)bde + \frac{1}{280}\left(35\,x^8\,\mathrm{arcsec}\,(cx) - \frac{5\,c^6x^7\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} + 21\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 35\,x\sqrt{-}\right.$$

$$c^7$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arcsec(c*x) - (
c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d^2 +
 1/45*(15*x^6*arcsec(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^
3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*d*e + 1/28
0*(35*x^8*arcsec(c*x) - (5*c^6*x^7*(-1/(c^2*x^2) + 1)^(7/2) + 21*c^4*x^5*(-
1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 35*x*sqrt(-1
/(c^2*x^2) + 1))/c^7)*b*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17666 vs. 2(212) = 424.

Time = 0.50 (sec) , antiderivative size = 17666, normalized size of antiderivative = 73.00

$$\int x^3(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/2520*(630*b*c^4*d^2*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*
x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2
) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*
c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c
*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2)
 - 1)^8/(1/(c*x) + 1)^16) + 630*a*c^4*d^2/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(
1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*
x^2) - 1)^8/(1/(c*x) + 1)^16) - 1260*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1)/((c^9
 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/
(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*
x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 +
 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1
/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)) +
 840*b*c^2*d*e*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/
(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1
)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8
/(1/(c*x) + 1)^16) - 2520*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c
^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^
2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/
(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^
4) + 7140*b*c^4*d^2*(-1/(c^2*x^2) + 1)^(3/2)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)
^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(
c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^3) + 840*a*c^2*d*e/(c^9 + 8
*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2)
```

- 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16) - 2520*a*c^4*d^2*(1/(c^2*x^2) - 1)^2/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^4) - 3360*b*c^2*d*e*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^2) - 1680*b*c^2*d*e*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)) - 18060*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^5) - 3360*a*c^2*d*e*(1/(c^2*x^2) - 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^2) + 315*b*e^2*arccos(1/(c*x))/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16) + 3360*b*c^2*d*e*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^4) + 3780*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^

4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1
/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1
)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^8) - 27300*b
*c^4*d^2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x
^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*
c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*
x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^
2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c
^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^7) + 7280*b*c^2*d*e*
(-1/(c^2*x^2) + 1)^(3/2)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*
x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12
+ 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(
c*x) + 1)^16)*(1/(c*x) + 1)^3) + 315*a*e^2/(c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(
1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8
+ 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/
(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2
*x^2) - 1)^8/(1/(c*x) + 1)^16) + 3360*a*c^2*d*e*(1/(c^2*x^2) - 1)^2/((c^9 +
 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 2
8*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(
c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^4) +
 3780*a*c^4*d^2*(1/(c^2*x^2) - 1)^4/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x
) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2)
 - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c
^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*
x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2)
- 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^8) - 2520*b*e^2*(1/(c^2*x^2) - 1)*ar
ccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(
c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^
6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5
/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/
(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^1
6)*(1/(c*x) + 1)^2) + 3360*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((
c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/
(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c
^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^1
0 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7
/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)
^6) - 630*b*e^2*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(
c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 5

6*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)) - 18256*b*c^2*d*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^5) - 27300*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^9) - 2520*a*e^2*(1/(c^2*x^2) - 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^2) + 3360*a*c^2*d*e*(1/(c^2*x^2) - 1)^3/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^6) + 8820*b*e^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^4) - 8400*b*c^2*d*e*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^8) - 2520*b*c^4*d^2*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^12) - 30128*b*c^2*d*e*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^

2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^7) - 18060*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^11) + 1890*b*e^2*(-1/(c^2*x^2) + 1)^(3/2)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^3) + 8820*a*e^2*(1/(c^2*x^2) - 1)^2/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^4) - 8400*a*c^2*d*e*(1/(c^2*x^2) - 1)^4/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^8) - 2520*a*c^4*d^2*(1/(c^2*x^2) - 1)^6/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^12) - 17640*b*e^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^6) + 3360*b*c^2*d*e*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^10) - 6678*b*e^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^

2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70
c^9(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c
*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x
^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/
(c*x) + 1)^5) - 30128*b*c^2*d*e*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/
((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/
(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)
^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)
^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) +
1)^9) - 7140*b*c^4*d^2*(1/(c^2*x^2) - 1)^6*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8
c^9(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*
c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*
x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^13) -
17640*a*e^2*(1/(c^2*x^2) - 1)^3/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) +
 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)
^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^6) + 3360*a*c^2*d*e*(1/(c^2*x^2) - 1)^5/
((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/
(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)
^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)
^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) +
1)^10) + 22050*b*e^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(
c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
+ 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(
1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c
^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^1
4 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^8) + 3360*b*c^2
*d*e*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(
1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*
x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^12) + 630*b*c^4*d^2*(1/(c^2*x^2
) - 1)^8*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 +
28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/
(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*
x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12
+ 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(
c*x) + 1)^16)*(1/(c*x) + 1)^16) - 9234*b*e^2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c

^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/
(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(
c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16
)*(1/(c*x) + 1)^7) - 18256*b*c^2*d*e*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2)
+ 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2)
- 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^
9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x)
 + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2)
 - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*
x) + 1)^11) - 1260*b*c^4*d^2*(1/(c^2*x^2) - 1)^7*sqrt(-1/(c^2*x^2) + 1)/((c
^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^
2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/
(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^
15) + 22050*a*e^2*(1/(c^2*x^2) - 1)^4/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56
c^9(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(
c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2
) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^8) + 3360*a*c^2*d*e*(1/(c^2*x^2) -
 1)^6/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c
^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x
) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2
) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c
*x) + 1)^12) + 630*a*c^4*d^2*(1/(c^2*x^2) - 1)^8/((c^9 + 8*c^9*(1/(c^2*x^2)
 - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9
*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x)
+ 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2)
- 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*
(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^16) - 17640*b*e^2*(1/(c
^2*x^2) - 1)^5*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1
)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)
^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1
/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) +
1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^
8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^10) - 3360*b*c^2*d*e*(1/(c^2*x^2) - 1)^7*
arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1
/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1
)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)
^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(
1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)

^16)*(1/(c*x) + 1)^14) - 9234*b*e^2*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^9) - 7280*b*c^2*d*e*(1/(c^2*x^2) - 1)^6*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^13) - 17640*a*e^2*(1/(c^2*x^2) - 1)^5/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^10) - 3360*a*c^2*d*e*(1/(c^2*x^2) - 1)^7/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^14) + 8820*b*e^2*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^12) + 840*b*c^2*d*e*(1/(c^2*x^2) - 1)^8*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^16) - 6678*b*e^2*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^11) - 1680*b*c^2*d*e*(1/(c^2*x^2) - 1)^7*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12

+ 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^15) + 8820*a*e^2*(1/(c^2*x^2) - 1)^6/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^12) + 840*a*c^2*d*e*(1/(c^2*x^2) - 1)^8/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^16) - 2520*b*e^2*(1/(c^2*x^2) - 1)^7*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^14) - 1890*b*e^2*(1/(c^2*x^2) - 1)^6*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^13) - 2520*a*e^2*(1/(c^2*x^2) - 1)^7/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^14) + 315*b*e^2*(1/(c^2*x^2) - 1)^8*arccos(1/(c*x))/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^16) - 630*b*e^2*(1/(c^2*x^2) - 1)^7*sqrt(-1/(c^2*x^2) + 1)/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^15) + 315*a*e^2*(1/(c^2*x^2) - 1)^8/((c^9 + 8*c^9*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 28*c^9*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 56*c^9*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 70*c^9*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 56*c^9*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + 28*c^9*(1/(c^2*x^2) - 1)^

```
 1)^6/(1/(c*x) + 1)^12 + 8*c^9*(1/(c^2*x^2) - 1)^7/(1/(c*x) + 1)^14 + c^9*(
1/(c^2*x^2) - 1)^8/(1/(c*x) + 1)^16)*(1/(c*x) + 1)^16))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^3\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

3.88 $\int x(d+ex^2)^2 (a+b\sec^{-1}(cx)) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 195

$$\int x(d+ex^2)^2 (a+b\sec^{-1}(cx)) \, dx = -\frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} - \frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} - \frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{(d+ex^2)^3 (a+b\sec^{-1}(cx))}{6e} - \frac{bcd^3x\arctan\left(\sqrt{-1+c^2x^2}\right)}{6e\sqrt{c^2x^2}}$$

```
[Out] 1/6*(e*x^2+d)^3*(a+b*arcsec(c*x))/e-1/18*b*e*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(3
/2)/c^5/(c^2*x^2)^(1/2)-1/30*b*e^2*x*(c^2*x^2-1)^(5/2)/c^5/(c^2*x^2)^(1/2)-
1/6*b*c*d^3*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-1/6*b*(3*c^4*d^2+
3*c^2*d*e+e^2)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used

$= \{5344, 457, 90, 65, 211\}$

$$\int x\left(d+ex^2\right)^2\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{\left(d+ex^2\right)^3\left(a+b\sec^{-1}(cx)\right)}{6e} - \frac{bcd^3x\arctan\left(\sqrt{c^2x^2-1}\right)}{6e\sqrt{c^2x^2}} - \frac{bex(c^2x^2-1)^{3/2}\left(3c^2d+2e\right)}{18c^5\sqrt{c^2x^2}} - \frac{be^2x(c^2x^2-1)^{5/2}}{30c^5\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1}(3c^4d^2+3c^2de+e^2)}{6c^5\sqrt{c^2x^2}}$$

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/6*(b*(3*c^4*d^2 + 3*c^2*d*e + e^2)*x*Sqrt[-1 + c^2*x^2])/(c^5*Sqrt[c^2*x
^2]) - (b*e*(3*c^2*d + 2*e)*x*(-1 + c^2*x^2)^(3/2))/(18*c^5*Sqrt[c^2*x^2])
- (b*e^2*x*(-1 + c^2*x^2)^(5/2))/(30*c^5*Sqrt[c^2*x^2]) + ((d + e*x^2)^3*(a
 + b*ArcSec[c*x]))/(6*e) - (b*c*d^3*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*e*Sqrt
[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5344

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^3\left(a+b\sec^{-1}(cx)\right)}{6e} - \frac{(bcx)\int\frac{(d+ex^2)^3}{x\sqrt{-1+c^2x^2}}\,dx}{6e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^3\left(a+b\sec^{-1}(cx)\right)}{6e} - \frac{(bcx)\text{Subst}\left(\int\frac{(d+ex)^3}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{12e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^3\left(a+b\sec^{-1}(cx)\right)}{6e} - \frac{(bcx)\text{Subst}\left(\int\left(\frac{e(3c^4d^2+3c^2de+e^2)}{c^4\sqrt{-1+c^2x}}+\frac{d^3}{x\sqrt{-1+c^2x}}+\frac{e^2(3c^2d+2e)\sqrt{-1+c^2x}}{c^4}+\frac{e^3(-1+c^2x)^{3/2}}{c^4}\right)dx,x,x^2\right)}{12e\sqrt{c^2x^2}}$$

$$= -\frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} - \frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} - \frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{(d+ex^2)^3\left(a+b\sec^{-1}(cx)\right)}{6e} - \frac{(bcd^3x)\,\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{12e\sqrt{c^2x^2}}$$

$$= -\frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} - \frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} - \frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{(d+ex^2)^3\left(a+b\sec^{-1}(cx)\right)}{6e} - \frac{(bd^3x)\,\text{Subst}\left(\int\frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{6ce\sqrt{c^2x^2}}$$

$$= -\frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} - \frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} - \frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{(d+ex^2)^3\left(a+b\sec^{-1}(cx)\right)}{6e} - \frac{bcd^3x\arctan\left(\sqrt{-1+c^2x^2}\right)}{6e\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64

$$\int x(d+ex^2)^2 (a+b\sec^{-1}(cx))\,dx$$
$$=\frac{1}{90}x\left(15ax(3d^2+3dex^2+e^2x^4)-\frac{b\sqrt{1-\frac{1}{c^2x^2}}(8e^2+2c^2e(15d+2ex^2)+3c^4(15d^2+5dex^2+e^2x^4))}{c^5}+15bx(3d^2+3dex^2+e^2x^4)\sec^{-1}(cx)\right)$$

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] (x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) - (b*Sqrt[1 - 1/(c^2*x^2)]*(8*e^2
+ 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 + 1
5*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcSec[c*x]))/90
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(169) = 338.

Time = 0.82 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.81

method	result
parts	$\frac{a(e\,x^2+d)^3}{6e}+\frac{b\,\mathrm{arcsec}(cx)e^2x^6}{6}+\frac{b\,\mathrm{arcsec}(cx)de\,x^4}{2}+\frac{b\,\mathrm{arcsec}(cx)d^2x^2}{2}+\frac{b\,\mathrm{arcsec}(cx)d^3}{6e}-\frac{b(c^2x^2-1)x^3e^2}{30c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-$
derivativedivides	$\frac{\frac{a(c^2e\,x^2+c^2d)^3}{6c^4e}+\frac{b\,c^2\,\mathrm{arcsec}(cx)d^3}{6e}+\frac{b\,\mathrm{arcsec}(cx)d^2c^2x^2}{2}+\frac{b\,c^2e\,\mathrm{arcsec}(cx)d\,x^4}{2}+\frac{b\,c^2e^2\,\mathrm{arcsec}(cx)x^6}{6}+\frac{bc\sqrt{c^2x^2-1}\,d^3\arctan\left(\frac{}{\sqrt{c}}\right.}{6e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}}{c^2}$
default	$\frac{\frac{a(c^2e\,x^2+c^2d)^3}{6c^4e}+\frac{b\,c^2\,\mathrm{arcsec}(cx)d^3}{6e}+\frac{b\,\mathrm{arcsec}(cx)d^2c^2x^2}{2}+\frac{b\,c^2e\,\mathrm{arcsec}(cx)d\,x^4}{2}+\frac{b\,c^2e^2\,\mathrm{arcsec}(cx)x^6}{6}+\frac{bc\sqrt{c^2x^2-1}\,d^3\arctan\left(\frac{}{\sqrt{c}}\right.}{6e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}}{c^2}$

```
[In] int(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*(e*x^2+d)^3/e+1/6*b*arcsec(c*x)*e^2*x^6+1/2*b*arcsec(c*x)*d*e*x^4+1/2
*b*arcsec(c*x)*d^2*x^2+1/6*b/e*arcsec(c*x)*d^3-1/30*b/c^3*(c^2*x^2-1)/((c^2
*x^2-1)/c^2/x^2)^(1/2)*x^3*e^2-1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^
(1/2)*x*d*e+1/6*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*a
rctan(1/(c^2*x^2-1)^(1/2))-2/45*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/
```

```
2)*x*e^2-1/2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2-1/3*b/c^5*
e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d-4/45*b/c^7*e^2*(c^2*x^2-1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78

$$\int x(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{15\,ac^6e^2x^6+45\,ac^6dex^4+45\,ac^6d^2x^2+15\left(bc^6e^2x^6+3\,bc^6dex^4+3\,bc^6d^2x^2\right)\operatorname{arcsec}(cx)-\left(3\,bc^4e^2x^4+45\,b\right.}{90\,c^6}$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^6*e^2*x^6 + 45*a*c^6*d*e*x^4 + 45*a*c^6*d^2*x^2 + 15*(b*c^6*e^
2*x^6 + 3*b*c^6*d*e*x^4 + 3*b*c^6*d^2*x^2)*arcsec(c*x) - (3*b*c^4*e^2*x^4 +
 45*b*c^4*d^2 + 30*b*c^2*d*e + 8*b*e^2 + (15*b*c^4*d*e + 4*b*c^2*e^2)*x^2)*
sqrt(c^2*x^2 - 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.81

$$\int x(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\operatorname{asec}(cx)}{2}+\frac{bdex^4\operatorname{asec}(cx)}{2}$$
$$+\frac{be^2x^6\operatorname{asec}(cx)}{6}-\frac{bd^2\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$
$$-\frac{bde\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{2c}$$
$$-\frac{be^2\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

```
[In] integrate(x*(e*x**2+d)**2*(a+b*asec(c*x)),x)
```

```
[Out] a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*asec(c*x)/2 + b*
d*e*x**4*asec(c*x)/2 + b*e**2*x**6*asec(c*x)/6 - b*d**2*Piecewise((sqrt(c**
2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
 - b*d*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/
(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqr
t(-c**2*x**2 + 1)/(3*c**3), True))/(2*c) - b*e**2*Piecewise((x**4*sqrt(c**2
*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2
- 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4
*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5)
, True))/(6*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.98

$$\int x(d+ex^2)^2\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}ae^2x^6+\frac{1}{2}adex^4+\frac{1}{2}ad^2x^2+\frac{1}{2}\left(x^2\operatorname{arcsec}(cx)-\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2$$
$$+\frac{1}{6}\left(3x^4\operatorname{arcsec}(cx)-\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bde$$
$$+\frac{1}{90}\left(15x^6\operatorname{arcsec}(cx)-\frac{3c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be^2$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arcsec(c*x) - x*sq
rt(-1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arcsec(c*x) - (c^2*x^3*(-1/(c^2*
x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*arc
sec(c*x) - (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) +
 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11858 vs. $2(169) = 338$.

Time = 0.45 (sec) , antiderivative size = 11858, normalized size of antiderivative = 60.81

$$\int x(d+ex^2)^2 (a+b\sec^{-1}(cx)) \, dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] 1/90*(45*b*c^4*d^2*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x)
+ 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) -
 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*
(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1
)^12) + 45*a*c^4*d^2/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^
7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x)
 + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) -
 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) + 90*b*c
^4*d^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1
/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2
*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 +
 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c
*x) + 1)^12)*(1/(c*x) + 1)^2) - 90*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1)/((c^7 +
 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^
7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)) + 90*a*c^4*d^2*(1/(c
^2*x^2) - 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(
1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2
) + 45*b*c^2*d*e*arccos(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1
/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^
12) - 45*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^
2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/
(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2
) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^4) + 450*b*c^4*d^2*(-1/(c^2*x^2) +
 1)^(3/2)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*
x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 +
15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(
c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^3) +
 45*a*c^2*d*e/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c
```

```
^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(
1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 45*a*c^4*d^2*
(1/(c^2*x^2) - 1)^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^
7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x)
 + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) -
 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x)
 + 1)^4) - 90*b*c^2*d*e*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c^7*(1/
(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4
 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/
(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*
x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2) - 180*b*c^4*d^2*(1/(c^2*x^2)
 - 1)^3*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 1
5*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(
c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^
2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(
c*x) + 1)^6) - 90*b*c^2*d*e*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^
2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c
^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x
) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) -
1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)) - 900*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*s
qrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*
c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*
x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2)
 - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*
x) + 1)^5) - 90*a*c^2*d*e*(1/(c^2*x^2) - 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)
/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(
c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^
8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1
/(c*x) + 1)^12)*(1/(c*x) + 1)^2) - 180*a*c^4*d^2*(1/(c^2*x^2) - 1)^3/((c^7
+ 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(
c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x
^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c
^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^6) + 15*b*e^2*arccos
(1/(c*x))/(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x
^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 1
5*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c
*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 45*b*c^2*d*e*(1/(
c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) +
1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1
)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1
/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^
12)*(1/(c*x) + 1)^4) - 45*b*c^4*d^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c
^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(
1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^
```

```
2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
+ c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^8) - 900*b*c^4*d^
2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(
1/(c*x) + 1)^12)*(1/(c*x) + 1)^7) + 330*b*c^2*d*e*(-1/(c^2*x^2) + 1)^(3/2)/
((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^
2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/
(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^
10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^3) + 15*a*e^2/
(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(
c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^1
0 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12) - 45*a*c^2*d*e*(1/(c^2*x^2) -
 1)^2/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c
^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x)
 + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^4) - 45*
a*c^4*d^2*(1/(c^2*x^2) - 1)^4/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12
)*(1/(c*x) + 1)^8) - 90*b*e^2*(1/(c^2*x^2) - 1)*arccos(1/(c*x))/((c^7 + 6*c
^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x)
+ 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) -
 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1
/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2) + 180*b*c^2*d*e*(1/(c^
2*x^2) - 1)^3*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12
)*(1/(c*x) + 1)^6) + 90*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^7
 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/
(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*
x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 +
c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^10) - 30*b*e^2*sqrt
(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x)
+ 1)) - 540*b*c^2*d*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*
c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x)
 + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2)
- 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(
```

1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^5) - 450*b*c^4*d^2*(1/(c
^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c
*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^
2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*
c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x)
 + 1)^12)*(1/(c*x) + 1)^9) - 90*a*e^2*(1/(c^2*x^2) - 1)/((c^7 + 6*c^7*(1/(c
^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 +
 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1
/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^
2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^2) + 180*a*c^2*d*e*(1/(c^2*x^2) -
 1)^3/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c
^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x)
 + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^6) + 90*
a*c^4*d^2*(1/(c^2*x^2) - 1)^5/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12
)*(1/(c*x) + 1)^10) + 225*b*e^2*(1/(c^2*x^2) - 1)^2*arccos(1/(c*x))/((c^7 +
 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^
7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^4) - 45*b*c^2*d*e*(1/
(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) +
 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) -
1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(
1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)
^12)*(1/(c*x) + 1)^8) + 45*b*c^4*d^2*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((
c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/
(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c
^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10
 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^12) - 540*b*c^2*
d*e*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) -
 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(
1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) +
1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6
/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^7) - 90*b*c^4*d^2*(1/(c^2*x^2) - 1)^5*sqrt
(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7
*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x)
+ 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) -
1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x)
+ 1)^11) + 70*b*e^2*(-1/(c^2*x^2) + 1)^(3/2)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1
)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/
(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)
^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(

1/(c*x) + 1)^12)*(1/(c*x) + 1)^3) + 225*a*e^2*(1/(c^2*x^2) - 1)^2/((c^7 + 6
c^7(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x
) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2)
 - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*
(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^4) - 45*a*c^2*d*e*(1/(c
^2*x^2) - 1)^4/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/
(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)
^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5
/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)
^8) + 45*a*c^4*d^2*(1/(c^2*x^2) - 1)^6/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(
c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x
^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6
c^7(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x
) + 1)^12)*(1/(c*x) + 1)^12) - 300*b*e^2*(1/(c^2*x^2) - 1)^3*arccos(1/(c*x)
)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1
)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(
1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1
)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^6) - 90*b*c^
2*d*e*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(
1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^
2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8
 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(
c*x) + 1)^12)*(1/(c*x) + 1)^10) - 156*b*e^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^
2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^
2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6
 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1
/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^5)
 - 330*b*c^2*d*e*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(
1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)
^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^
4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^
2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^9) - 300*a*e^2*(1/(c^2*x^2) -
 1)^3/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2)
 - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c
^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x)
 + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^6) - 90*
a*c^2*d*e*(1/(c^2*x^2) - 1)^5/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)
^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^
3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(
c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12
)*(1/(c*x) + 1)^10) + 225*b*e^2*(1/(c^2*x^2) - 1)^4*arccos(1/(c*x))/((c^7 +
 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c
*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^
2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^
7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^8) + 45*b*c^2*d*e*(1/

(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1) ^12)*(1/(c*x) + 1)^12) - 156*b*e^2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7* (1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^7) - 90*b*c ^2*d*e*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^ 7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^11) + 225*a*e^2*(1/(c^2*x^2) - 1)^4/((c ^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^ 2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^8) + 45*a*c^2*d*e *(1/(c^2*x^2) - 1)^6/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c ^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^12) - 90*b*e^2*(1/(c^2*x^2) - 1)^5*arccos(1/(c*x))/((c^7 + 6*c^7*(1/ (c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/ (1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2* x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^10) - 70*b*e^2*(1/(c^2*x^2) - 1)^4*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2 *x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^9) - 90*a*e^2*(1/(c^2*x^2) - 1)^5/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6 /(1/(c*x) + 1)^12)*(1/(c*x) + 1)^10) + 15*b*e^2*(1/(c^2*x^2) - 1)^6*arccos(1/(c*x))/((c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x ^2) - 1)^2/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 1 5*c^7*(1/(c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c *x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^12) - 30*b*e^2*(1/(c^2*x^2) - 1)^5*sqrt(-1/(c^2*x^2) + 1)/((c^7 + 6*c^7*(1/(c^2* x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2/(1/(c*x) + 1)^4 + 20 *c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(c^2*x^2) - 1)^4/(1/(c *x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^10 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^11) + 15*a*e^2*(1/(c^2*x^2) - 1)^6/(

```
(c^7 + 6*c^7*(1/(c^2*x^2) - 1)/(1/(c*x) + 1)^2 + 15*c^7*(1/(c^2*x^2) - 1)^2
/(1/(c*x) + 1)^4 + 20*c^7*(1/(c^2*x^2) - 1)^3/(1/(c*x) + 1)^6 + 15*c^7*(1/(
c^2*x^2) - 1)^4/(1/(c*x) + 1)^8 + 6*c^7*(1/(c^2*x^2) - 1)^5/(1/(c*x) + 1)^1
0 + c^7*(1/(c^2*x^2) - 1)^6/(1/(c*x) + 1)^12)*(1/(c*x) + 1)^12))*c
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^2 (a+b\sec^{-1}(cx)) \, dx = \int x \left(e\,x^2+d\right)^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

3.89 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 186

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x}\,dx = -\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c} - \frac{1}{2}ibd^2\csc^{-1}(cx)^2 + dex^2(a+b\sec^{-1}(cx)) + \frac{1}{4}e^2x^4(a+b\sec^{-1}(cx)) + bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d^2(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - \frac{1}{2}ibd^2\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

```
[Out] -1/2*I*b*d^2*arccsc(c*x)^2+d*e*x^2*(a+b*arcsec(c*x))+1/4*e^2*x^4*(a+b*arcse
c(c*x))+b*d^2*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-b*d^2*arccsc(
c*x)*ln(1/x)-d^2*(a+b*arcsec(c*x))*ln(1/x)-1/2*I*b*d^2*polylog(2,(I/c/x+(1-
1/c^2/x^2)^(1/2))^2)-1/6*b*e*(6*c^2*d+e)*x*(1-1/c^2/x^2)^(1/2)/c^3-1/12*b*e
^2*x^3*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {5348, 272, 45, 4816, 6874, 464, 270, 2363, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x} dx = -d^2 \log\left(\frac{1}{x}\right)(a+b\sec^{-1}(cx)) + dex^2(a+b\sec^{-1}(cx)) + \frac{1}{4}e^2x^4(a+b\sec^{-1}(cx)) - \frac{be^2x^3\sqrt{1-\frac{1}{c^2x^2}}}{12c} - \frac{bex\sqrt{1-\frac{1}{c^2x^2}}(6c^2d+e)}{6c^3} - \frac{1}{2}ibd^2\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right) - \frac{1}{2}ibd^2\csc^{-1}(cx)^2 + bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - bd^2\log\left(\frac{1}{x}\right)\csc^{-1}(cx)$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x,x]
```

```
[Out] -1/6*(b*e*(6*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x)/c^3 - (b*e^2*Sqrt[1 - 1/(c
^2*x^2)]*x^3)/(12*c) - (I/2)*b*d^2*ArcCsc[c*x]^2 + d*e*x^2*(a + b*ArcSec[c*
x]) + (e^2*x^4*(a + b*ArcSec[c*x]))/4 + b*d^2*ArcCsc[c*x]*Log[1 - E^((2*I)*
ArcCsc[c*x])] - b*d^2*ArcCsc[c*x]*Log[x^(-1)] - d^2*(a + b*ArcSec[c*x])*Log
[x^(-1)] - (I/2)*b*d^2*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4816

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCos[c*x], u, x] + Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(e+dx^2)^2\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{x^5}\,dx, x, \frac{1}{x}\right)\\
&= dex^2\left(a+b\sec^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\sec^{-1}(cx)\right)\\
&\quad -d^2\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-\frac{b\text{Subst}\left(\int \frac{-\frac{e\left(e+4dx^2\right)}{4x^4}+d^2\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= dex^2\left(a+b\sec^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\sec^{-1}(cx)\right)-d^2\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)\\
&\quad -\frac{b\text{Subst}\left(\int\left(-\frac{e(e+4dx^2)}{4x^4\sqrt{1-\frac{x^2}{c^2}}}+\frac{d^2\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\right)dx, x, \frac{1}{x}\right)}{c}\\
&= dex^2\left(a+b\sec^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\sec^{-1}(cx)\right)-d^2\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)\\
&\quad -\frac{(bd^2)\,\text{Subst}\left(\int\frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}+\frac{(be)\text{Subst}\left(\int\frac{e+4dx^2}{x^4\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4c}
\end{aligned}
$$

$$= -\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + dex^2\left(a+b\sec^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\sec^{-1}(cx)\right) - bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d^2\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \left(bd^2\right)\text{Subst}\left(\int\frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right) + \frac{\left(be(6c^2d+e)\right)\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{6c^3}$$

$$= -\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + dex^2\left(a+b\sec^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\sec^{-1}(cx)\right) - bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d^2\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \left(bd^2\right)\text{Subst}\left(\int x\cot(x)\,dx,x,\csc^{-1}(cx)\right)$$

$$= -\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} - \frac{1}{2}ibd^2\csc^{-1}(cx)^2 + dex^2\left(a+b\sec^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\sec^{-1}(cx)\right) - bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d^2\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \left(2ibd^2\right)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx,x,\csc^{-1}(cx)\right)$$

$$= -\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} - \frac{1}{2}ibd^2\csc^{-1}(cx)^2 + dex^2\left(a+b\sec^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\sec^{-1}(cx)\right) + bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d^2\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - \left(bd^2\right)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx,x,\csc^{-1}(cx)\right)$$

$$= -\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} - \frac{1}{2}ibd^2\csc^{-1}(cx)^2 + dex^2\left(a+b\sec^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\sec^{-1}(cx)\right) + bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d^2\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{1}{2}\left(ibd^2\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\csc^{-1}(cx)}\right)$$

$$= -\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} - \frac{1}{2}ibd^2\csc^{-1}(cx)^2 + dex^2(a+b\sec^{-1}(cx)) + \frac{1}{4}e^2x^4(a+b\sec^{-1}(cx)) + bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d^2(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - \frac{1}{2}ibd^2\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x}\,dx = adex^2 + \frac{1}{4}ae^2x^4 - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x(2+c^2x^2)}{12c^3} + \frac{1}{4}be^2x^4\sec^{-1}(cx) + \frac{bdex\left(-\sqrt{1-\frac{1}{c^2x^2}}+cx\sec^{-1}(cx)\right)}{c} + ad^2\log(x) + \frac{1}{2}ibd^2\left(\sec^{-1}(cx)\left(\sec^{-1}(cx)+2i\log\left(1+e^{2i\sec^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)\right)$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 - (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + c^2*x^2))/(
12*c^3) + (b*e^2*x^4*ArcSec[c*x])/4 + (b*d*e*x*(-Sqrt[1 - 1/(c^2*x^2)] + c*
x*ArcSec[c*x]))/c + a*d^2*Log[x] + (I/2)*b*d^2*(ArcSec[c*x]*(ArcSec[c*x] +
(2*I)*Log[1 + E^((2*I)*ArcSec[c*x])]) + PolyLog[2, -E^((2*I)*ArcSec[c*x])])
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.24

method	result
parts	$a\left(\frac{e^2x^4}{4}+de\,x^2+d^2\ln(x)\right)+b\left(\frac{id^2\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(12c^4d\operatorname{arcsec}(cx)x^2+3\operatorname{arcsec}(cx)e\,c^4x^4-12\sqrt{\frac{c^2x^2}{c^2x}}\right.}{}\right.$
derivativedivides	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+\frac{b\left(\frac{ic^4d^2\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(12c^4d\operatorname{arcsec}(cx)x^2+3\operatorname{arcsec}(cx)e\,c^4x^4-12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3d\right.}{12}\right.}{}$
default	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+\frac{b\left(\frac{ic^4d^2\operatorname{arcsec}(cx)^2}{2}+\frac{e\left(12c^4d\operatorname{arcsec}(cx)x^2+3\operatorname{arcsec}(cx)e\,c^4x^4-12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3d\right.}{12}\right.}{}$

```
[In] int((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/4*e^2*x^4+d*e*x^2+d^2*ln(x))+b*(1/2*I*d^2*arcsec(c*x)^2+1/12/c^4*e*(12
*c^4*d*arcsec(c*x)*x^2+3*arcsec(c*x)*e*c^4*x^4-12*((c^2*x^2-1)/c^2/x^2)^(1/
2)*c^3*d*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c^3*x^3-12*I*c^2*d-2*((c^2*x^2-1)/
c^2/x^2)^(1/2)*e*c*x-2*I*e)-d^2*arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/
2))^2)+1/2*I*d^2*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x}\,dx=\int \frac{(ex^2+d)^2(b\operatorname{arcsec}(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arcsec(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2\,(a+b\sec^{-1}(cx))}{x}\,dx=\int \frac{(a+b\operatorname{asec}(cx))\,(d+ex^2)^2}{x}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*asec(c*x))/x,x)
```

```
[Out] Integral((a + b*asec(c*x))*(d + e*x**2)**2/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) - 1/8*(-2*I*b*c^4*e^2*x^4*log(c) -
 4*I*b*c^4*d^2*log(-c*x + 1)*log(x) - 4*I*b*c^4*d^2*log(x)^2 - 4*I*b*c^4*d^
2*dilog(c*x) - 4*I*b*c^4*d^2*dilog(-c*x) + I*(b*e^2*(x^2/c^2 + log(c*x + 1)
/c^4 + log(c*x - 1)/c^4) + 4*b*d*e*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) +
32*b*d^2*integrate(1/4*log(x)/(c^2*x^3 - x), x))*c^4 + 8*c^4*integrate(1/4*
(b*e^2*x^4 + 4*b*d*e*x^2 + 4*b*d^2*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2
*x^3 - x), x) + (-8*I*b*c^4*d*e*log(c) - I*b*c^2*e^2)*x^2 - 2*(b*c^4*e^2*x^
4 + 4*b*c^4*d*e*x^2 + 4*b*c^4*d^2*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1
)) + (I*b*c^4*e^2*x^4 + 4*I*b*c^4*d*e*x^2 + 4*I*b*c^4*d^2*log(x))*log(c^2*x
^2) + (-4*I*b*c^4*d^2*log(x) - 4*I*b*c^2*d*e - I*b*e^2)*log(c*x + 1) + (-4*
I*b*c^2*d*e - I*b*e^2)*log(c*x - 1) - 2*(I*b*c^4*e^2*x^4 + 4*I*b*c^4*d*e*x^
2 + 4*I*b*c^4*d^2*log(c))*log(x))/c^4
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expansi
on Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x, x)
```

3.90 $\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 189

$$\int \frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^3}\,dx = \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\csc^{-1}(cx) - ibde\csc^{-1}(cx)^2 - \frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\sec^{-1}(cx)) + 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - ibde\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

```
[Out] -1/4*b*c^2*d^2*arccsc(c*x)-I*b*d*e*arccsc(c*x)^2-1/2*d^2*(a+b*arcsec(c*x))/
x^2+1/2*e^2*x^2*(a+b*arcsec(c*x))+2*b*d*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/
x^2)^(1/2))^2)-2*b*d*e*arccsc(c*x)*ln(1/x)-2*d*e*(a+b*arcsec(c*x))*ln(1/x)-
I*b*d*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+1/4*b*c*d^2*(1-1/c^2/x^2)^
(1/2)/x-1/2*b*e^2*x*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5348, 272, 45, 4816, 12, 6874, 270, 327, 222, 2363, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3}\,dx = -\frac{d^2(a+b\sec^{-1}(cx))}{2x^2} - 2de\log\left(\frac{1}{x}\right)(a+b\sec^{-1}(cx)) + \frac{1}{2}e^2x^2(a+b\sec^{-1}(cx)) + \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d^2\csc^{-1}(cx) - \frac{be^2x\sqrt{1-\frac{1}{c^2x^2}}}{2c} - ibde\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right) - ibde\csc^{-1}(cx)^2 + 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - 2bde\log\left(\frac{1}{x}\right)\csc^{-1}(cx)$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^3,x]
```

```
[Out] (b*c*d^2*Sqrt[1 - 1/(c^2*x^2)])/(4*x) - (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*
c) - (b*c^2*d^2*ArcCsc[c*x])/4 - I*b*d*e*ArcCsc[c*x]^2 - (d^2*(a + b*ArcSec
[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcSec[c*x]))/2 + 2*b*d*e*ArcCsc[c*x]*Log
[1 - E^((2*I)*ArcCsc[c*x])] - 2*b*d*e*ArcCsc[c*x]*Log[x^(-1)] - 2*d*e*(a +
b*ArcSec[c*x])*Log[x^(-1)] - I*b*d*e*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4816

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCos[c*x], u, x] + Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{\left(e+dx^2\right)^2 \left(a+b\arccos\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\sec^{-1}(cx)\right) - 2de\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-\frac{b\text{Subst}\left(\int \frac{-\frac{e^2}{x^2}+d^2x^2+4de\log(x)}{2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{d^2\left(a+b\sec^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\sec^{-1}(cx)\right) - 2de\left(a+b\sec^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-\frac{b\text{Subst}\left(\int \frac{-\frac{e^2}{x^2}+d^2x^2+4de\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\big(a+b\sec^{-1}(cx)\big) - 2de\big(a+b\sec^{-1}(cx)\big)\log\left(\frac{1}{x}\right) - \frac{b\text{Subst}\left(\int\left(-\frac{e^2}{x^2\sqrt{1-\frac{x^2}{c^2}}} + \frac{d^2x^2}{\sqrt{1-\frac{x^2}{c^2}}} + \frac{4de\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\right)dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\big(a+b\sec^{-1}(cx)\big) - 2de\big(a+b\sec^{-1}(cx)\big)\log\left(\frac{1}{x}\right) - \frac{(bd^2)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{2c} - \frac{(2bde)\text{Subst}\left(\int\frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{c} + \frac{(be^2)\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{2c}$$

$$= \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\big(a+b\sec^{-1}(cx)\big) - 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de\big(a+b\sec^{-1}(cx)\big)\log\left(\frac{1}{x}\right) - \frac{1}{4}(bcd^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right) + (2bde)\text{Subst}\left(\int\frac{\arcsin\left(\frac{x}{c}\right)}{x}dx, x, \frac{1}{x}\right)$$

$$= \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\csc^{-1}(cx) - \frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\big(a+b\sec^{-1}(cx)\big) - 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de\big(a+b\sec^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + (2bde)\text{Subst}\left(\int x\cot(x)\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\csc^{-1}(cx) - ibde\csc^{-1}(cx)^2 - \frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\big(a+b\sec^{-1}(cx)\big) - 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de\big(a+b\sec^{-1}(cx)\big)\log\left(\frac{1}{x}\right) - (4ibde)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\csc^{-1}(cx) - ibde\csc^{-1}(cx)^2 - \frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\big(a+b\sec^{-1}(cx)\big) + 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de\big(a+b\sec^{-1}(cx)\big)\log\left(\frac{1}{x}\right) - (2bde)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x}{2c} - \frac{1}{4}bc^2d^2\csc^{-1}(cx) - ibde\csc^{-1}(cx)^2 - \frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\sec^{-1}(cx)) + 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) + (ibde)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)$$

$$= \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x}{2c} - \frac{1}{4}bc^2d^2\csc^{-1}(cx) - ibde\csc^{-1}(cx)^2 - \frac{d^2(a+b\sec^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\sec^{-1}(cx)) + 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de(a+b\sec^{-1}(cx))\log\left(\frac{1}{x}\right) - ibde\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int\frac{(d+ex^2)^2(a+b\sec^{-1}(cx))}{x^3}\,dx$$

$$= \frac{1}{4}\left(-\frac{2ad^2}{x^2} + 2ae^2x^2 - \frac{2bd^2\sec^{-1}(cx)}{x^2} + \frac{2be^2x\left(-\sqrt{1-\frac{1}{c^2x^2}} + cx\sec^{-1}(cx)\right)}{c} + \frac{bd^2\left(-1+c^2x^2+c^2x^2\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)\right)}{c\sqrt{1-\frac{1}{c^2x^2}}x^3} + 8ade\log(x) + 4ibde\left(\sec^{-1}(cx)\left(\sec^{-1}(cx)+2i\log\left(1+e^{2i\sec^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2, -e^{2i\sec^{-1}(cx)}\right)\right)\right)$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSec[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcSec[c*x])/x^2 + (2*b*e^2*x*(-Sq
rt[1 - 1/(c^2*x^2)] + c*x*ArcSec[c*x]))/c + (b*d^2*(-1 + c^2*x^2 + c^2*x^2*
Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]]))/(c*Sqrt[1 - 1/(c^2*x^2)]*x^
3) + 8*a*d*e*Log[x] + (4*I)*b*d*e*(ArcSec[c*x]*(ArcSec[c*x] + (2*I)*Log[1 +
 E^((2*I)*ArcSec[c*x])]) + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/4
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.14

method	result
parts	$a\left(\frac{e^{2} x^{2}}{2}-\frac{d^{2}}{2 x^{2}}+2 d e \ln\left(x\right)\right)+i b d e \,\mathrm{arcsec}\left(c x\right)^{2}+\frac{b c \,d^{2} \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}{4 x}+\frac{b \,c^{2} d^{2} \,\mathrm{arcsec}\left(c x\right)}{4}-\frac{b \,\mathrm{arcsec}(cx}{2 x^{2}}$
derivativedivides	$c^{2}\left(\frac{a \,x^{2} e^{2}}{2 c^{2}}+\frac{2 a d e \ln\left(c x\right)}{c^{2}}-\frac{a \,d^{2}}{2 c^{2} x^{2}}+\frac{i b d e \,\mathrm{arcsec}\left(c x\right)^{2}}{c^{2}}+\frac{b \,d^{2} \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}{4 c x}+\frac{b \,\mathrm{arcsec}\left(c x\right) d^{2}}{4}-\frac{b \,\mathrm{arcsec}\left(c x\right) d^{2}}{2 c^{2} x^{2}}+\right.$
default	$c^{2}\left(\frac{a \,x^{2} e^{2}}{2 c^{2}}+\frac{2 a d e \ln\left(c x\right)}{c^{2}}-\frac{a \,d^{2}}{2 c^{2} x^{2}}+\frac{i b d e \,\mathrm{arcsec}\left(c x\right)^{2}}{c^{2}}+\frac{b \,d^{2} \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}{4 c x}+\frac{b \,\mathrm{arcsec}\left(c x\right) d^{2}}{4}-\frac{b \,\mathrm{arcsec}\left(c x\right) d^{2}}{2 c^{2} x^{2}}+\right.$

```
[In] int((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x,method=_RETURNVERBOSE)
[Out] a*(1/2*e^2*x^2-1/2*d^2/x^2+2*d*e*ln(x))+I*b*d*e*arcsec(c*x)^2+1/4*b*c*d^2/x
*((c^2*x^2-1)/c^2/x^2)^(1/2)+1/4*b*c^2*d^2*arcsec(c*x)-1/2*b*arcsec(c*x)*d^
2/x^2+1/2*b*e^2*arcsec(c*x)*x^2-1/2*b/c*e^2*((c^2*x^2-1)/c^2/x^2)^(1/2)*x-1
/2*I*b/c^2*e^2-2*b*d*e*arcsec(c*x)*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)+I*
b*d*e*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2 (b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arcsec(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)^2}{x^3}\,dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*asec(c*x))/x**3,x)
[Out] Integral((a + b*asec(c*x))*(d + e*x**2)**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")

[Out] 1/2*a*e^2*x^2 - 1/4*b*d^2*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c + 2*arcsec(c*x)/x^2) + 2*a*d*e*log(x) - 1/2*a*d^2/x^2 - 1/4*(-2*I*b*c^2*e^2*x^2*log(c) - 4*I*b*c^2*d*e*log(-c*x + 1)*log(x) - 4*I*b*c^2*d*e*log(x)^2 - 4*I*b*c^2*d*e*dilog(c*x) - 4*I*b*c^2*d*e*dilog(-c*x) - I*b*e^2*log(c*x - 1) + I*(b*e^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 16*b*d*e*integrate(1/2*log(x)/(c^2*x^3 - x), x))*c^2 + 4*c^2*integrate(1/2*(b*e^2*x^2 + 4*b*d*e*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) - 2*(b*c^2*e^2*x^2 + 4*b*c^2*d*e*log(x))*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) + (I*b*c^2*e^2*x^2 + 4*I*b*c^2*d*e*log(x))*log(c^2*x^2) + (-4*I*b*c^2*d*e*log(x) - I*b*e^2)*log(c*x + 1) - 2*(I*b*c^2*e^2*x^2 + 4*I*b*c^2*d*e*log(c))*log(x))/c^2

Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

[In] integrate((e*x^2+d)^2*(a+b*arcsec(c*x))/x^3,x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2*(b*arcsec(c*x) + a)/x^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

[In] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^3,x)

[Out] int(((d + e*x^2)^2*(a + b*acos(1/(c*x))))/x^3, x)

3.91 $\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 546

$$\begin{aligned}\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx &= \frac{x(a+b\sec^{-1}(cx))}{e}-\frac{b\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}\\
&+\frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}\\
&-\frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}\\
&+\frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}\\
&-\frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}\\
&+\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}\\
&-\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}\\
&+\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}\\
&-\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}\end{aligned}$$

```
[Out] x*(a+b*arcsec(c*x))/e-b*arctanh((1-1/c^2/x^2)^(1/2))/c/e+1/2*(a+b*arcsec(c*
x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
```

```
)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*ar
csec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+
e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*
I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*I*b*polylo
g(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*
(-d)^(1/2)/e^(3/2)-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5348, 4818, 4724, 272, 65, 214, 4758, 4826, 4616, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = {} & \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ & -\frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ & +\frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}} \\ & -\frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}} \\ & +\frac{x(a+b\sec^{-1}(cx))}{e}-\frac{b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} \\ & +\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}} \\ & -\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}} \\ & +\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}} \\ & -\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}\end{aligned}$$

```
[In] Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

```
[Out] (x*(a + b*ArcSec[c*x]))/e - (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e) + (Sqr
t[-d]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSec[c*x])*Log[1 + (c*
Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) + (Sq
rt[-d]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e]
+ Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSec[c*x])*Log[1 + (c
*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) + ((
I/2)*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt
[c^2*d + e]))])/e^(3/2) - ((I/2)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^(I*Arc
Sec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(3/2) + ((I/2)*b*Sqrt[-d]*PolyLo
g[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/e^(3/2
) - ((I/2)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] +
Sqrt[c^2*d + e])])/e^(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
```

```
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)}\,dx,x,\frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{ex^2}-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x^2}\,dx,x,\frac{1}{x}\right)}{e}+\frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)}{e}\\
&= \frac{x\left(a+b\sec^{-1}(cx)\right)}{e}+\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{ce}\\
&\quad+\frac{d\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx,x,\frac{1}{x}\right)}{e}\\
&= \frac{x\left(a+b\sec^{-1}(cx)\right)}{e}+\frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{2e^{3/2}}\\
&\quad+\frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{2e^{3/2}}+\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2ce}\\
&= \frac{x\left(a+b\sec^{-1}(cx)\right)}{e}-\frac{d\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx,x,\sec^{-1}(cx)\right)}{2e^{3/2}}\\
&\quad-\frac{d\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx,x,\sec^{-1}(cx)\right)}{2e^{3/2}}\\
&\quad-\frac{(bc)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}
\end{aligned}$$

$$= \frac{x(a+b\sec^{-1}(cx))}{e} - \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} + \frac{(id)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^{3/2}} + \frac{(id)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^{3/2}} + \frac{(id)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^{3/2}} + \frac{(id)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^{3/2}}$$

$$= \frac{x(a+b\sec^{-1}(cx))}{e} - \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2e^{3/2}} + \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2e^{3/2}} - \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2e^{3/2}} + \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2e^{3/2}}$$

$$= \frac{x(a+b\sec^{-1}(cx))}{e} - \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{\left(ib\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e^{3/2}} - \frac{\left(ib\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e^{3/2}} + \frac{\left(ib\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e^{3/2}} - \frac{\left(ib\sqrt{-d}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e^{3/2}}$$

$$= \frac{x(a+b\sec^{-1}(cx))}{e} - \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 1.88 (sec) , antiderivative size = 1023, normalized size of antiderivative = 1.87

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \frac{ax}{e} - \frac{a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} + b\left(\frac{cx\sec^{-1}(cx)+\log\left(\cos\left(\frac{1}{2}\sec^{-1}(cx)\right)-\sin\left(\frac{1}{2}\sec^{-1}(cx)\right)\right)-\log\left(\cos\left(\frac{1}{2}\sec^{-1}(cx)\right)+\sin\left(\frac{1}{2}\sec^{-1}(cx)\right)\right.}{ce}\right. - \frac{\sqrt{d}\left(8\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-2i\sec^{-1}(cx)\log\left(1+\frac{i\left(\sqrt{e}-\sqrt{c^2d+e}\right)e^{i\sec^{-1}(cx}}{c\sqrt{d}}\right.\right.}{} + \frac{\sqrt{d}\left(8\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-2i\sec^{-1}(cx)\log\left(1+\frac{i\left(-\sqrt{e}+\sqrt{c^2d+e}\right)e^{i\sec^{-1}}}{c\sqrt{d}}\right.\right.}{}$$

```
[In] Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

```
[Out] (a*x)/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + b*((c*x*ArcSec[
c*x] + Log[Cos[ArcSec[c*x]/2] - Sin[ArcSec[c*x]/2]] - Log[Cos[ArcSec[c*x]/2
] + Sin[ArcSec[c*x]/2]])/(c*e) - (Sqrt[d]*(8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqr
t[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^
(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*S
qrt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*Ar
cSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/S
qrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d
])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*PolyLog[2, (I*(-
Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*PolyLog[2, (
(-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + PolyLog[
2, -E^((2*I)*ArcSec[c*x])]))/(4*e^(3/2)) + (Sqrt[d]*(8*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec
[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 - (I*Sqrt
[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*Arc
Sec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*
d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c
*x]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*P
olyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])
] - 2*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[
d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/(4*e^(3/2)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 34.29 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.68

method	result
parts	$\frac{ax}{e}-\frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+\frac{b\,\mathrm{arcsec}(cx)x}{e}+\frac{2ib\arctan\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}-\frac{ibcd\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_\right.}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^3x}{e}-\frac{a\,c^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+b\,c^2\left(\frac{\mathrm{arcsec}(cx)cx}{e}+\frac{2i\arctan\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{ic^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c\right.}\right.}{}\right.}{}$
default	$\frac{\frac{a\,c^3x}{e}-\frac{a\,c^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+b\,c^2\left(\frac{\mathrm{arcsec}(cx)cx}{e}+\frac{2i\arctan\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{ic^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c\right.}\right.}{}\right.}{}$

```
[In] int(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/e*x-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*arcsec(c*x)/e*x+2*I*b/c/e
*arctan(1/c/x+I*(1-1/c^2/x^2)^(1/2))-1/8*I*b*c/e^2*d*sum((_R1^2*c^2*d+4*_R1
^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1
/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/8*I*b*c/e^2*d*sum((_R1^2*c^2*d+
c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/
c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Root
Of(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{ex^2+d}\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsec(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x^2(a+b\,\mathrm{asec}\,(cx))}{d+ex^2}\,dx$$

```
[In] integrate(x**2*(a+b*asec(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*asec(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{d+ex^2}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

```
[In] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2),x)
[Out] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2), x)
```

3.92 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 487

$$\begin{aligned}\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx &= \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e} \\ &- \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}\end{aligned}$$

```
[Out] -(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e+1/2*(a+b*arcsec(
c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
```

```
/2)))/e+1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e+1/2*(a+b*arcsec(c*x
))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)
))/e+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e-1/2*I*b*polylog(
2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e-
1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*
d+e)^(1/2)))/e-1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5348, 4818, 4722, 3800, 2221, 2317, 2438, 4826, 4616}

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e} - \frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))}{e} - \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e} - \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}$$

```
[In] Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt
[c^2*d + e])])/(2*e) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec
```

```
[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSec[c*x])*Log[1 -
(c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) + ((a +
b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d
 + e])])/(2*e) - ((a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcSec[c*x])])/e - (
(I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
 e]))])/e - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - S
qrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/
(Sqrt[e] + Sqrt[c^2*d + e]))])/e - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*Arc
Sec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e + ((I/2)*b*PolyLog[2, -E^((2*I)*
ArcSec[c*x])])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x\,(e+dx^2)}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{ex}-\frac{dx\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e\,(e+dx^2)}\right)dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e}+\frac{d\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}\\
&= \frac{\text{Subst}\left(\int (a+bx)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{e}\\
&\quad+\frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e}\\
&= \frac{i(a+b\sec^{-1}(cx))^2}{2be}-\frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}\,dx, x, \sec^{-1}(cx)\right)}{e}\\
&\quad-\frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e}+\frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e}
\end{aligned}
$$

$$= \frac{i(a+b\sec^{-1}(cx))^2}{2be} - \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e} + \frac{b\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{e} + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2e} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2e}$$

$$= -\frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\sec^{-1}(cx)}\right)}{2e} - \frac{\left(i\sqrt{-d}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e} - \frac{\left(i\sqrt{-d}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e} + \frac{\left(i\sqrt{-d}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e} + \frac{\left(i\sqrt{-d}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e}$$

$$
\begin{aligned}
&= \frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&- \frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e} \\
&- \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e} \\
&- \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e} \\
&- \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e} \\
&- \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&\quad + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&\quad + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&\quad + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&\quad - \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e} \\
&\quad + \frac{(ib)\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e} \\
&\quad + \frac{(ib)\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e} \\
&\quad + \frac{(ib)\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e} \\
&\quad + \frac{(ib)\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e}
\end{aligned}
$$

$$= \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} - \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.83

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx$$

$$= \frac{4ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)+4ib\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec\right.}{\sqrt{c^2d+e}}\right.}{}$$

```
[In] Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

```
[Out] ((4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*
Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + (4*I)*b*ArcSin[Sq
rt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Ta
n[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sq
rt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 2*b*ArcSin[Sqrt[1 + (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*Arc
Sec[c*x]))/(c*Sqrt[d])] + b*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d +
 e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x])
)/(c*Sqrt[d])] + b*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*
ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/S
qrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d
])] + b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]
```

```
))/(c*Sqrt[d])] - 2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*A
rcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] + a*Log[d + e*x^2] - I*b*PolyLog[
2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - I*b
*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]
 - I*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*S
qrt[d])] - I*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))
/(c*Sqrt[d])] + I*b*PolyLog[2, -E^((2*I)*ArcSec[c*x])])/(2*e)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.71 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.92

method	result
parts	$\frac{a\ln(e\,x^2+d)}{2e}-\frac{i b c^2 d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2 d\,\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\left(\_R1^2+1\right)\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-i\sqrt{1-\frac{1}{c^2 \ldots}}}{\_R1}\right.\right.}{\_R1^2c^2d+c^2d \ldots}\right.}{4e}$
derivativedivides	$\frac{a\,c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}+b\,c^2\left(-\frac{\mathrm{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}-\frac{\mathrm{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}+\frac{i\,\mathrm{dilog}\left(1+i\left(\frac{1}{cx}+\ldots\right.\right.}{e}\right.$
default	$\frac{a\,c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}+b\,c^2\left(-\frac{\mathrm{arcsec}(cx)\ln\left(1+i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}-\frac{\mathrm{arcsec}(cx)\ln\left(1-i\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}+\frac{i\,\mathrm{dilog}\left(1+i\left(\frac{1}{cx}+\ldots\right.\right.}{e}\right.$

```
[In] int(x*(a+b*arcsec(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/e*ln(e*x^2+d)-1/4*I*b*c^2*d/e*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(
I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*
(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
-b/e*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-b/e*arcsec(c*x)*ln(1
```

```
-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I*b/e*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1
/2)))+I*b/e*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/4*I*b/e*sum((_R1^2*c
^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1
/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{ex^2+d}\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x*arcsec(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx=\int \frac{x(a+b\,\mathrm{asec}\,(cx))}{d+ex^2}\,dx$$

```
[In] integrate(x*(a+b*asec(c*x))/(e*x**2+d),x)
[Out] Integral(x*(a + b*asec(c*x))/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{ex^2+d}\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] b*integrate(x*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + 1/2*a*l
og(e*x^2 + d)/e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

```
[In] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2), x)
```

3.93 $\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 509

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx =& \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+ \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+ \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[Out] 1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/
x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e
^(1/2)+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsec(c*x))*ln(1+
c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(1/2)+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,c*(1/c/x+I
*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1
```

```
/2)+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5338, 4758, 4826, 4616, 2221, 2317, 2438}

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx = \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt
[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[
-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) +
 ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqr
t[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt
[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e])
+ ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*
d + e]))])/(Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec
[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog
[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/(Sqrt[-
d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] +
 Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5338

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{e}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{e}}$$

$$= -\frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{e}}$$

$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{e}}$$

$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{e}}$$

$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{e}}$$

$$= \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$+\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$-\frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}}$$

$$+\frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}}$$

$$-\frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}}$$

$$+\frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$
\begin{aligned}
&= \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.71

$$
\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx
$$

$$
= \frac{2a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - 4b\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right) + 4b\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan}{}
$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x^2),x]
```

```
[Out] (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[
d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c
^2*d + e]] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(
(I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - I*b*ArcSec[c
*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]
- (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqr
t[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*L
og[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2
*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e
] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*Log[
1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*
b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + S
qrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - I*b*ArcSec[c*x]*Log[1 + (
```

```
I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*b*Arc
Sin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c
^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + b*PolyLog[2, ((-I)*(-Sqrt[e] +
 Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - b*PolyLog[2, (I*(-Sqrt[
e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - b*PolyLog[2, ((-I)*
(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + b*PolyLog[2,
(I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/(2*Sqrt[d]*
Sqrt[e])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.53

method	result
parts	$\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{ibc\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}d\_Z^{4}+\left(2c^{2}d+4e\right)\_Z^{2}+c^{2}d\right)}\frac{\_R1\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-i\sqrt{1-\frac{1}{c^{2}x^{2}}}}{\_R1}\right)+d\right.}{\_R1^{2}c^{2}d+c^{2}d+2e}\right.}{2}$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^{2}\left(\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}d\_Z^{4}+\left(2c^{2}d+4e\right)\_Z^{2}+c^{2}d\right)}\frac{\_R1\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-i\sqrt{1-\frac{1}{c^{2}x^{2}}}}{\_R1}\right)+\right.}{\_R1^{2}c^{2}d+c^{2}d+}\right.}{2}\right.}{}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^{2}\left(\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}d\_Z^{4}+\left(2c^{2}d+4e\right)\_Z^{2}+c^{2}d\right)}\frac{\_R1\left(i\,\mathrm{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-i\sqrt{1-\frac{1}{c^{2}x^{2}}}}{\_R1}\right)+\right.}{\_R1^{2}c^{2}d+c^{2}d+}\right.}{2}\right.}{}$

```
[In] int((a+b*arcsec(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*I*b*c*sum(_R1/(_R1^2*c^2*d+c^2*d+
2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/
c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c
^2*d))-1/2*I*b*c*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1
```

```
/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R
1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{ex^2+d}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{d+ex^2}\,dx$$

```
[In] integrate((a+b*asec(c*x))/(e*x**2+d),x)
[Out] Integral((a + b*asec(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{d + ex^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{d + ex^2}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{e\,x^2 + d}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x^2),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x^2), x)
```

3.94 $\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 459

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)}\,dx &= \frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}\\&-\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}\\&-\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\\&-\frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\\&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}\\&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\end{aligned}$$

```
[Out] 1/2*I*(a+b*arcsec(c*x))^2/b/d-1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsec(c*x))
*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/d-1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,-c*(1/
c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d+1/2*I*b*
polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/
2)))/d+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5348, 4818, 4826, 4616, 2221, 2317, 2438}

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)}\,dx = -\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d} + \frac{i(a+b\sec^{-1}(cx))^2}{2bd} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}$$

```
[In] Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] ((I/2)*(a + b*ArcSec[c*x])^2)/(b*d) - ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[
-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSec
[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])
/(2*d) - ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[
e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^
(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) + ((I/2)*b*PolyLog[2,
-((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d + ((I/2)*
b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d
 + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2
*d + e]))])/d + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e]
 + Sqrt[c^2*d + e])])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}} + \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{i(a+b\sec^{-1}(cx))^2}{2bd} - \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{-d}}$$
$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{-d}}$$
$$+ \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{-d}}$$
$$+ \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{i(a+b\sec^{-1}(cx))^2}{2bd} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \sec^{-1}(cx)\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \sec^{-1}(cx)\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \sec^{-1}(cx)\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \sec^{-1}(cx)\right)}{2d}$$

$$= \frac{i(a+b\sec^{-1}(cx))^2}{2bd} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d}$$

$$= \frac{i(a+b\sec^{-1}(cx))^2}{2bd} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.91

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)}\,dx$$
$$= \frac{i\left(b\sec^{-1}(cx)^2-4b\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-4b\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\right.}{}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] ((I/2)*(b*ArcSec[c*x]^2 - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt
[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]
] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt
[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + I*b*ArcSec[c*x]*Log[1
 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*b
*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sq
rt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*Log[1 + (I
*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*b*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*Log[1 - (I*(S
qrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*b*ArcSin[
Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d
 + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + I*b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e
] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*b*ArcSin[Sqrt[
1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]
)*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*a*Log[x] + I*a*Log[d + e*x^2] + b
*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d
])] + b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sq
rt[d])] + b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))
/(c*Sqrt[d])] + b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x
]))/(c*Sqrt[d])]))/d
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.33 (sec) , antiderivative size = 1970, normalized size of antiderivative = 4.29

method	result	size
parts	Expression too large to display	1970
derivativedivides	Expression too large to display	1997
default	Expression too large to display	1997

```
[In] int((a+b*arcsec(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*ln(x)-1/2*a/d*ln(e*x^2+d)+b*(-1/2*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d
*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^
(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/d^3/(c^2*d+e)/c^4+1/2*I/d*sum(
(_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/
c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1
)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/4*I*(e*(c^2*d+e))^(1/
2)/e/(c^2*d+e)*arcsec(c*x)^2*c^2+1/2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*po
lylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)
-2*e))*e/d^3/c^4-1/4*I*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*polylog(2,d*c^2*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/4*(-(e*(c
^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)/e/(c^2*d+e)/d
*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2
*e))*arcsec(c*x)+1/4*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*c^2*arcsec(c*x)*ln(1-d
*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/
2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arcsec(c*x)^2/c^2/d^2+1/4*I*(c^2*d-2*
(e*(c^2*d+e))^(1/2)+2*e)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-
c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/c^2/d^2+(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2
*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)/d^3*e/(c^2*d+e)/c^4*ln(1-d*c^2*(1/c/x+I
*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-1/2
*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*pol
ylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-
2*e))/(c^2*d+e)/c^2/d^2-1/8*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c
^2*d+e))^(1/2)*e+2*e^2)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c
^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/d/e/(c^2*d+e)-1/2*(c^2*d-2*(e*(c^2*d+e))^(
1/2)+2*e)/c^2/d^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(
c^2*d+e))^(1/2)-2*e))*arcsec(c*x)+1/2*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arcse
c(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(
1/2)-2*e))-I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+
2*e^2)*arcsec(c*x)^2/(c^2*d+e)/c^2/d^2+I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*
arcsec(c*x)^2*e/d^3/c^4-I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d
+e))^(1/2)*e+2*e^2)*e*arcsec(c*x)^2/d^3/(c^2*d+e)/c^4+1/2*I/d*arcsec(c*x)^2
-1/8*I*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^
2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))*c^2-1/4*I*(-(e*(c^2*d+e))^(
1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arcsec(c*x)^2/d/e/(c^2*
d+e)+(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)/(
c^2*d+e)/c^2/d^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c
^2*d+e))^(1/2)-2*e))*arcsec(c*x)-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^3/c^4*
e*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-
2*e))*arcsec(c*x)-1/2*I*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arcsec(c*x)^2)
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)}\,dx = \int \frac{a+b\,\mathrm{asec}\,(cx)}{x\,(d+ex^2)}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x/(e*x**2+d),x)
[Out] Integral((a + b*asec(c*x))/(x*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d),x, algorithm="maxima")
[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(arctan(sqrt(c*x + 1)*s
qrt(c*x - 1))/(e*x^3 + d*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{x(d + ex^2)}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2 + d\right)}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)), x)
```

3.95 $\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 551

$$\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)}\,dx &= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}-\frac{a}{dx}-\frac{b\sec^{-1}(cx)}{dx} \\
&+\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&-\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&+\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&-\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&+\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&-\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&+\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
&-\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}
\end{aligned}$$

```
[Out] -a/d/x-b*arcsec(c*x)/d/x+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*
arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*
d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/
2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2
*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*polyl
og(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*
e^(1/2)/(-d)^(3/2)+b*c*(1-1/c^2/x^2)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5348, 4818, 4716, 267, 4758, 4826, 4616, 2221, 2317, 2438}

$$
\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)}\,dx = {} & \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
& -\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\
& +\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}} \\
& -\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}} \\
& -\frac{a}{dx}+\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}} \\
& -\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}} \\
& +\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}} \\
& -\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}-\frac{b\sec^{-1}(cx)}{dx}
\end{aligned}
$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] (b*c*Sqrt[1 - 1/(c^2*x^2)])/d - a/(d*x) - (b*ArcSec[c*x])/(d*x) + (Sqrt[e]*
(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[
c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt
[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqr
t[e]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] +
Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSec[c*x])*Log[1 + (c
*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) +
 ((I/2)*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sq
rt[c^2*d + e]))])/(-d)^(3/2) - ((I/2)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^(I
*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*
PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/
(-d)^(3/2) - ((I/2)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sq
rt[e] + Sqrt[c^2*d + e])])/(-d)^(3/2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
```

```
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{d} - \frac{e\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \left(a+b\arccos\left(\frac{x}{c}\right)\right) dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{a}{dx} - \frac{b\text{Subst}\left(\int \arccos\left(\frac{x}{c}\right) dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{a}{dx} - \frac{b\sec^{-1}(cx)}{dx} - \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{cd} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x} dx, x, \frac{1}{x}\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x} dx, x, \frac{1}{x}\right)}{2d}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\sec^{-1}(cx)}{dx} - \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)} dx, x, \sec^{-1}(cx)\right)}{2d} - \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)} dx, x, \sec^{-1}(cx)\right)}{2d}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\sec^{-1}(cx)}{dx} + \frac{\left(i\sqrt{e}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}} dx, x, \sec^{-1}(cx)\right)}{2d} + \frac{\left(i\sqrt{e}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}} dx, x, \sec^{-1}(cx)\right)}{2d} + \frac{\left(i\sqrt{e}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}} dx, x, \sec^{-1}(cx)\right)}{2d} + \frac{\left(i\sqrt{e}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}} dx, x, \sec^{-1}(cx)\right)}{2d}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\sec^{-1}(cx)}{dx} + \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$- \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$+ \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$- \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$- \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$+ \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$- \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$+ \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$=\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}-\frac{a}{dx}-\frac{b\sec^{-1}(cx)}{dx}+\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$-\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$+\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$-\frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$+\frac{\left(ib\sqrt{e}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2(-d)^{3/2}}$$

$$-\frac{\left(ib\sqrt{e}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2(-d)^{3/2}}$$

$$+\frac{\left(ib\sqrt{e}\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2(-d)^{3/2}}$$

$$-\frac{\left(ib\sqrt{e}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2(-d)^{3/2}}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\sec^{-1}(cx)}{dx} + \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$- \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$+ \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$- \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$+ \frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$+ \frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 997, normalized size of antiderivative = 1.81

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)}\,dx = -\frac{a}{dx} - \frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + b\left(\frac{c\sqrt{1-\frac{1}{c^2x^2}} - \frac{\sec^{-1}(cx)}{x}}{d}\right.$$
$$- \frac{\sqrt{e}\left(8\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right) - 2i\sec^{-1}(cx)\log\left(1+\frac{i\left(\sqrt{e}-\sqrt{c^2d+e}\right)e^{i\sec^{-1}(cx)}}{c\sqrt{d}}\right.\right.}{}$$
$$+ \frac{\sqrt{e}\left(8\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right) - 2i\sec^{-1}(cx)\log\left(1+\frac{i\left(-\sqrt{e}+\sqrt{c^2d+e}\right)e^{i\sec^{-1}}}{c\sqrt{d}}\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*((c*Sqrt[1
 - 1/(c^2*x^2)] - ArcSec[c*x]/x)/d - (Sqrt[e]*(8*ArcSin[Sqrt[1 + (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])
/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]
```

```
)*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/
(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d]
)]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sq
rt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*PolyLog[2, (
I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*PolyLog[
2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + Poly
Log[2, -E^((2*I)*ArcSec[c*x])]))/(4*d^(3/2)) + (Sqrt[e]*(8*ArcSin[Sqrt[1 -
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[Ar
cSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + S
qrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 - (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I
*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 - (I*Sqrt
[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcS
ec[c*x]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] -
 2*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt
[d])] - 2*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*S
qrt[d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/(4*d^(3/2)))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.44 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.60

method	result
parts	$-\frac{a}{dx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}+\frac{bc\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\text{arcsec}(cx)}{dx}-\frac{ibce\left(\sum_{\_R1=\text{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1}{}\right.}{}$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}+\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\text{arcsec}(cx)}{dcx}-\frac{ibe\left(\sum_{\_R1=\text{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\right.}{}\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}+\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\text{arcsec}(cx)}{dcx}-\frac{ibe\left(\sum_{\_R1=\text{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\right.}{}\right.$

```
[In] int((a+b*arcsec(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x-a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*c/d*((c^2*x^2-1)/c^2/x^2
)^(1/2)-b*arcsec(c*x)/d/x-1/2*I*b*c*e/d*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*
arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1
-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1
/2*I*b*c*e/d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x
-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),
_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{b\,\text{arcsec}\left(cx\right)+a}{(ex^2+d)x^2}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsec(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x^2\,(d + ex^2)}\,dx = \int \frac{a + b\,\mathrm{asec}\,(cx)}{x^2\,(d + ex^2)}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asec(c*x))/(x**2*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{x^2\,(d + ex^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{x^2\,(d + ex^2)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \sec^{-1}(cx)}{x^2 (d + ex^2)} \, dx = \int \frac{a + b \operatorname{acos}\left(\frac{1}{c x}\right)}{x^2 \left(e x^2 + d\right)} \, dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

3.96 $\int \frac{x^5 (a+b \sec^{-1}(cx))}{(d+ex^2)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 608

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2}+\frac{d(a+b\sec^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2(a+b\sec^{-1}(cx))}{2e^2}+\frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}}-\frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}-\frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}+\frac{2d(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3}+\frac{ibd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}+\frac{ibd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}+\frac{ibd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}+\frac{ibd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}-\frac{ibd\operatorname{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{e^3}$$

```
[Out] 1/2*d*(a+b*arcsec(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arcsec(c*x))/e^2+2*d*(a+
b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e^3-d*(a+b*arcsec(c*x)
)*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2))
)/e^3-d*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(
e^(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-d*(a+b*arcsec(c*x))*l
n(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
^3-I*b*d*polylog(2,-(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2)/e^3+I*b*d*polylog(2,-c
```

```
*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+I*
b*d*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)
^(1/2)))/e^3+I*b*d*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/e^3+I*b*d*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*b*d*arctan((c^2*d+e)^(1/2)/
c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^(5/2)/(c^2*d+e)^(1/2)-1/2*b*x*(1-1/c^2/x
^2)^(1/2)/c/e^2
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5348, 4818, 4724, 270, 4722, 3800, 2221, 2317, 2438, 4814, 385, 211, 4826, 4616}

$$\begin{aligned}\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = & -\frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} \\ & -\frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} \\ & -\frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3} \\ & -\frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3} \\ & +\frac{2d\log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))}{e^3} \\ & +\frac{d(a+b\sec^{-1}(cx))}{2e^2\left(\frac{d}{x^2}+e\right)}+\frac{x^2(a+b\sec^{-1}(cx))}{2e^2} \\ & +\frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}}+\frac{ibd\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3} \\ & +\frac{ibd\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3} \\ & +\frac{ibd\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3} \\ & +\frac{ibd\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3} \\ & -\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2ce^2}-\frac{ibd\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{e^3}\end{aligned}$$

```
[In] Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(b*Sqrt[1 - 1/(c^2*x^2)]*x)/(c*e^2) + (d*(a + b*ArcSec[c*x]))/(2*e^2*(
e + d/x^2)) + (x^2*(a + b*ArcSec[c*x]))/(2*e^2) + (b*d*ArcTan[Sqrt[c^2*d +
e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]) - (d*(
a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c
^2*d + e])])/e^3 - (d*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c
*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcSec[c*x])*Log[1 - (c
*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (d*(a + b*
ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d +
 e])])/e^3 + (2*d*(a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcSec[c*x])])/e^3 +
 (I*b*d*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
 e]))])/e^3 + (I*b*d*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - S
qrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/
(Sqrt[e] + Sqrt[c^2*d + e]))])/e^3 + (I*b*d*PolyLog[2, (c*Sqrt[-d]*E^(I*Arc
Sec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (I*b*d*PolyLog[2, -E^((2*I)*
ArcSec[c*x])])/e^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4814

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])/(2*e*(p + 1))), x]
 + Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
```

```
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x^3\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{e^2x^3}-\frac{2d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e^3x}+\frac{d^2x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2}\right.\right.\\
&\qquad\left.\left.+\frac{2d^2x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e^3\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)\\
&= \frac{(2d)\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3}-\frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3}\\
&\quad-\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x^3}\,dx, x, \frac{1}{x}\right)}{e^2}-\frac{d^2\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e^2}\\
&= \frac{d\left(a+b\sec^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\left(a+b\sec^{-1}(cx)\right)}{2e^2}\\
&\quad-\frac{(2d)\text{Subst}\left(\int (a+bx)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{e^3}\\
&\quad-\frac{\left(2d^2\right)\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e^3}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce^2}+\frac{(bd)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{2ce^2}
\end{aligned}$$

$$= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\sec^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\sec^{-1}(cx))}{2e^2} - \frac{id(a+b\sec^{-1}(cx))^2}{be^3} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{(4id)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}\,dx, x, \sec^{-1}(cx)\right)}{e^3} + \frac{(bd)\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2ce^2}$$

$$= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\sec^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\sec^{-1}(cx))}{2e^2} - \frac{id(a+b\sec^{-1}(cx))^2}{be^3} + \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} + \frac{2d(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} + \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{e^3} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{e^3} - \frac{(2bd)\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \sec^{-1}(cx)\right)}{e^3}$$

$$= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\sec^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\sec^{-1}(cx))}{2e^2} + \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} + \frac{2d(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} - \frac{\left(i(-d)^{3/2}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{e^3} - \frac{\left(i(-d)^{3/2}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{e^3} + \frac{\left(i(-d)^{3/2}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{e^3} + \frac{\left(i(-d)^{3/2}\right)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{e^3} + \frac{(ibd)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\sec^{-1}(cx)}\right)}{e^3}$$

$$= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\sec^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\sec^{-1}(cx))}{2e^2}$$

$$+ \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

$$+ \frac{2d(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} - \frac{ibd\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{e^3}$$

$$+ \frac{(bd)\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{e^3}$$

$$+ \frac{(bd)\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{e^3}$$

$$+ \frac{(bd)\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{e^3}$$

$$+ \frac{(bd)\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{e^3}$$

$$= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\sec^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\sec^{-1}(cx))}{2e^2}$$

$$+ \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

$$+ \frac{2d(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} - \frac{ibd\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{e^3}$$

$$- \frac{(ibd)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{e^3}$$

$$- \frac{(ibd)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{e^3}$$

$$- \frac{(ibd)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{e^3}$$

$$- \frac{(ibd)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{e^3}$$

$$= -\frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\sec^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\sec^{-1}(cx))}{2e^2} + \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} + \frac{2d(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} + \frac{ibd\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{ibd\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{e^3}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1255 vs. 2(608) = 1216.

Time = 3.18 (sec) , antiderivative size = 1255, normalized size of antiderivative = 2.06

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx =$$

$$-\frac{-2aex^2 + \frac{2ad^2}{d+ex^2} + 4ad\log(d+ex^2) + b\left(\frac{2e\sqrt{1-\frac{1}{c^2x^2}}x}{c} - 2ex^2\sec^{-1}(cx) + \frac{d^{3/2}\sec^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} + \frac{d^{3/2}\sec^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} + 2d\,\cdots\right.}{\cdots}$$

```
[In] Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*((2*e*S
qrt[1 - 1/(c^2*x^2)]*x)/c - 2*e*x^2*ArcSec[c*x] + (d^(3/2)*ArcSec[c*x])/(Sq
rt[d] - I*Sqrt[e]*x) + (d^(3/2)*ArcSec[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + 2*d*
ArcSin[1/(c*x)] + (16*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]
]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] +
```

```
(16*I)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sq
rt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + 4*d*ArcSec[c*x]*Log
[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 8*d*A
rcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt
[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*d*ArcSec[c*x]*Log[1 + (I*(
-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 8*d*ArcSin[Sq
rt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d
+ e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*d*ArcSec[c*x]*Log[1 - (I*(Sqrt[e]
 + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 8*d*ArcSin[Sqrt[1 - (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*d*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c
^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 8*d*ArcSin[Sqrt[1 + (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[
c*x]))/(c*Sqrt[d])] - 8*d*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - (d*S
qrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e
]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])
/Sqrt[-(c^2*d) - e] - (d*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*
Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]
*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] - (4*I)*d*PolyLog[2, ((-I)*(
-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*d*PolyL
og[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*
I)*d*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqr
t[d])] - (4*I)*d*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]
))/(c*Sqrt[d])] + (4*I)*d*PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/e^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.59 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.15

method	result
parts	$\frac{a\,x^2}{2e^2}-\frac{a\,d^2}{2e^3(e\,x^2+d)}-\frac{ad\ln(e\,x^2+d)}{e^3}+\frac{b\left(\frac{c^4\left(2c^4d\,\operatorname{arcsec}(cx)x^2+\operatorname{arcsec}(cx)e\,c^4x^4-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c^3x^3-ic^2\right.}{2\left(c^2e\,x^2+c^2d\right)e^2}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^6x^2}{2e^2}-\frac{a\,c^8d^2}{2e^3\left(c^2e\,x^2+c^2d\right)}-\frac{a\,c^6d\ln\left(c^2e\,x^2+c^2d\right)}{e^3}+b\,c^4\left(\frac{2c^4d\,\operatorname{arcsec}(cx)x^2+\operatorname{arcsec}(cx)e\,c^4x^4-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c^3x^3-}{2\left(c^2e\,x^2+c^2d\right)e^2}\right.}{}$
default	$\frac{\frac{a\,c^6x^2}{2e^2}-\frac{a\,c^8d^2}{2e^3\left(c^2e\,x^2+c^2d\right)}-\frac{a\,c^6d\ln\left(c^2e\,x^2+c^2d\right)}{e^3}+b\,c^4\left(\frac{2c^4d\,\operatorname{arcsec}(cx)x^2+\operatorname{arcsec}(cx)e\,c^4x^4-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx-\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c^3x^3-}{2\left(c^2e\,x^2+c^2d\right)e^2}\right.}{}$

```
[In] int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2/e^2-1/2*a*d^2/e^3/(e*x^2+d)-a*d/e^3*ln(e*x^2+d)+b/c^6*(1/2*c^4*(2
*c^4*d*arcsec(c*x)*x^2+arcsec(c*x)*e*c^4*x^4-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^
3*d*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c^3*x^3-I*c^2*d-I*e*c^2*x^2)/(c^2*e*x^2
+c^2*d)/e^2-1/2*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/e^3*arctanh(1/4*(2*c^2*d*(1
/c/x+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))*d*c^6-2*I/e
^3*d*c^6*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+1/2*I/e^3*d^2*c^8*sum((_R
1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)
^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d
*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*I/e^3*d*c^6*sum((_R1^2*c^2*d+c^2*d+4*e
)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2
))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4
+(2*c^2*d+4*e)*_Z^2+c^2*d))-2*I/e^3*d*c^6*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^
(1/2)))+2/e^3*d*c^6*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+2/e^3
*d*c^6*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2))))
```

Fricas [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^5*arcsec(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^5(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**5*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] -1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*integ
rate(x^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x
)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.97 $\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 570

$$
\begin{aligned}
\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx &= -\frac{a+b\sec^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}} \\
&+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} \\
&+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} \\
&+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} \\
&+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} \\
&- \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^2} \\
&- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} \\
&- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} \\
&- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} \\
&- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^2}
\end{aligned}
$$

```
[Out] 1/2*(-a-b*arcsec(c*x))/e/(e+d/x^2)-(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2
/x^2)^(1/2))^2)/e^2+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsec(c*x))*ln(1+c*
(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2
*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*I*b*polylog(2,-(1/c/x+
I*(1-1/c^2/x^2)^(1/2))^2)/e^2-1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,c*(1/c/x+
I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*po
lylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/e^2-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/e^2-1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/
x^2)^(1/2))/e^(3/2)/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules

used = {5348, 4818, 4722, 3800, 2221, 2317, 2438, 4814, 385, 211, 4826, 4616}

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2} - \frac{a+b\sec^{-1}(cx)}{2e\left(\frac{d}{x^2}+e\right)} - \frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))}{e^2} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{3/2}\sqrt{c^2d+e}} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^2}$$

```
[In] Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcSec[c*x])/(e*(e + d/x^2)) - (b*ArcTan[Sqrt[c^2*d + e]/(c*Sqr
t[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(3/2)*Sqrt[c^2*d + e]) + ((a + b*ArcSe
c[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]
)/(2*e^2) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sq
rt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-
d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcSe
c[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(2*e^2) - ((a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcSec[c*x])])/e^2 - ((I/
2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]
))])/e^2 - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sq
rt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))
/(Sqrt[e] + Sqrt[c^2*d + e]))])/e^2 - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*
ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 + ((I/2)*b*PolyLog[2, -E^((
2*I)*ArcSec[c*x])])/e^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 4814

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])/(2*e*(p + 1))), x]
 + Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{e^2x}-\frac{dx\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2}-\frac{dx\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right)dx,x,\frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)}{e^2}+\frac{d\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)}{e^2}\\
&\quad+\frac{d\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)}{e}
\end{aligned}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{\text{Subst}\left(\int (a+bx)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{e^2} + \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e^2} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}(e+dx^2)}dx, x, \frac{1}{x}\right)}{2ce}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2be^2} - \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}dx, x, \sec^{-1}(cx)\right)}{e^2} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}dx, x, \frac{1}{x}\right)}{2e^2} + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}dx, x, \frac{1}{x}\right)}{2e^2} - \frac{b\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2ce}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2be^2} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}} - \frac{\left(a+b\sec^{-1}(cx)\right)\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^2} + \frac{b\text{Subst}\left(\int \log\left(1+e^{2ix}\right)dx, x, \sec^{-1}(cx)\right)}{e^2} + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}dx, x, \sec^{-1}(cx)\right)}{2e^2} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}dx, x, \sec^{-1}(cx)\right)}{2e^2}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$

$$-\frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^2} - \frac{(ib)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{2i\sec^{-1}(cx)}\right)}{2e^2}$$

$$-\frac{\left(i\sqrt{-d}\right)\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^2}$$

$$-\frac{\left(i\sqrt{-d}\right)\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^2}$$

$$+\frac{\left(i\sqrt{-d}\right)\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^2}$$

$$+\frac{\left(i\sqrt{-d}\right)\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^2}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^2} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^2}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2e^2}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2e^2}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2e^2}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2e^2}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^2} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^2}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2e^2}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2e^2}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2e^2}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2e^2}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^2}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1213 vs. 2(570) = 1140.

Time = 1.17 (sec) , antiderivative size = 1213, normalized size of antiderivative = 2.13

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$= \frac{\frac{2ad}{d+ex^2} + \frac{b\sqrt{d}\sec^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} + \frac{b\sqrt{d}\sec^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} + 2b\arcsin\left(\frac{1}{cx}\right) + 8ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right.}{}$$

```
[In] Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (b
*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + 2*b*ArcSin[1/(c*x)] + (8*I)
*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d
] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] + (8*I)*b*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcS
ec[c*x]/2])/Sqrt[c^2*d + e]] + 2*b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c
```

```
^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[
c*x]))/(c*Sqrt[d])] + 2*b*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e
])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/
(c*Sqrt[d])] + 2*b*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*
ArcSec[c*x]))/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/S
qrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d
])] + 2*b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*
x]))/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*L
og[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 4*b
*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - (b*Sqrt[e]*Log[(2*Sqrt[d]*Sqr
t[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*
x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] - (b*
Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) -
 e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))
])/Sqrt[-(c^2*d) - e] + 2*a*Log[d + e*x^2] - (2*I)*b*PolyLog[2, ((-I)*(-Sqr
t[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*b*PolyLog[2
, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*b
*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d]
)] - (2*I)*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(
c*Sqrt[d])] + (2*I)*b*PolyLog[2, -E^((2*I)*ArcSec[c*x])])/(4*e^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.81 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.01

method	result
parts	$\frac{ad}{2e^2(e\,x^2+d)}+\frac{a\ln(e\,x^2+d)}{2e^2}-\frac{b\,c^2x^2\,\mathrm{arcsec}(cx)}{2(c^2e\,x^2+c^2d)e}+\frac{ib\sqrt{e(c^2d+e)}\,\mathrm{arctanh}\left(\frac{2c^2d\left(\frac{1}{cx}+i\sqrt{1-\frac{1}{c^2x^2}}\right)^2+2c^2d+4e}{4\sqrt{c^2de+e^2}}\right)}{2(c^2d+e)e^2}+\frac{ib}{}$
derivativedivides	$\frac{\frac{a\,c^6d}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^4\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}+b\,c^4\left(-\frac{c^2x^2\,\mathrm{arcsec}(cx)}{2\left(c^2e\,x^2+c^2d\right)e}-\frac{ic^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{(\_R}{}\right.}{}\right.}{}$
default	$\frac{\frac{a\,c^6d}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^4\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}+b\,c^4\left(-\frac{c^2x^2\,\mathrm{arcsec}(cx)}{2\left(c^2e\,x^2+c^2d\right)e}-\frac{ic^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{(\_R}{}\right.}{}\right.}{}$

```
[In] int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*d/e^2/(e*x^2+d)+1/2*a/e^2*ln(e*x^2+d)-1/2*b*c^2*x^2*arcsec(c*x)/(c^2*
e*x^2+c^2*d)/e+1/2*I*b*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/e^2*arctanh(1/4*(2*c^2
*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))+I*b/e^
2*dilog(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/4*I*b*c^2/e^2*d*sum((_R1^2+1)/
(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))
/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(
2*c^2*d+4*e)*_Z^2+c^2*d))-1/4*I*b/e^2*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^
2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dil
og((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4
*e)*_Z^2+c^2*d))+I*b/e^2*dilog(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-b/e^2*arc
sec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-b/e^2*arcsec(c*x)*ln(1-I*(1/
c/x+I*(1-1/c^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^3*arcsec(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*arctan(s
qrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\text{Timed out}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.98 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=-\frac{a+b\sec^{-1}(cx)}{2e\,(d+ex^2)}+\frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}}-\frac{bcx\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}$$

```
[Out] 1/2*(-a-b*arcsec(c*x))/e/(e*x^2+d)+1/2*b*c*x*arctan((c^2*x^2-1)^(1/2))/d/e/
(c^2*x^2)^(1/2)-1/2*b*c*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2))
/d/e^(1/2)/(c^2*d+e)^(1/2)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5344, 457, 88, 65, 211}

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=-\frac{a+b\sec^{-1}(cx)}{2e\,(d+ex^2)}+\frac{bcx\arctan\left(\sqrt{c^2x^2-1}\right)}{2de\sqrt{c^2x^2}}-\frac{bcx\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2x^2}\sqrt{c^2d+e}}$$

```
[In] Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcSec[c*x])/(e*(d + e*x^2)) + (b*c*x*ArcTan[Sqrt[-1 + c^2*x^2]
])/(2*d*e*Sqrt[c^2*x^2]) - (b*c*x*ArcTan[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[
c^2*d + e]])/(2*d*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5344

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{(bcx)\int\frac{1}{x\sqrt{-1+c^2x^2}\left(d+ex^2\right)}\,dx}{2e\sqrt{c^2x^2}}\\
&= -\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{(bcx)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}(d+ex)}\,dx,x,x^2\right)}{4e\sqrt{c^2x^2}}\\
&= -\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)}-\frac{(bcx)\text{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}(d+ex)}\,dx,x,x^2\right)}{4d\sqrt{c^2x^2}}\\
&\quad+\frac{(bcx)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{4de\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{(bx)\text{Subst}\left(\int \frac{1}{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{2cd\sqrt{c^2x^2}} + \frac{(bx)\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{2cde\sqrt{c^2x^2}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}} - \frac{bcx\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.18

$$\int \frac{x(a+b\sec^{-1}(cx))}{\left(d+ex^2\right)^2}\,dx$$

$$= \frac{-\frac{2a}{d+ex^2} - \frac{2b\sec^{-1}(cx)}{d+ex^2} - \frac{2b\arcsin\left(\frac{1}{cx}\right)}{d} + \frac{b\sqrt{e}\log\left(\frac{4ide+4cd\sqrt{e}\left(c\sqrt{d}-i\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x}{b\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d-e}} + \frac{b\sqrt{e}\log\left(\frac{-4ide+4cd\sqrt{e}\left(c\sqrt{d}+i\sqrt{-c^2d-e}\cdots\right.}{b\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}\cdots\right.}\right.}{d\sqrt{-c^2d-e}}}{4e}$$

```
[In] Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-2*a)/(d + e*x^2) - (2*b*ArcSec[c*x])/(d + e*x^2) - (2*b*ArcSin[1/(c*x)])
/d + (b*Sqrt[e]*Log[((4*I)*d*e + 4*c*d*Sqrt[e]*(c*Sqrt[d] - I*Sqrt[-(c^2*d)
 - e]*Sqrt[1 - 1/(c^2*x^2)])*x)/(b*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*
x))])/(d*Sqrt[-(c^2*d) - e]) + (b*Sqrt[e]*Log[((-4*I)*d*e + 4*c*d*Sqrt[e]*(
c*Sqrt[d] + I*Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)/(b*Sqrt[-(c^2*d)
 - e]*(Sqrt[d] - I*Sqrt[e]*x))])/(d*Sqrt[-(c^2*d) - e]))/(4*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(112) = 224.

Time = 6.50 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.02

method	result
parts	$-\frac{a}{2e(e\,x^2+d)}+\frac{b\left(-\frac{c^4\,\mathrm{arcsec}(cx)}{2e\left(c^2e\,x^2+c^2d\right)}-\frac{c\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(\frac{2\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e-2\sqrt{-c^2de}\,cx}{cex+\sqrt{-c^2de}}\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd\sqrt{-\frac{c^2d+e}{e}}}\right.}{c^2}$
derivativedivides	$\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\mathrm{arcsec}(cx)}{2e\left(c^2e\,x^2+c^2d\right)}-\frac{\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e+\sqrt{-c^2}\right.}{-cex+\sqrt{-c^2de}}\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3xd\sqrt{-}}\right.}{c^2}$
default	$\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\mathrm{arcsec}(cx)}{2e\left(c^2e\,x^2+c^2d\right)}-\frac{\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e+\sqrt{-c^2}\right.}{-cex+\sqrt{-c^2de}}\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3xd\sqrt{-}}\right.}{c^2}$

```
[In] int(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/e/(e*x^2+d)+b/c^2*(-1/2*c^4/e/(c^2*e*x^2+c^2*d)*arcsec(c*x)-1/4*c/e*
(c^2*x^2-1)^(1/2)*(2*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)-ln(2*
((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-
c^2*d*e)^(1/2)))-ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)
^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2))))/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(
-(c^2*d+e)/e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.93

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$
$$= \left[-\frac{2\,ac^2d^2+2\,ade+\sqrt{-c^2de-e^2}(bex^2+bd)\log\left(\frac{c^2ex^2-c^2d+2\sqrt{-c^2de-e^2}\sqrt{c^2x^2-1}-2\,e}{ex^2+d}\right)+2\,(bc^2d^2+bde)\,\mathrm{arcsec}}{4\,(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)x^2)}\right.$$
$$-\frac{ac^2d^2+ade+\sqrt{c^2de+e^2}(bex^2+bd)\arctan\left(\frac{\sqrt{c^2de+e^2}\sqrt{c^2x^2-1}}{c^2d+e}\right)+(bc^2d^2+bde)\,\mathrm{arcsec}\,(cx)-2\,(bc^2d^2+}{2\,(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)x^2)}$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 + 2*a*d*e + sqrt(-c^2*d*e - e^2)*(b*e*x^2 + b*d)*log((c^
2*e*x^2 - c^2*d + 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2 +
d)) + 2*(b*c^2*d^2 + b*d*e)*arcsec(c*x) - 4*(b*c^2*d^2 + b*d*e + (b*c^2*d*e
 + b*e^2)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^
2*d^2*e^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 + a*d*e + sqrt(c^2*d*e + e^2)*(b*e
*x^2 + b*d)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2 - 1)/(c^2*d + e)) + (b*
c^2*d^2 + b*d*e)*arcsec(c*x) - 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x
^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 +
 d*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x*(a+b*asec(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(c^2*e^2*x^2 + c^2*d*e)*integrate(1/2*x*e^(1/2*log(c*x + 1) + 1/2*lo
g(c*x - 1))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c^2*
d*e - e^2)*x^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) - arctan(sqrt(c*
x + 1)*sqrt(c*x - 1)))*b/(e^2*x^2 + d*e) - 1/2*a/(e^2*x^2 + d*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.99 $\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 546

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^2}\,dx = &-\frac{e(a+b\sec^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i(a+b\sec^{-1}(cx))^2}{2bd^2}\\
&-\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}}\\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\\
&+\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&+\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\end{aligned}$$

```
[Out] -1/2*e*(a+b*arcsec(c*x))/d^2/(e+d/x^2)+1/2*I*(a+b*arcsec(c*x))^2/b/d^2-1/2*
(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsec(c*x))*ln(1-
```

```
c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2-1
/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,c*(1/c/x+I*(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog
(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
^2+1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))/d^2-1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^
(1/2))*e^(1/2)/d^2/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5348, 4818, 4814, 385, 211, 4826, 4616, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = &-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\ &-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\ &-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}\\ &-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-\frac{e(a+b\sec^{-1}(cx))}{2d^2\left(\frac{d}{x^2}+e\right)}\\ &+\frac{i(a+b\sec^{-1}(cx))^2}{2bd^2}-\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\sqrt{c^2d+e}}\\ &+\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}\\ &+\frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}\end{aligned}$$

```
[In] Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(e*(a + b*ArcSec[c*x]))/(d^2*(e + d/x^2)) + ((I/2)*(a + b*ArcSec[c*x])
^2)/(b*d^2) - (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*
x^2)]*x)])/(2*d^2*Sqrt[c^2*d + e]) - ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-
d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcSe
c[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]
)/(2*d^2) - ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sq
```

```
rt[e] + Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-
d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) + ((I/2)*b*Poly
Log[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d^2
+ ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d
+ e])])/d^2 + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e]
 + Sqrt[c^2*d + e]))])/d^2 + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*
x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4814

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])/(2*e*(p + 1))), x]
 + Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^3\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(-\frac{ex\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)dx,x,\frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)}{d}\\
&= -\frac{e\left(a+b\sec^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}-\frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)}{d}\\
&\quad -\frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx,x,\frac{1}{x}\right)}{2cd^2}
\end{aligned}
$$

$$= -\frac{e(a+b\sec^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}} - \frac{(be)\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2cd^2}$$

$$= -\frac{e(a+b\sec^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$= -\frac{e(a+b\sec^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\sec^{-1}(cx))^2}{2bd^2} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}} + \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}} + \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}} - \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}} - \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$= -\frac{e(a+b\sec^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\sec^{-1}(cx))^2}{2bd^2} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2d^2}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2d^2}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2d^2}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2d^2}$$

$$= -\frac{e(a+b\sec^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\sec^{-1}(cx))^2}{2bd^2} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d^2}$$

$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d^2}$$

$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d^2}$$

$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d^2}$$

$$= -\frac{e(a+b\sec^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\sec^{-1}(cx))^2}{2bd^2} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$
$$+ \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$
$$+ \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1190 vs. $2(546) = 1092$.

Time = 0.98 (sec) , antiderivative size = 1190, normalized size of antiderivative = 2.18

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

$$= \frac{\frac{2ad}{d+ex^2} + \frac{b\sqrt{d}\sec^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} + \frac{b\sqrt{d}\sec^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} + 2ib\sec^{-1}(cx)^2 + 2b\arcsin\left(\frac{1}{cx}\right) - 8ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}\right.}{}\right.}{}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] ((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (b
*Sqrt[d]*ArcSec[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (2*I)*b*ArcSec[c*x]^2 + 2*b
*ArcSin[1/(c*x)] - (8*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]
]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] -
 (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqr
t[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - 2*b*ArcSec[c*x]*Log[
1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 4*b*Ar
cSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSec[c*x]*Log[1 + (I*(-
```

```
Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 4*b*ArcSin[Sqr
t[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d +
 e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSec[c*x]*Log[1 - (I*(Sqrt[e]
+ Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I
*ArcSec[c*x]))/(c*Sqrt[d])] - 2*b*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^
2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c
*x]))/(c*Sqrt[d])] + 4*a*Log[x] - (b*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e
] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(
c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] - (b*Sqrt[e]*Log[
(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 -
 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^
2*d) - e] - 2*a*Log[d + e*x^2] + (2*I)*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*b*PolyLog[2, (I*(-Sqrt[
e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*b*PolyLog[2,
((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*b
*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])
/(4*d^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.06 (sec) , antiderivative size = 2108, normalized size of antiderivative = 3.86

method	result	size
parts	Expression too large to display	2108
derivativedivides	Expression too large to display	2157
default	Expression too large to display	2157

```
[In] int((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a/d^2*ln(x)+1/2*a/d/(e*x^2+d)-1/2*a/d^2*ln(e*x^2+d)+b*(-1/2*I*(-(e*(c^2*d+e
))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*polylog(2,d*c^2*(
1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/d^4/(c^2
*d+e)/c^4-I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2
*e^2)*e*arcsec(c*x)^2/d^4/(c^2*d+e)/c^4+(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d
*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)/d^4*e/(c^2*d+e)/c^4*ln(1-d*c^2*(1/c/x+I*(
1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-1/4*I
*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*arcsec(c*x)^2*c^2+1/4*(e*(c^2*d+e))^(1/2
)/d/e/(c^2*d+e)*c^2*arcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/
(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))-1/8*I*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)
*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1
/2)-2*e))*c^2+1/2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arcsec(c*x)^2/c^2/d^3
```

```
+1/2*I*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arctanh(1/4*(2*c^2*d*(1/c/x+I*(1-1
/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))-1/2*(c^2*d-2*(e*(c^2*d
+e))^(1/2)+2*e)/d^3/c^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-
2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-1/4*I*(e*(c^2*d+e))^(1/2)/d^2/(c^2*
d+e)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e)
)^(1/2)-2*e))-1/2*I*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arcsec(c*x)^2+1/2*(e*
(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+1/4*I*(c^2*d-2*(e*(c^2*d+e))
^(1/2)+2*e)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c
^2*d+e))^(1/2)-2*e))/c^2/d^3+1/2*I/d^2*sum((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2
*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+
dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*
d+4*e)*_Z^2+c^2*d))+1/2*I*arcsec(c*x)^2/d^2-1/4*I*(-(e*(c^2*d+e))^(1/2)*c^2
*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arcsec(c*x)^2/d^2/e/(c^2*d+e)-1
/2*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*p
olylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2
)-2*e))/d^3/(c^2*d+e)/c^2+(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d
+e))^(1/2)*e+2*e^2)/d^3/(c^2*d+e)/c^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e)/d^4/c^4*e*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2
*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)+I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*
arcsec(c*x)^2*e/d^4/c^4-1/8*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c
^2*d+e))^(1/2)*e+2*e^2)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c
^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/d^2/e/(c^2*d+e)+1/2*I*(c^2*d-2*(e*(c^2*d+e
))^(1/2)+2*e)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*
(c^2*d+e))^(1/2)-2*e))*e/d^4/c^4+1/4*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-
2*(e*(c^2*d+e))^(1/2)*e+2*e^2)/d^2/e/(c^2*d+e)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2
/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-1/2*x^2*c^2*
e*arcsec(c*x)/(c^2*e*x^2+c^2*d)/d^2-I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e
-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arcsec(c*x)^2/d^3/(c^2*d+e)/c^2)
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*asec(c*x))/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\,\text{arcsec}\,(cx)+a}{\left(ex^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrate
(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

3.100 $\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 784

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{x(a+b\sec^{-1}(cx))}{e^2}$$
$$-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2}-\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}$$
$$-\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}$$
$$+\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$-\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$-\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$-\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$-\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

```
[Out] x*(a+b*arcsec(c*x))/e^2-b*arctanh((1-1/c^2/x^2)^(1/2))/c/e^2+3/4*(a+b*arcse
c(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^
(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+
b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^
2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+
3/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*I*b*po
```

```
lylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))*(-d)^(1/2)/e^(5/2)-3/4*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-1/4*d*(a+b*arcsec(c*
x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arcsec(c*x))/e^2/(d/x+(-d)^(1/
2)*e^(1/2))-1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^
(1/2)/(1-1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2*d+e)^(1/2)-1/4*b*arctanh((c^2*d
+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))*d^(1/
2)/e^2/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5348, 4818, 4724, 272, 65, 214, 4758, 4828, 739, 212, 4826, 4616, 2221, 2317, 2438}

$$\begin{aligned}
\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx &= \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&- \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+ \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4e^{5/2}} \\
&- \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4e^{5/2}} \\
&- \frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+ \frac{x(a+b\sec^{-1}(cx))}{e^2} - \frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4e^2\sqrt{c^2d+e}} \\
&- \frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4e^2\sqrt{c^2d+e}} - \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} \\
&+ \frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}} \\
&- \frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}} \\
&+ \frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}} \\
&- \frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}
\end{aligned}$$

```
[In] Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(d*(a + b*ArcSec[c*x]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*Arc
Sec[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcSec[c*x]))/e^2 -
 (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e^2) - (b*Sqrt[d]*ArcTanh[(c^2*d - (
Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4
*e^2*Sqrt[c^2*d + e]) - (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(
c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*e^2*Sqrt[c^2*d + e])
+ (3*Sqrt[-d]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(S
qrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSec[c*x])*L
og[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5
/2)) + (3*Sqrt[-d]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]
))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSec[c*
x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4
*e^(5/2)) + (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]
))/(Sqrt[e] - Sqrt[c^2*d + e]))])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2
, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(5/2) + ((
(3*I)/4)*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] +
Sqrt[c^2*d + e]))])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*
E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{e^2x^2} - \frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2}\\
&\quad + \frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e}
\end{aligned}
$$

$$= \frac{x(a+b\sec^{-1}(cx))}{e^2} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce^2} + \frac{d\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \left(-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2e(-de-d^2x^2)}\right)\,dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x(a+b\sec^{-1}(cx))}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} + \frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ce^2} - \frac{d^2\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e^2} - \frac{d^2\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e^2} - \frac{d^2\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2e^2}$$

$$= -\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\sec^{-1}(cx))}{e^2} - \frac{d\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2e^{5/2}} - \frac{d\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2e^{5/2}} - \frac{(bc)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)}{e^2} - \frac{(bd)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce^2} + \frac{(bd)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce^2} - \frac{d^2\text{Subst}\left(\int \left(-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{2e^2}$$

$$= -\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\sec^{-1}(cx))}{e^2}$$

$$- \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} + \frac{(id)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{(id)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{(id)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{(id)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4e^{5/2}} + \frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4e^{5/2}}$$

$$+ \frac{(bd)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4ce^2}$$

$$- \frac{(bd)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4ce^2}$$

$$= -\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\sec^{-1}(cx))}{e^2} - \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} - \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} - \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} - \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e^{5/2}} + \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e^{5/2}} - \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e^{5/2}} + \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e^{5/2}} - \frac{d\text{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}dx,x,\sec^{-1}(cx)\right)}{4e^{5/2}} - \frac{d\text{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}dx,x,\sec^{-1}(cx)\right)}{4e^{5/2}}$$

$$= -\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+ \frac{x(a+b\sec^{-1}(cx))}{e^2} - \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2}$$
$$- \frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} - \frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}$$
$$+ \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$
$$- \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$
$$+ \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$
$$- \frac{\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}$$
$$+ \frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e^{5/2}}$$
$$- \frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e^{5/2}}$$
$$+ \frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e^{5/2}}$$
$$- \frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2e^{5/2}}$$
$$+ \frac{(id)\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{4e^{5/2}}$$
$$+ \frac{(id)\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{4e^{5/2}}$$
$$+ \frac{(id)\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{4e^{5/2}}$$
$$+ \frac{(id)\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{4e^{5/2}}$$

$$
\begin{aligned}
&= -\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+\frac{x(a+b\sec^{-1}(cx))}{e^2}-\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} \\
&-\frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}-\frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} \\
&+\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&-\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&-\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&-\frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4e^{5/2}} \\
&+\frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4e^{5/2}} \\
&-\frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4e^{5/2}} \\
&+\frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+\frac{x(a+b\sec^{-1}(cx))}{e^2}-\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} \\
&-\frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}-\frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} \\
&+\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&-\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&-\frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&+\frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{4e^{5/2}} \\
&-\frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{4e^{5/2}} \\
&+\frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{4e^{5/2}} \\
&-\frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{4e^{5/2}}
\end{aligned}
$$

$$= -\frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\sec^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\sec^{-1}(cx))}{e^2} - \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} - \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} - \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} + \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} - \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} + \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} - \frac{3\sqrt{-d}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} + \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} - \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} + \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} - \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 1.80 (sec) , antiderivative size = 1331, normalized size of antiderivative = 1.70

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$= \frac{4a\sqrt{e}x + \frac{2ad\sqrt{e}x}{d+ex^2} - 6a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + b\left(4\sqrt{e}x\sec^{-1}(cx) + \frac{d\sec^{-1}(cx)}{-i\sqrt{d}+\sqrt{e}x} + \frac{d\sec^{-1}(cx)}{i\sqrt{d}+\sqrt{e}x} + 12\sqrt{d}\arcsin\left(\frac{\sqrt{1-\frac{i}{c}}}{\sqrt{2}}\right.\right.}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (4*a*Sqrt[e]*x + (2*a*d*Sqrt[e]*x)/(d + e*x^2) - 6*a*Sqrt[d]*ArcTan[(Sqrt[e
]*x)/Sqrt[d]] + b*(4*Sqrt[e]*x*ArcSec[c*x] + (d*ArcSec[c*x])/((-I)*Sqrt[d]
+ Sqrt[e]*x) + (d*ArcSec[c*x])/(I*Sqrt[d] + Sqrt[e]*x) + 12*Sqrt[d]*ArcSin[
Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e
])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - 12*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]
```

```
/2])/Sqrt[c^2*d + e]] + (3*I)*Sqrt[d]*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqr
t[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (6*I)*Sqrt[d]*ArcSin[Sqrt[1
 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])
*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[d]*ArcSec[c*x]*Log[1 + (I*(-S
qrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (6*I)*Sqrt[d]*A
rcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqr
t[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[d]*ArcSec[c*x]*L
og[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (6*
I)*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sq
rt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (3*I)*Sqrt[d]*Ar
cSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt
[d])] - (6*I)*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (I*Sq
rt[d]*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2
*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e
]*x))]])/Sqrt[-(c^2*d) - e] - (I*Sqrt[d]*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(-Sq
rt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqr
t[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]])/Sqrt[-(c^2*d) - e] + (4*Sqrt[e]*
Log[Cos[ArcSec[c*x]/2] - Sin[ArcSec[c*x]/2]])/c - (4*Sqrt[e]*Log[Cos[ArcSec
[c*x]/2] + Sin[ArcSec[c*x]/2]])/c - 3*Sqrt[d]*PolyLog[2, ((-I)*(-Sqrt[e] +
Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 3*Sqrt[d]*PolyLog[2, (I*
(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 3*Sqrt[d]*Po
lyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]
- 3*Sqrt[d]*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c
*Sqrt[d])]))/(4*e^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 88.11 (sec) , antiderivative size = 941, normalized size of antiderivative = 1.20

method	result	size
parts	Expression too large to display	941
derivativedivides	Expression too large to display	966
default	Expression too large to display	966

```
[In] int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/e^2*x-1/e^2*d*(-1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
)))+b/c^5*(1/2*x*c^5*arcsec(c*x)*(2*c^2*e*x^2+3*c^2*d)/(c^2*e*x^2+c^2*d)/e^
2+2*I/e^2*c^4*arctan(1/c/x+I*(1-1/c^2/x^2)^(1/2))+3/16*I/e^3*c^6*d*sum((_R1
^2*c^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/
x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1))
,_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-3/16*I/e^3*c^6*d*sum((_R1
```

```
^2*c^2*d+4*_R1^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R
1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))
/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*I*((c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c
*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2
))/d^2/e^2+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(
c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2
*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^2/e^2-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1
/2)*e+2*e^2)*c*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d
+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^2-1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1
/2)*e+2*e^2)*c*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2
*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^2)
```

Fricas [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^4*arcsec(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^4(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**4*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

```
[In] int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.101 $\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 745

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

```
[Out] 1/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arcsec(c*x))*ln(1+c*(1/c/
x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)
^(1/2)+1/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arcsec(c*x))*ln(1+
c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/
2)/(-d)^(1/2)+1/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*I*b*polylog(2,c*(1/c/x+I
*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1
/2)+1/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a
```

```
+b*arcsec(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arcsec(c*x))/e/(d/x+(
-d)^(1/2)*e^(1/2))+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^
2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d+e)^(1/2)+1/4*b*arctanh((
c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/
e/d^(1/2)/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5348, 4758, 4828, 739, 212, 4826, 4616, 2221, 2317, 2438}

$$\begin{aligned}
\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx =& \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&- \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} \\
&+ \frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} \\
&+ \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&- \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&+ \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&- \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}
\end{aligned}$$

```
[In] Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcSec[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSec[c*x])/(4
*e*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c
*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d +
 e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]
*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d + e]) + ((a + b*ArcSec[c*
x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4
*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c
*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcSec[c
*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(
4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[
c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLo
g[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/(Sqrt[
-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, -((c*Sqrt[-d]
*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/(Sqrt[-d]*e^(3/2)) - ((I
/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])
])/(Sqrt[-d]*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a + b\arccos\left(\frac{x}{c}\right)}{\left(e + dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx,x,\frac{1}{x}\right)$$

$$= \frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}dx,x,\frac{1}{x}\right)}{4e}+\frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}dx,x,\frac{1}{x}\right)}{4e}+\frac{d\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{-de-d^2x^2}dx,x,\frac{1}{x}\right)}{2e}$$

$$= \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{4ce}-\frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{4ce}+\frac{d\text{Subst}\left(\int \left(-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)}{2e}$$

$$= \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{4e^{3/2}}-\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{4e^{3/2}}-\frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}dx,x,\frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4ce}+\frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}dx,x,\frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4ce}$$

$$= \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}}+\frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}dx,x,\sec^{-1}(cx)\right)}{4e^{3/2}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}dx,x,\sec^{-1}(cx)\right)}{4e^{3/2}}$$

$$= \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4e^{3/2}} - \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4e^{3/2}} - \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4e^{3/2}} - \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4e^{3/2}}$$

$$= \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\text{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\text{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\text{Subst}\left(\int \log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\text{Subst}\left(\int \log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}}$$

$$
\begin{aligned}
&= \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} \\
&+ \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
&+ \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}} \\
&- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}} \\
&+ \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}} \\
&- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}}
\end{aligned}
$$

$$= \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}}$$
$$+ \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 1.27 (sec) , antiderivative size = 1245, normalized size of antiderivative = 1.67

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$$
$$= \frac{-\frac{2a\sqrt{e}x}{d+ex^2} + \frac{2a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + b\left(\frac{\sec^{-1}(cx)}{i\sqrt{d}-\sqrt{e}x} - \frac{\sec^{-1}(cx)}{i\sqrt{d}+\sqrt{e}x} - \frac{4\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)}{\sqrt{d}} + \frac{4\arcsin\left(\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-2*a*Sqrt[e]*x)/(d + e*x^2) + (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 b*(ArcSec[c*x]/(I*Sqrt[d] - Sqrt[e]*x) - ArcSec[c*x]/(I*Sqrt[d] + Sqrt[e]*
x) - (4*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*S
qrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d] + (4*ArcSin
[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])
*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[d] - (I*ArcSec[c*x]*Log[1 + (I*
(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d] - ((2*
I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] -
Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d] + (I*ArcSec[c*x]*
```

```
Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]])/Sq
rt[d] + ((2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I
*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[d] + (I
*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*S
qrt[d])]])/Sqrt[d] - ((2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]
]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/S
qrt[d] - (I*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[
c*x]))/(c*Sqrt[d])]])/Sqrt[d] + ((2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*S
qrt[d])]])/Sqrt[d] - (I*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqr
t[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(S
qrt[d] - I*Sqrt[e]*x))]])/(Sqrt[d]*Sqrt[-(c^2*d) - e]) + (I*Sqrt[e]*Log[(2*S
qrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(
c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]])/(Sqrt[d]*Sqrt
[-(c^2*d) - e]) + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec
[c*x]))/(c*Sqrt[d])]/Sqrt[d] - PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E
^(I*ArcSec[c*x]))/(c*Sqrt[d])]/Sqrt[d] - PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c
^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]/Sqrt[d] + PolyLog[2, (I*(Sqrt[e]
 + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]/Sqrt[d]))/(4*e^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 30.53 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.14

method	result
parts	$-\frac{ax}{2e(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+\frac{b\left(-\frac{c^5\,\mathrm{arcsec}(cx)x}{2e\left(c^2e\,x^2+c^2d\right)}-\frac{i\sqrt{\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\,\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)\arctan\left(\frac{}{\sqrt{}}\right.}{2c\,d^3e}\right.}{}$
derivativedivides	$\frac{-\frac{a\,c^5x}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\,c^4\left(-\frac{\mathrm{arcsec}(cx)cx}{2e\left(c^2e\,x^2+c^2d\right)}-\frac{i\sqrt{\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\,\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)\arctan\left(\right.}{2e\,c^5d^3}\right.}{}$
default	$\frac{-\frac{a\,c^5x}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\,c^4\left(-\frac{\mathrm{arcsec}(cx)cx}{2e\left(c^2e\,x^2+c^2d\right)}-\frac{i\sqrt{\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\,\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)\arctan\left(\right.}{2e\,c^5d^3}\right.}{}$

```
[In] int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/e*x/(e*x^2+d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c^3*(-1/
2*c^5*arcsec(c*x)/e*x/(c^2*e*x^2+c^2*d)-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)
+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+I*(1-1/c
^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/c/d^3/e+1/2*I*(
(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^
2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2
))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^3/c-1/2*I*(-(
c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)
*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2
*e)*d)^(1/2))/c/d^3/e+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((
e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c
*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/
2))/e/(c^2*d+e)/d^3/c+1/4*I/e*c^4*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec
(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2
```

```
/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/4*I/e
*c^4*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/
c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Root
Of(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2*arcsec(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**2*(a + b*asec(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

[In] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)

[Out] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)

3.102 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 739

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx = &-\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&-\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}-\frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}\\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\end{aligned}$$

```
[Out] -1/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsec(c*x))*ln(1+c*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/
```

```
e^(1/2)-1/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arcsec(c*x))*ln(1
+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)
^(3/2)/e^(1/2)-1/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*I*b*polylog(2,c*(1/c/x+
I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(
1/2)-1/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(
-a-b*arcsec(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arcsec(c*x))/d/(d/x+
(-d)^(1/2)*e^(1/2))-1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c
^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)-1/4*b*arctanh((c
^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d
^(3/2)/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5338, 4818, 4758, 4828, 739, 212, 4826, 4616, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx = &-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\&+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\&-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\&+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\&-\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\&-\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4d^{3/2}\sqrt{c^2d+e}}-\frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4d^{3/2}\sqrt{c^2d+e}}\\&-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\&-\frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\end{aligned}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(a + b*ArcSec[c*x])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcSec[c*x])
/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)
/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d
+ e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e
]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d + e]) - ((a + b*ArcSec[c*x
])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*
(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[
c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSe
c[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*Ar
cSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((I/4)*b
*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])
/((-d)^(3/2)*Sqrt[e]) + ((I/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/
(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, -(
(c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/((-d)^(3/2)*S
qrt[e]) + ((I/4)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqr
t[c^2*d + e])])/((-d)^(3/2)*Sqrt[e])
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 739</u>

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

<u>Rule 2317</u>

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

<u>Rule 2438</u>

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5338

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{a+b\arccos\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{d}$$
$$+\frac{e\text{Subst}\left(\int \left(-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{d}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)\right)$$
$$-\frac{1}{4}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)$$
$$-\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d\sqrt{e}}-\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d\sqrt{e}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$-\frac{1}{2}\text{Subst}\left(\int \left(-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)$$
$$-\frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4cd}+\frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4cd}$$
$$+\frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2d\sqrt{e}}$$
$$+\frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cd} - \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cd}$$

$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$+ \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d\sqrt{e}} + \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d\sqrt{e}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}$$

$$- \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}dx, x, \sec^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}dx, x, \sec^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}$$
$$- \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{4d\sqrt{e}}$$
$$+ \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{4d\sqrt{e}}$$
$$+ \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{4d\sqrt{e}}$$
$$+ \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}$$

$$- \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}$$

$$- \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\sec^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\sec^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\sec^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\sec^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$
\begin{aligned}
&= -\frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\sec^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} \\
&- \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&- \frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&- \frac{ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 1.60 (sec) , antiderivative size = 1239, normalized size of antiderivative = 1.68

$$
\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx = \frac{1}{2}\left(\frac{ax}{d^2+dex^2} + \frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}} + \frac{b\left(\frac{\sqrt{d}\sec^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex} + \frac{\sqrt{d}\sec^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex} - \frac{4\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)}{\sqrt{e}} + \frac{4\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{(ic\cdots}{}\right)}{\sqrt{e}}\cdots\right)}{}\right.
$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*((Sqrt[d]*ArcSec[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (Sqrt[d]*ArcSec[
c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - (4*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d]
)]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2
*d + e]])/Sqrt[e] + (4*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Ar
cTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]])/Sqrt[e]
 - (I*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))
/(c*Sqrt[d])])/Sqrt[e] - ((2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d]
)])/Sqrt[e] + (I*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*A
rcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]
))/(c*Sqrt[d])])/Sqrt[e] + (I*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d
+ e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] - ((2*I)*ArcSin[Sqrt[1 - (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*
ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] - (I*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] +
Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*ArcSin[S
qrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d
+ e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])])/Sqrt[e] + (I*Log[(2*Sqrt[d]*Sqrt[e]*
(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(
Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] - (I*Log[(
2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 -
1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2
*d) - e] + PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))
/(c*Sqrt[d])]/Sqrt[e] - PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*Arc
Sec[c*x]))/(c*Sqrt[d])]/Sqrt[e] - PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d +
e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]/Sqrt[e] + PolyLog[2, (I*(Sqrt[e] + Sqrt
[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]/Sqrt[e]))/(2*d^(3/2)))/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 45.23 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.14

method	result
parts	$\frac{ax}{2d(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3\,\mathrm{arcsec}(cx)x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ic^2\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(i\,\mathrm{arcse}\right.}{}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\mathrm{arcsec}(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}+\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(i\,\mathrm{arcs}\right.}{}\right.}{}\right.}{}$
default	$\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\mathrm{arcsec}(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}+\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\_R1\left(i\,\mathrm{arcs}\right.}{}\right.}{}\right.}{}$

```
[In] int((a+b*arcsec(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c^
3*arcsec(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4*I/d*c^2*sum(_R1/(_R1^2*c^2*d+c^2*d+
2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/
c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c
^2*d))+1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d
+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2))/d^4/c^3-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e
^2)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+
2*e)*d)^(1/2))/d^4/(c^2*d+e)/c^3+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^
2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/c^3-1/2*I*(-(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*
e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/
```

```
((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)/c^3-1/4*I/d*c^2
*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/
x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c
^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^2}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{(ex^2 + d)^2}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^2}\,dx = \int \frac{a + b\,\mathrm{asec}\,(cx)}{(d + ex^2)^2}\,dx$$

```
[In] integrate((a+b*asec(c*x))/(e*x**2+d)**2,x)
[Out] Integral((a + b*asec(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^2}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x^2)^2, x)
```

3.103 $\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 785

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^2}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2}-\frac{a}{d^2x}-\frac{b\sec^{-1}(cx)}{d^2x}+\frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{be\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}+\frac{be\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}-\frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

```
[Out] -a/d^2/x-b*arcsec(c*x)/d^2/x-3/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(
a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2
)+3/4*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*I*b*polylog(2,-c*(1/c/x+I*(1
-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)
```

```
+3/4*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*I*b*p
olylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))*e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arcsec(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2)
)-1/4*e*(a+b*arcsec(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))+1/4*b*e*arctanh((c^2
*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(
5/2)/(c^2*d+e)^(1/2)+1/4*b*e*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)
/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+b*c*(1-1/c^2/
x^2)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5348, 4818, 4716, 267, 4758, 4828, 739, 212, 4826, 4616, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = &-\frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}\\&+\frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}\\&-\frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{5/2}}\\&+\frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{5/2}}\\&+\frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{a}{d^2x}\\&+\frac{be\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4d^{5/2}\sqrt{c^2d+e}}+\frac{be\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4d^{5/2}\sqrt{c^2d+e}}\\&+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2}-\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}\\&+\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}\\&-\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}\\&+\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{b\sec^{-1}(cx)}{d^2x}\end{aligned}$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] (b*c*Sqrt[1 - 1/(c^2*x^2)])/d^2 - a/(d^2*x) - (b*ArcSec[c*x])/(d^2*x) + (e*
(a + b*ArcSec[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcSec[c*
x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqr
t[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*Sqr
t[c^2*d + e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt
[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c^2*d + e]) - (3*Sqrt[
e]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sq
rt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSec[c*x])*Log[1 + (c
*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) -
 (3*Sqrt[e]*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSec[c*x])*L
og[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)
^(5/2)) - (((3*I)/4)*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/
(Sqrt[e] - Sqrt[c^2*d + e]))])/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2,
 (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(5/2) -
(((3*I)/4)*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] +
 Sqrt[c^2*d + e]))])/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-
d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^4\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{d^2}+\frac{e^2\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2}-\frac{2e\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \left(a+b\arccos\left(\frac{x}{c}\right)\right)dx, x, \frac{1}{x}\right)}{d^2}\\
&\quad+\frac{(2e)\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2}-\frac{e^2\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2}\\
&= -\frac{a}{d^2x}-\frac{b\text{Subst}\left(\int \arccos\left(\frac{x}{c}\right)dx, x, \frac{1}{x}\right)}{d^2}\\
&\quad+\frac{(2e)\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{d^2}\\
&\quad-\frac{e^2\text{Subst}\left(\int \left(-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{d^2}
\end{aligned}
$$

$$= -\frac{a}{d^2x} - \frac{b\sec^{-1}(cx)}{d^2x} - \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd^2}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{d^2}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{e\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4d}$$
$$+ \frac{e\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4d} + \frac{e\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2d}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\sec^{-1}(cx)}{d^2x} + \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{d^2}$$
$$- \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{d^2}$$
$$+ \frac{(be)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4cd^2}$$
$$- \frac{(be)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4cd^2}$$
$$+ \frac{e\text{Subst}\left(\int \left(-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{2d}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\sec^{-1}(cx)}{d^2x} + \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{(i\sqrt{e})\,\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{d^2}$$
$$+ \frac{(i\sqrt{e})\,\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{d^2}$$
$$+ \frac{(i\sqrt{e})\,\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{d^2}$$
$$+ \frac{(i\sqrt{e})\,\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{d^2}$$
$$- \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d^2} - \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d^2}$$
$$- \frac{(be)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cd^2}$$
$$+ \frac{(be)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cd^2}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\sec^{-1}(cx)}{d^2x} + \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{be\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}$$
$$+ \frac{be\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} - \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$
$$+ \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$
$$- \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$
$$+ \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$
$$+ \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{(-d)^{5/2}}$$
$$- \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{(-d)^{5/2}}$$
$$+ \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{(-d)^{5/2}}$$
$$- \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{(-d)^{5/2}}$$
$$+ \frac{\sqrt{e}\,\mathrm{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}dx, x, \sec^{-1}(cx)\right)}{4d^2}$$
$$+ \frac{\sqrt{e}\,\mathrm{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}dx, x, \sec^{-1}(cx)\right)}{4d^2}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\sec^{-1}(cx)}{d^2x} + \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$- \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{be\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}$$

$$+ \frac{be\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} - \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{(ib\sqrt{e})\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{(-d)^{5/2}}$$

$$+ \frac{(ib\sqrt{e})\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{(-d)^{5/2}}$$

$$- \frac{(ib\sqrt{e})\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{(-d)^{5/2}}$$

$$+ \frac{(ib\sqrt{e})\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\sec^{-1}(cx)}\right)}{(-d)^{5/2}}$$

$$- \frac{(i\sqrt{e})\,\mathrm{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4d^2}$$

$$- \frac{(i\sqrt{e})\,\mathrm{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4d^2}$$

$$- \frac{(i\sqrt{e})\,\mathrm{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4d^2}$$

$$- \frac{(i\sqrt{e})\,\mathrm{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4d^2}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\sec^{-1}(cx)}{d^2x} + \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$- \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{be\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}$$

$$+ \frac{be\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} - \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$+ \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$- \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$+ \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

$$- \frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} + \frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} + \frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4(-d)^{5/2}}$$

$$+ \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4(-d)^{5/2}}$$

$$- \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4(-d)^{5/2}}$$

$$+ \frac{(b\sqrt{e})\,\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{4(-d)^{5/2}}$$

$$
\begin{aligned}
&= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\sec^{-1}(cx)}{d^2x} + \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&- \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{be\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} \\
&+ \frac{be\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} - \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} \\
&+ \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} \\
&- \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} \\
&+ \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} \\
&- \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} + \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} \\
&- \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} + \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} \\
&+ \frac{(ib\sqrt{e})\,\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{4(-d)^{5/2}} \\
&- \frac{(ib\sqrt{e})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{4(-d)^{5/2}} \\
&+ \frac{(ib\sqrt{e})\,\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{4(-d)^{5/2}} \\
&- \frac{(ib\sqrt{e})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{4(-d)^{5/2}}
\end{aligned}
$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\sec^{-1}(cx)}{d^2x} + \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e(a+b\sec^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{be\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} + \frac{be\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} - \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 1.65 (sec) , antiderivative size = 1291, normalized size of antiderivative = 1.64

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$$

$$= \frac{-\frac{4a\sqrt{d}}{x} - \frac{2a\sqrt{d}ex}{d+ex^2} - 6a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + b\left(4c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}} - \frac{4\sqrt{d}\sec^{-1}(cx)}{x} - \frac{\sqrt{d}e\sec^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex} - \frac{\sqrt{d}e\sec^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex} + 12\right.}{}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-4*a*Sqrt[d])/x - (2*a*Sqrt[d]*e*x)/(d + e*x^2) - 6*a*Sqrt[e]*ArcTan[(Sqr
t[e]*x)/Sqrt[d]] + b*(4*c*Sqrt[d]*Sqrt[1 - 1/(c^2*x^2)] - (4*Sqrt[d]*ArcSec
[c*x])/x - (Sqrt[d]*e*ArcSec[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (Sqrt[d]*
e*ArcSec[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + 12*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[
c*x]/2])/Sqrt[c^2*d + e]] - 12*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[
```

```
d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*
d + e]] + (3*I)*Sqrt[e]*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*
E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (6*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c
*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[e]*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[
c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (6*I)*Sqrt[e]*ArcSin[Sqrt[1 -
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*
E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (3*I)*Sqrt[e]*ArcSec[c*x]*Log[1 - (I*(Sqr
t[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (6*I)*Sqrt[e]*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c
^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (3*I)*Sqrt[e]*ArcSec[c*x]*Log[
1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (6*I)*
Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[
e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (I*e*Log[(2*Sqrt[d]
*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2
)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] +
 (I*e*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e
]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])
/Sqrt[-(c^2*d) - e] - 3*Sqrt[e]*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e
])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 3*Sqrt[e]*PolyLog[2, (I*(-Sqrt[e] + Sq
rt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + 3*Sqrt[e]*PolyLog[2, ((-I)
*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 3*Sqrt[e]*Po
lyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]))/(
4*d^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 81.58 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.19

method	result	size
parts	Expression too large to display	933
derivativedivides	Expression too large to display	960
default	Expression too large to display	960

```
[In] int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/d^2/x-e/d^2*(1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
)+b*c*(-1/2*(arcsec(c*x)+I)/d^2*(I*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x+1)/x/c+1
/2*(I*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x-1)*(arcsec(c*x)-I)/d^2/x/c-1/2*x*c*ar
csec(c*x)*e/(c^2*e*x^2+c^2*d)/d^2-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))*e/d^5/c^5+1/2*I*((c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*
```

```
e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))
/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/c^5/(c^2*d+e)-1/2*I*(-(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*a
rctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e
)*d)^(1/2))*e/d^5/c^5+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((
e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctanh
(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(
1/2))/d^5/c^5/(c^2*d+e)+3/4*I*e/d^2*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*ar
csec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1
/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-3/4
*I*e/d^2*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-
1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Ro
otOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{asec}\left(cx\right)}{x^2\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**2,x)
[Out] Integral((a + b*asec(c*x))/(x**2*(d + e*x**2)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\,(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\,(d+ex^2)^2}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^2\,(e\,x^2+d)^2}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

3.104 $\int \frac{x^5 \left(a+b \sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 707

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\sec^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\sec^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{8e^{5/2}(c^2d+e)^{3/2}} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} - \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} - \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^3}$$

```
[Out] 1/4*(-a-b*arcsec(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arcsec(c*x))/e^2/(e+d/x^2)-1
/8*b*(c^2*d+2*e)*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^
(5/2)/(c^2*d+e)^(3/2)-(a+b*arcsec(c*x))*ln(1+(1/c/x+I*(1-1/c^2/x^2)^(1/2))^
2)/e^3+1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arcsec(
c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1
/2)))/e^3+1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*I*b*polylog(2,-(1/c/x+I*(1-1/c^2/x^
2)^(1/2))^2)/e^3-1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1
```

```
/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,-c*(1
/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*I
*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^
(1/2)))/e^3-1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e
^(5/2)/(c^2*d+e)^(1/2)-1/8*b*c*d*(1-1/c^2/x^2)^(1/2)/e^2/(c^2*d+e)/(e+d/x^2
)/x
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules

used = {5348, 4818, 4722, 3800, 2221, 2317, 2438, 4814, 390, 385, 211, 4826, 4616}

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3} - \frac{a+b\sec^{-1}(cx)}{2e^2\left(\frac{d}{x^2}+e\right)} - \frac{a+b\sec^{-1}(cx)}{4e\left(\frac{d}{x^2}+e\right)^2} - \frac{\log\left(1+e^{2i\sec^{-1}(cx)}\right)(a+b\sec^{-1}(cx))}{e^3} - \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8e^{5/2}(c^2d+e)^{3/2}} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3} - \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2x(c^2d+e)\left(\frac{d}{x^2}+e\right)} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^3}$$

```
[In] Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*c*d*Sqrt[1 - 1/(c^2*x^2)])/(e^2*(c^2*d + e)*(e + d/x^2)*x) - (a + b
*ArcSec[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcSec[c*x])/(2*e^2*(e + d/x^2))
 - (b*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/
2)*Sqrt[c^2*d + e]) - (b*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sq
rt[1 - 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)) + ((a + b*ArcSec[c*x
])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*
e^3) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^
(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcSec[c*x
```

```
])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*
e^3) - ((a + b*ArcSec[c*x])*Log[1 + E^((2*I)*ArcSec[c*x])])/e^3 - ((I/2)*b*
PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/
e^3 - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^
2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e]))])/e^3 - ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSe
c[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + ((I/2)*b*PolyLog[2, -E^((2*I)*
ArcSec[c*x])])/e^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 4814

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])/(2*e*(p + 1))), x]
 + Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
```

`&& IntegerQ[m] && IntegerQ[p]`

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{e^3x} - \frac{dx\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^3} - \frac{dx\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} - \frac{dx\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e^3\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{a+b\sec^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\sec^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{\text{Subst}\left(\int (a+bx)\tan(x)\,dx, x, \sec^{-1}(cx)\right)}{e^3} + \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e^3} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{2ce^2} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{4ce}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\sec^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\sec^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\sec^{-1}(cx)\right)^2}{2be^3} - \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}\,dx, x, \sec^{-1}(cx)\right)}{e^3} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^3} + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^3} - \frac{b\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2ce^2} - \frac{\left(b\left(c^2d+2e\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{8ce^2\left(c^2d+e\right)}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\sec^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\sec^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\sec^{-1}(cx))^2}{2be^3} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} + \frac{b\text{Subst}\left(\int\log\left(1+e^{2ix}\right)dx, x, \sec^{-1}(cx)\right)}{e^3} + \frac{\sqrt{-d}\text{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}dx, x, \sec^{-1}(cx)\right)}{2e^3} - \frac{\sqrt{-d}\text{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}dx, x, \sec^{-1}(cx)\right)}{2e^3} - \frac{(b(c^2d+2e))\text{Subst}\left(\int\frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8ce^2(c^2d+e)}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\sec^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\sec^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}(c^2d+e)^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} - \frac{(ib)\text{Subst}\left(\int\frac{\log(1+x)}{x}dx, x, e^{2i\sec^{-1}(cx)}\right)}{2e^3} - \frac{(i\sqrt{-d})\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{2e^3} - \frac{(i\sqrt{-d})\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{2e^3} + \frac{(i\sqrt{-d})\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{2e^3} + \frac{(i\sqrt{-d})\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}dx, x, \sec^{-1}(cx)\right)}{2e^3}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\sec^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\sec^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$

$$-\frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}(c^2d+e)^{3/2}}$$

$$+\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$

$$+\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$

$$-\frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^3}$$

$$-\frac{b\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e^3}$$

$$-\frac{b\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e^3}$$

$$-\frac{b\mathrm{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e^3}$$

$$-\frac{b\mathrm{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\sec^{-1}(cx)\right)}{2e^3}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\sec^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\sec^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$
$$- \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}(c^2d+e)^{3/2}}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^3}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2e^3}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2e^3}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2e^3}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\sec^{-1}(cx)}\right)}{2e^3}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\sec^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\sec^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$
$$- \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}(c^2d+e)^{3/2}}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+e^{2i\sec^{-1}(cx)}\right)}{e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{2e^3}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1805 vs. $2(707) = 1414$.

Time = 7.51 (sec) , antiderivative size = 1805, normalized size of antiderivative = 2.55

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{ad^2}{4e^3(d+ex^2)^2}+\frac{ad}{e^3(d+ex^2)}+\frac{a\log(d+ex^2)}{2e^3}$$

$$+b\left(-\frac{7i\sqrt{d}\left(-\frac{\sec^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex}+\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}\right)}{16e^{5/2}}\right.$$

$$+\frac{7i\sqrt{d}\left(-\frac{\sec^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex}-\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(-\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}\right)}{16e^{5/2}}$$

$$-\frac{d\left(-\frac{\sec^{-1}(cx)}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}x\right)^2}+\frac{\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-i\left(\frac{c\sqrt{d}\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}{(c^2d+e)\left(-i\sqrt{d}+\sqrt{e}x\right)}+\frac{(2c^2d+e)\log\left(-\frac{4d\sqrt{e}\sqrt{c^2d+e}\left(i\sqrt{e}+c\left(c\sqrt{d}-\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{(2c^2d+e)\left(-i\sqrt{d}+\sqrt{e}x\right)}\right)}{(c^2d+e)^{3/2}}\right)}{d}\right)}{16e^{5/2}}$$

$$-\frac{d\left(\frac{ic\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}{\sqrt{d}(c^2d+e)\left(i\sqrt{d}+\sqrt{e}x\right)}-\frac{\sec^{-1}(cx)}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}x\right)^2}+\frac{\arcsin\left(\frac{1}{cx}\right)}{d\sqrt{e}}-\frac{i(2c^2d+e)\log\left(\frac{4d\sqrt{e}\sqrt{c^2d+e}\left(-i\sqrt{e}+c\left(c\sqrt{d}+\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{(2c^2d+e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d(c^2d+e)^{3/2}}\right)}{16e^{5/2}}$$

$$+\frac{i\left(8\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-2i\sec^{-1}(cx)\log\left(1+\frac{i\left(\sqrt{e}-\sqrt{c^2d+e}\right)e^{i\sec^{-1}(cx)}}{c\sqrt{d}}\right.\right.}{}$$

```
[In] Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x
^2])/(2*e^3) + b*((((-7*I)/16)*Sqrt[d]*(-(ArcSec[c*x]/(I*Sqrt[d]*Sqrt[e] +
e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(
I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d)
- e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2) + (((
7*I)/16)*Sqrt[d]*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[
1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt
[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*
Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2) - (d*(-(ArcSec[c*x]/(Sq
rt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] - I*((c*Sqr
t[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*
x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*Sq
rt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*Sqr
t[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/(16*e^(5/2)) - (d*((I*c*Sqrt[e
]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) -
ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/(c*x)]/(d*Sqrt[e
]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c*(c
*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I*Sqr
t[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)) + ((I/4)*(8*ArcSi
n[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e]
)*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt
[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[
1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]
)*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] +
 Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*
x])] - 2*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*S
qrt[d])] - 2*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x])
)/(c*Sqrt[d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/e^3 + ((I/4)*(8*ArcSi
n[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt
[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(-
Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[S
qrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d
 + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 - (I*(Sqrt
[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[
1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]
)*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcS
ec[c*x])] - 2*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x
]))/(c*Sqrt[d])] - 2*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[
c*x]))/(c*Sqrt[d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/e^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.69 (sec) , antiderivative size = 1396, normalized size of antiderivative = 1.97

method	result	size
parts	Expression too large to display	1396
derivativedivides	Expression too large to display	1409
default	Expression too large to display	1409

```
[In] int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/4*d^2/e^3/(e*x^2+d)^2+1/2/e^3*ln(e*x^2+d)+d/e^3/(e*x^2+d))+b/c^6*(-1/
8*c^6*(4*c^6*d^2*arcsec(c*x)*x^2+6*c^6*d*e*arcsec(c*x)*x^4+((c^2*x^2-1)/c^2
/x^2)^(1/2)*c^5*d^2*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^5*d*e*x^3+4*c^4*d*e*arc
sec(c*x)*x^2+6*arcsec(c*x)*e^2*c^4*x^4+I*c^4*d^2+2*I*c^4*d*e*x^2+I*e^2*c^4*
x^4)/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2+I/(c^2*d+e)/e^2*c^6*dilog(1+I*(1/c/x
+I*(1-1/c^2/x^2)^(1/2)))+I/(c^2*d+e)/e^2*c^6*dilog(1-I*(1/c/x+I*(1-1/c^2/x^
2)^(1/2)))-1/(c^2*d+e)/e^2*c^6*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1
/2)))-1/(c^2*d+e)/e^2*c^6*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))
-1/4*I/(c^2*d+e)/e^2*c^6*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e
)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x
-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*
d))-1/4*I/(c^2*d+e)/e^2*c^8*d*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcs
ec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c
^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/(c^
2*d+e)/e^3*c^8*d*arcsec(c*x)*ln(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/(c^2*d
+e)/e^3*c^8*d*arcsec(c*x)*ln(1-I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))-1/4*I/(c^2*
d+e)/e^3*c^8*d*sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcse
c(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^
2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/4*I/
(c^2*d+e)/e^3*c^10*d^2*sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)
*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)
^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+5/8*I*(e*(c^
2*d+e))^(1/2)/(c^2*d+e)^2/e^3*arctanh(1/4*(2*c^2*d*(1/c/x+I*(1-1/c^2/x^2)^(
1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))*c^8*d+I/(c^2*d+e)/e^3*c^8*d*dilog
(1+I*(1/c/x+I*(1-1/c^2/x^2)^(1/2)))+I/(c^2*d+e)/e^3*c^8*d*dilog(1-I*(1/c/x+
I*(1-1/c^2/x^2)^(1/2)))+3/4*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/e^2*arctanh(1
/4*(2*c^2*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2
))*c^6)
```

Fricas [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*x^5*arcsec(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + b*integrate(x^5*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3*x^6 +
3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

3.105 $\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bcx\sqrt{-1+c^2x^2}}{8e\,(c^2d+e)\,\sqrt{c^2x^2}\,(d+ex^2)} + \frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2} - \frac{bc(c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\,(c^2d+e)^{3/2}\,\sqrt{c^2x^2}}$$

```
[Out] 1/4*x^4*(a+b*arcsec(c*x))/d/(e*x^2+d)^2-1/8*b*c*(c^2*d+2*e)*x*arctan(e^(1/2
)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2))/d/e^(3/2)/(c^2*d+e)^(3/2)/(c^2*x^2)^(1
/2)+1/8*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5346, 12, 457, 79, 65, 211}

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2} - \frac{bcx(c^2d+2e)\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\sqrt{c^2x^2}\,(c^2d+e)^{3/2}} + \frac{bcx\sqrt{c^2x^2-1}}{8e\sqrt{c^2x^2}\,(c^2d+e)\,(d+ex^2)}$$

```
[In] Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*x*Sqrt[-1 + c^2*x^2])/(8*e*(c^2*d + e)*Sqrt[c^2*x^2]*(d + e*x^2)) + (x
^4*(a + b*ArcSec[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*(c^2*d + 2*e)*x*ArcTan[(
```

```
Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*d + e]])/(8*d*e^(3/2)*(c^2*d + e)^(3/2
)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2} - \frac{(bcx)\int \frac{x^3}{4d\sqrt{-1+c^2x^2}(d+ex^2)^2}\,dx}{\sqrt{c^2x^2}} \\
&= \frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2} - \frac{(bcx)\int \frac{x^3}{\sqrt{-1+c^2x^2}(d+ex^2)^2}\,dx}{4d\sqrt{c^2x^2}} \\
&= \frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2} - \frac{(bcx)\text{Subst}\left(\int \frac{x}{\sqrt{-1+c^2x}(d+ex)^2}\,dx, x, x^2\right)}{8d\sqrt{c^2x^2}} \\
&= \frac{bcx\sqrt{-1+c^2x^2}}{8e\,(c^2d+e)\sqrt{c^2x^2}\,(d+ex^2)} + \frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2} \\
&\quad - \frac{(bc(c^2d+2e)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}(d+ex)}\,dx, x, x^2\right)}{16de\,(c^2d+e)\sqrt{c^2x^2}} \\
&= \frac{bcx\sqrt{-1+c^2x^2}}{8e\,(c^2d+e)\sqrt{c^2x^2}\,(d+ex^2)} + \frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2} \\
&\quad - \frac{(b(c^2d+2e)\,x)\,\text{Subst}\left(\int \frac{1}{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{8cde\,(c^2d+e)\sqrt{c^2x^2}} \\
&= \frac{bcx\sqrt{-1+c^2x^2}}{8e\,(c^2d+e)\sqrt{c^2x^2}\,(d+ex^2)} + \frac{x^4(a+b\sec^{-1}(cx))}{4d\,(d+ex^2)^2} - \frac{bc(c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\,(c^2d+e)^{3/2}\sqrt{c^2x^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.48

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx =$$

$$-\frac{-\frac{4ad}{(d+ex^2)^2} + \frac{8a}{d+ex^2} - \frac{2bce\sqrt{1-\frac{1}{c^2x^2}}\,x}{(c^2d+e)(d+ex^2)} + \frac{4b(d+2ex^2)\sec^{-1}(cx)}{(d+ex^2)^2} + \frac{4b\arcsin\left(\frac{1}{cx}\right)}{d} + \frac{b\sqrt{e}(c^2d+2e)\log\left(-\frac{16d\sqrt{-c^2d-e}e^{3/2}\left(\sqrt{e}+c\left(ic\cdots\right.\right.}{b\left(c^2d+2e\right)\left(\cdots\right.}\cdots\right.}{d(-c^2d-e)^{3/2}}\cdots}{16e^2}$$

```
[In] Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 - 1/(c^
2*x^2)]*x)/((c^2*d + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcSec[c*x])/(d +
 e*x^2)^2 + (4*b*ArcSin[1/(c*x)])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(-16*d*S
qrt[-(c^2*d) - e]*e^(3/2)*(Sqrt[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sq
rt[1 - 1/(c^2*x^2)])*x))/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(-(
c^2*d) - e)^(3/2)) + (b*Sqrt[e]*(c^2*d + 2*e)*Log[((16*I)*d*Sqrt[-(c^2*d) -
 e]*e^(3/2)*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2
*x^2)])*x))/(b*(c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(-(c^2*d) - e)^(
3/2)))/e^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(135) = 270$.

Time = 5.55 (sec) , antiderivative size = 941, normalized size of antiderivative = 5.99

method	result
parts	$a\left(-\frac{1}{2e^2(e\,x^2+d)}+\frac{d}{4e^2(e\,x^2+d)^2}\right)+\frac{b\left(\frac{c^8\,\mathrm{arcsec}(cx)d}{4e^2\left(c^2e\,x^2+c^2d\right)^2}-\frac{c^6\,\mathrm{arcsec}(cx)}{2e^2\left(c^2e\,x^2+c^2d\right)}-\frac{c^3\sqrt{c^2x^2-1}\left(-4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{a\,c^6\left(-\frac{1}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}\right)+b\,c^6\left(-\frac{\mathrm{arcsec}(cx)}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{\mathrm{arcsec}(cx)d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}-\frac{\sqrt{c^2x^2-1}\left(-4\arctan\left(\frac{1}{\sqrt{c^2x\cdots}}\right.\right.}{\cdots}\right.}{\cdots}$
default	$\frac{a\,c^6\left(-\frac{1}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}\right)+b\,c^6\left(-\frac{\mathrm{arcsec}(cx)}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{\mathrm{arcsec}(cx)d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}-\frac{\sqrt{c^2x^2-1}\left(-4\arctan\left(\frac{1}{\sqrt{c^2x\cdots}}\right.\right.}{\cdots}\right.}{\cdots}$

```
[In] int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/e^2/(e*x^2+d)+1/4*d/e^2/(e*x^2+d)^2)+b/c^4*(1/4*c^8*arcsec(c*x)*d/e
^2/(c^2*e*x^2+c^2*d)^2-1/2*c^6*arcsec(c*x)/e^2/(c^2*e*x^2+c^2*d)-1/16*c^3*(
c^2*x^2-1)^(1/2)/e*(-4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4
*d^2-4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*d*e*x^2+ln(2*((
c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^
2*d*e)^(1/2)))*c^4*d^2+ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2
```

```
*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2+ln(-2*((c^2*x^2-1)
^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1
/2)))*c^4*d^2+ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1
/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2+2*(c^2*x^2-1)^(1/2)*(-(c^
2*d+e)/e)^(1/2)*c^2*d*e-4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*
c^2*d*e-4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*e^2*c^2*x^2+2*ln
(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x
+(-c^2*d*e)^(1/2)))*c^2*d*e+2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*
e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2+2*ln(-2*((c
^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^
2*d*e)^(1/2)))*c^2*d*e+2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-
c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x/d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(c*e*x+(-c^2*d*e)^(1/2))/
(-c*e*x+(-c^2*d*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(135) = 270$.

Time = 0.51 (sec) , antiderivative size = 1015, normalized size of antiderivative = 6.46

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{(d + ex^2)^3}\,dx$$

$$= \left[-\frac{4\,ac^4d^4 + 8\,ac^2d^3e + 4\,ad^2e^2 + 8\,(ac^4d^3e + 2\,ac^2d^2e^2 + ade^3)x^2 + (bc^2d^3 + (bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e + \cdots}{\cdots}\right.$$

$$-\frac{2\,ac^4d^4 + 4\,ac^2d^3e + 2\,ad^2e^2 + 4\,(ac^4d^3e + 2\,ac^2d^2e^2 + ade^3)x^2 + (bc^2d^3 + (bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e + \cdots}{\cdots}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e + 2*a*c^
2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d
^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*log((c^2*e*x^2
 - c^2*d + 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2 + d)) + 4
*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2
+ b*d*e^3)*x^2)*arcsec(c*x) - 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b
*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^
2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^2*d^3*e + b*d^2
*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^5*e^2 + 2*c
^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d
^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e +
 2*a*d^2*e^2 + 4*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3
 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^
```

```
2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2 - 1)/(c^2*d
+ e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2
*d^2*e^2 + b*d*e^3)*x^2)*arcsec(c*x) - 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2
*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c
^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (b*c^2*d^3*e
+ b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^5*e^
2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2
*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^3}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] -1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/4*((2*e*x^2 + d)
*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 4*(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)*
integrate(1/4*(2*c^2*e*x^3 + c^2*d*x)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1
))/(c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^
3)*x^2 + (c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 -
2*d*e^3)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x))*b/(e^4*x^4 + 2*d*e^3*x^
2 + d^2*e^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

```
[In] int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

3.106 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 193

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bcx\sqrt{-1+c^2x^2}}{8d\,(c^2d+e)\,\sqrt{c^2x^2}\,(d+ex^2)} - \frac{a+b\sec^{-1}(cx)}{4e\,(d+ex^2)^2} + \frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}} - \frac{bc(3c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\,(c^2d+e)^{3/2}\,\sqrt{c^2x^2}}$$

```
[Out] 1/4*(-a-b*arcsec(c*x))/e/(e*x^2+d)^2+1/4*b*c*x*arctan((c^2*x^2-1)^(1/2))/d^
2/e/(c^2*x^2)^(1/2)-1/8*b*c*(3*c^2*d+2*e)*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2
)/(c^2*d+e)^(1/2))/d^2/(c^2*d+e)^(3/2)/e^(1/2)/(c^2*x^2)^(1/2)-1/8*b*c*x*(c
^2*x^2-1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5344, 457, 105, 162, 65, 211}

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{a+b\sec^{-1}(cx)}{4e\,(d+ex^2)^2} + \frac{bcx\arctan\left(\sqrt{c^2x^2-1}\right)}{4d^2e\sqrt{c^2x^2}} - \frac{bcx(3c^2d+2e)\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\sqrt{c^2x^2}\,(c^2d+e)^{3/2}} - \frac{bcx\sqrt{c^2x^2-1}}{8d\sqrt{c^2x^2}\,(c^2d+e)\,(d+ex^2)}$$

```
[In] Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*c*x*Sqrt[-1 + c^2*x^2])/(d*(c^2*d + e)*Sqrt[c^2*x^2]*(d + e*x^2)) -
 (a + b*ArcSec[c*x])/(4*e*(d + e*x^2)^2) + (b*c*x*ArcTan[Sqrt[-1 + c^2*x^2]
])/(4*d^2*e*Sqrt[c^2*x^2]) - (b*c*(3*c^2*d + 2*e)*x*ArcTan[(Sqrt[e]*Sqrt[-1
 + c^2*x^2])/Sqrt[c^2*d + e]])/(8*d^2*Sqrt[e]*(c^2*d + e)^(3/2)*Sqrt[c^2*x^
2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5344

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\int\frac{1}{x\sqrt{-1+c^2x^2}(d+ex^2)^2}\,dx}{4e\sqrt{c^2x^2}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}(d+ex)^2}\,dx,x,x^2\right)}{8e\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{(bcx)\text{Subst}\left(\int\frac{-c^2d-e+\frac{1}{2}c^2ex}{x\sqrt{-1+c^2x}(d+ex)}\,dx,x,x^2\right)}{8de\left(c^2d+e\right)\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{8d^2e\sqrt{c^2x^2}}-\frac{(bc(3c^2d+2e)\,x)\,\text{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}(d+ex)}\,dx,x,x^2\right)}{16d^2\left(c^2d+e\right)\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bx)\text{Subst}\left(\int\frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{4cd^2e\sqrt{c^2x^2}}-\frac{(b(3c^2d+2e)\,x)\,\text{Subst}\left(\int\frac{1}{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{8cd^2\left(c^2d+e\right)\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}-\frac{a+b\sec^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}}-\frac{bc(3c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\left(c^2d+e\right)^{3/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.00

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{1}{16}\left(-\frac{4a}{e(d+ex^2)^2}-\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}x}{d(c^2d+e)(d+ex^2)}-\frac{4b\sec^{-1}(cx)}{e(d+ex^2)^2}-\frac{4b\arcsin\left(\frac{1}{cx}\right)}{d^2e}\right.$$
$$-\frac{b(3c^2d+2e)\log\left(-\frac{16d^2\sqrt{-c^2d-e}\sqrt{e}\left(\sqrt{e}+c\left(ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2(-c^2d-e)^{3/2}\sqrt{e}}$$
$$\left.-\frac{b(3c^2d+2e)\log\left(\frac{16id^2\sqrt{-c^2d-e}\sqrt{e}\left(-\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d^2(-c^2d-e)^{3/2}\sqrt{e}}\right)$$

```
[In] Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
[Out] ((-4*a)/(e*(d + e*x^2)^2) - (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d + e)*
(d + e*x^2)) - (4*b*ArcSec[c*x])/(e*(d + e*x^2)^2) - (4*b*ArcSin[1/(c*x)])/
(d^2*e) - (b*(3*c^2*d + 2*e)*Log[(-16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(Sqrt[
e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(3*c
^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e]) -
 (b*(3*c^2*d + 2*e)*Log[((16*I)*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(-Sqrt[e] +
c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(3*c^2*d
+ 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e]))/16
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. 2(168) = 336.

Time = 5.55 (sec) , antiderivative size = 894, normalized size of antiderivative = 4.63

method	result
parts	$-\frac{a}{4e(e\,x^2+d)^2}+\frac{b\left(-\frac{c^6\,\mathrm{arcsec}(cx)}{4e\left(c^2e\,x^2+c^2d\right)^2}+\frac{c\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4de\,x^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d}{e}}\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a\,c^6}{4e\left(c^2e\,x^2+c^2d\right)^2}+b\,c^6\left(-\frac{\mathrm{arcsec}(cx)}{4e\left(c^2e\,x^2+c^2d\right)^2}+\frac{\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4d^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d}{e}}\right.}{}\right.}{}$
default	$\frac{-\frac{a\,c^6}{4e\left(c^2e\,x^2+c^2d\right)^2}+b\,c^6\left(-\frac{\mathrm{arcsec}(cx)}{4e\left(c^2e\,x^2+c^2d\right)^2}+\frac{\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4d^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d}{e}}\right.}{}\right.}{}$

```
[In] int(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arcsec(c*x)+1/16
*c*(c^2*x^2-1)^(1/2)*(4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^
4*d*e*x^2+4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*d^2-3*ln(-
2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x
+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-3*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(
1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-3*ln(2*((
c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^
2*d*e)^(1/2)))*c^4*d*e*x^2-3*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e
-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2+4*arctan(1/(c^2*
x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*e^2*c^2*x^2+4*arctan(1/(c^2*x^2-1)^(1/2)
)*(-(c^2*d+e)/e)^(1/2)*c^2*d*e+2*(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*c^2
*d*e-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x
-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^
2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*
e-2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/
(c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)
/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/x/d^2/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-c*e*x+(-c^2*
d*e)^(1/2))/(c*e*x+(-c^2*d*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(165) = 330.

Time = 0.49 (sec) , antiderivative size = 888, normalized size of antiderivative = 4.60

$$\int \frac{x(a + b\sec^{-1}(cx))}{(d + ex^2)^3}\, dx$$
$$= \left[-\frac{4\,ac^4d^4 + 8\,ac^2d^3e + 4\,ad^2e^2 + (3\,bc^2d^3 + (3\,bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e + 2\,(3\,bc^2d^2e + 2\,bde^2)x^2)\sqrt{-c^2d}}{} \right.$$
$$-\frac{2\,ac^4d^4 + 4\,ac^2d^3e + 2\,ad^2e^2 + (3\,bc^2d^3 + (3\,bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e + 2\,(3\,bc^2d^2e + 2\,bde^2)x^2)\sqrt{c^2de +}}{}$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^2
*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt
(-c^2*d*e - e^2)*log((c^2*e*x^2 - c^2*d + 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x
^2 - 1) - 2*e)/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*arc
sec(c*x) - 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*
c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*a
rctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e
^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3
+ (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*
e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*e^2 + (3*b
*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2
*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2
- 1)/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*arcsec(c*x) -
 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3
+ b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x
 + sqrt(c^2*x^2 - 1)) + (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3
)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e
^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^
4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(c^2*e^3*x^4 + 2*c^2*d*e^2*x^2 + c^2*d^2*e)*integrate(1/4*x*e^(1/2*l
og(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 - d^
2*e + (c^2*d^2*e - 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 -
d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) - a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e) - 1/4
*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

3.107 $\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 685

$$\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)^3}\,dx &= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\,(c^2d+e)\,\left(e+\frac{d}{x^2}\right)x}+\frac{e^2(a+b\sec^{-1}(cx))}{4d^3\,\left(e+\frac{d}{x^2}\right)^2}-\frac{e(a+b\sec^{-1}(cx))}{d^3\,\left(e+\frac{d}{x^2}\right)} \\
&+\frac{i(a+b\sec^{-1}(cx))^2}{2bd^3}-\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}} \\
&+\frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3\,(c^2d+e)^{3/2}} \\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} \\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} \\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} \\
&-\frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} \\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} \\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}
\end{aligned}$$

```
[Out] 1/4*e^2*(a+b*arcsec(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arcsec(c*x))/d^3/(e+d/x^2)
+1/2*I*(a+b*arcsec(c*x))^2/b/d^3-1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arcsec(
c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))/d^3-1/2*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arcsec(c*x))*ln(1+c*(1/c/x+I*(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polyl
og(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/d^3+1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,c*(1/c/x+I*(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/8*b*(c^2*d+2*e)*
arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))*e^(1/2)/d^3/(c^2*d+
e)^(3/2)-b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))*e^(1/2)/
d^3/(c^2*d+e)^(1/2)+1/8*b*c*e*(1-1/c^2/x^2)^(1/2)/d^2/(c^2*d+e)/(e+d/x^2)/x
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules

used = {5348, 4818, 4814, 390, 385, 211, 4826, 4616, 2221, 2317, 2438}

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = -\frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3} + \frac{e^2(a+b\sec^{-1}(cx))}{4d^3\left(\frac{d}{x^2}+e\right)^2} - \frac{e(a+b\sec^{-1}(cx))}{d^3\left(\frac{d}{x^2}+e\right)} + \frac{i(a+b\sec^{-1}(cx))^2}{2bd^3} + \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8d^3\left(c^2d+e\right)^{3/2}} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d^3\sqrt{c^2d+e}} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2x\left(c^2d+e\right)\left(\frac{d}{x^2}+e\right)}$$

```
[In] Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] (b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(8*d^2*(c^2*d + e)*(e + d/x^2)*x) + (e^2*(a +
 b*ArcSec[c*x]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcSec[c*x]))/(d^3*(e +
d/x^2)) + ((I/2)*(a + b*ArcSec[c*x])^2)/(b*d^3) - (b*Sqrt[e]*ArcTan[Sqrt[c^
2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^2*d + e]) + (b*S
qrt[e]*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)
]*x)])/(8*d^3*(c^2*d + e)^(3/2)) - ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]
*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSec[
c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/
(2*d^3) - ((a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt
[e] + Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]
*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) + ((I/2)*b*PolyLo
g[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/d^3 +
((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
e])])/d^3 + ((I/2)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] +
```

```
 Sqrt[c^2*d + e]))])/d^3 + ((I/2)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]
))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
```

```
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4814

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])/(2*e*(p + 1))), x]
 + Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^5\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{e^2x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3}-\frac{2ex\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2}\right.\right.\\
&\qquad\left.\left.+\frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2}+\frac{(2e)\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2}\\
&\quad-\frac{e^2\text{Subst}\left(\int \frac{x\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d^2}
\end{aligned}$$

$$= \frac{e^2(a+b\sec^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\sec^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int\left(-\frac{\sqrt{-d}(a+b\arccos(\frac{x}{c}))}{2d(\sqrt{e}-\sqrt{-d}x)}+\frac{\sqrt{-d}(a+b\arccos(\frac{x}{c}))}{2d(\sqrt{e}+\sqrt{-d}x)}\right)dx,x,\frac{1}{x}\right)}{d^2} - \frac{(be)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{c^2}}(e+dx^2)}dx,x,\frac{1}{x}\right)}{cd^3} + \frac{(be^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{c^2}}(e+dx^2)^2}dx,x,\frac{1}{x}\right)}{4cd^3}$$

$$= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\sec^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\sec^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int\frac{a+b\arccos(\frac{x}{c})}{\sqrt{e}-\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{2(-d)^{5/2}} + \frac{\text{Subst}\left(\int\frac{a+b\arccos(\frac{x}{c})}{\sqrt{e}+\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{2(-d)^{5/2}} - \frac{(be)\text{Subst}\left(\int\frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}dx,x,\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{cd^3} + \frac{(be(c^2d+2e))\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{c^2}}(e+dx^2)}dx,x,\frac{1}{x}\right)}{8cd^3(c^2d+e)}$$

$$= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\sec^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\sec^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}} + \frac{\text{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}dx,x,\sec^{-1}(cx)\right)}{2(-d)^{5/2}} - \frac{\text{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}dx,x,\sec^{-1}(cx)\right)}{2(-d)^{5/2}} + \frac{(be(c^2d+2e))\,\text{Subst}\left(\int\frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}dx,x,\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8cd^3(c^2d+e)}$$

$$= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\sec^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\sec^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$+ \frac{i(a+b\sec^{-1}(cx))^2}{2bd^3} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}}$$

$$+ \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3(c^2d+e)^{3/2}}$$

$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2(-d)^{5/2}}$$

$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2(-d)^{5/2}}$$

$$+ \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2(-d)^{5/2}}$$

$$+ \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2(-d)^{5/2}}$$

$$= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\sec^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\sec^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$+ \frac{i(a+b\sec^{-1}(cx))^2}{2bd^3} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}}$$

$$+ \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3(c^2d+e)^{3/2}}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \sec^{-1}(cx)\right)}{2d^3}$$

$$= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\sec^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\sec^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)}$$
$$+ \frac{i(a+b\sec^{-1}(cx))^2}{2bd^3} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}}$$
$$+ \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3(c^2d+e)^{3/2}}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d^3}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d^3}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d^3}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{2d^3}$$

$$= \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\sec^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\sec^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\sec^{-1}(cx))^2}{2bd^3} - \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}} + \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d+e\right)^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1871 vs. $2(685) = 1370$.

Time = 6.06 (sec) , antiderivative size = 1871, normalized size of antiderivative = 2.73

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\frac{a}{4d\left(d+ex^2\right)^2}+\frac{a}{2d^2\left(d+ex^2\right)}+\frac{a\log(x)}{d^3}-\frac{a\log\left(d+ex^2\right)}{2d^3}$$

$$+b\left(-\frac{5i\sqrt{e}\left(-\frac{\sec^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex}+\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}\right)}{16d^{5/2}}\right.$$

$$+\frac{5i\sqrt{e}\left(-\frac{\sec^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex}-\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(-\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}\right)}{16d^{5/2}}$$

$$+\frac{\sqrt{e}\left(-\frac{\sec^{-1}(cx)}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}x\right)^2}+\frac{\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-i\left(\frac{c\sqrt{d}\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}{\left(c^2d+e\right)\left(-i\sqrt{d}+\sqrt{e}x\right)}+\frac{\left(2c^2d+e\right)\log\left(-\frac{4d\sqrt{e}\sqrt{c^2d+e}\left(i\sqrt{e}+c\left(c\sqrt{d}-\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\left(2c^2d+e\right)\left(-i\sqrt{d}+\sqrt{e}x\right)}\right)}{\left(c^2d+e\right)^{3/2}}\right)}{d}\right.}{16d^2}$$

$$+\frac{\sqrt{e}\left(\frac{ic\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}{\sqrt{d}(c^2d+e)\left(i\sqrt{d}+\sqrt{e}x\right)}-\frac{\sec^{-1}(cx)}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}x\right)^2}+\frac{\arcsin\left(\frac{1}{cx}\right)}{d\sqrt{e}}-\frac{i(2c^2d+e)\log\left(\frac{4d\sqrt{e}\sqrt{c^2d+e}\left(-i\sqrt{e}+c\left(c\sqrt{d}+\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\left(2c^2d+e\right)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d(c^2d+e)^{3/2}}\right.}{16d^2}$$

$$+\frac{\frac{1}{2}i\sec^{-1}(cx)^2-\sec^{-1}(cx)\log\left(1+e^{2i\sec^{-1}(cx)}\right)+\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}(cx)}\right)}{d^3}$$

$$i\left(8\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-2i\sec^{-1}(cx)\log\left(1+\frac{i\left(\sqrt{e}-\sqrt{c^2d+e}\right)e^{i\sec^{-1}(cx)}}{c\sqrt{d}}\right.\right.$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^3),x]

[Out] a/(4*d*(d + e*x^2)^2) + a/(2*d^2*(d + e*x^2)) + (a*Log[x])/d^3 - (a*Log[d +
 e*x^2])/(2*d^3) + b*((((-5*I)/16)*Sqrt[e]*(-(ArcSec[c*x]/(I*Sqrt[d]*Sqrt[e
] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] +
 c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2
*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/d^(5/2) +
 (((5*I)/16)*Sqrt[e]*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(Arc
Sin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] +
Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d]
+ I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/d^(5/2) + (Sqrt[e]*(-(ArcSe
c[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] -
 I*((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d]
+ Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e]
 + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)
*((-I)*Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/(16*d^2) + (Sqrt[e]*
((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sq
rt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/(c*x
)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sq
rt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d
 + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*d^2) + ((I/2)*
ArcSec[c*x]^2 - ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] + (I/2)*PolyLog[
2, -E^((2*I)*ArcSec[c*x])])/d^3 - ((I/4)*(8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt
[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d]
)]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sq
rt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*Arc
Sec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d]
)] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*PolyLog[2, (I*(-S
qrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*PolyLog[2, ((
-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + PolyLog[2
, -E^((2*I)*ArcSec[c*x])]))/d^3 - ((I/4)*(8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/S
qrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])
*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/
(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d]
)]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sq
rt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*PolyLog[2, (
(-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*PolyL
og[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + Poly
Log[2, -E^((2*I)*ArcSec[c*x])]))/d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.87 (sec) , antiderivative size = 3533, normalized size of antiderivative = 5.16

method	result	size
parts	Expression too large to display	3533
derivativedivides	Expression too large to display	3607
default	Expression too large to display	3607

```
[In] int((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a/d^3*ln(x)+1/2*a/d^2/(e*x^2+d)-1/2*a/d^3*ln(e*x^2+d)+1/4*a/d/(e*x^2+d)^2+b
*(1/4*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)/
e/(c^4*d^2+2*c^2*d*e+e^2)*c^2/d^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^
2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-I*(-(e*(c^2*d+e))^(1/2)*c
^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*polylog(2,d*c^2*(1/c/x+I*(1
-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))/d^4/(c^4*d^2+2*c^2
*d*e+e^2)/c^2+3/4*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*polylog(2,d*c^2*(1/c/
x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e/c^2/d^4/(c
^2*d+e)+2*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e
^2)/d^4*e/(c^4*d^2+2*c^2*d*e+e^2)/c^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/
2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-I*(-(e*(c^2*d+e))^(1/
2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e^2*arcsec(c*x)^2/(c^4*d^
2+2*c^2*d*e+e^2)/d^5/c^4-1/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d/e*polylog(
2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))
*c^4+3/2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arcsec(c*x)^2*e/c^2/d^4/(c^2*d
+e)+I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arcsec(c*x)^2*e^2/c^4/d^5/(c^2*d+e)
-1/2*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)
*e^2*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e)
)^(1/2)-2*e))/(c^4*d^2+2*c^2*d*e+e^2)/d^5/c^4-2*I*(-(e*(c^2*d+e))^(1/2)*c^2
*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arcsec(c*x)^2/d^4/(c^4*d^2+2*
c^2*d*e+e^2)/c^2+5/4*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^
(1/2)*e+2*e^2)/d^3/(c^4*d^2+2*c^2*d*e+e^2)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2
)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)+1/2*I*arcsec(c*x
)^2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^3/(c^2*d+e)-5/4*I*(-(e*(c^2*d+e))^(
1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arcsec(c*x)^2/d^3/(c^4*
d^2+2*c^2*d*e+e^2)-5/8*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+
e))^(1/2)*e+2*e^2)*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-
2*(e*(c^2*d+e))^(1/2)-2*e))/d^3/(c^4*d^2+2*c^2*d*e+e^2)-1/8*e*(8*c^6*d^2*ar
csec(c*x)*x^2+6*c^6*d*e*arcsec(c*x)*x^4-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^5*d^2
*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^5*d*e*x^3+8*c^4*d*e*arcsec(c*x)*x^2+6*arcs
ec(c*x)*e^2*c^4*x^4-I*c^4*d^2-2*I*c^4*d*e*x^2-I*e^2*c^4*x^4)/d^3/(c^2*d+e)/
(c^2*e*x^2+c^2*d)^2+1/4*I*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(
-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/d^3/(c
```

```
^2*d+e)+1/2*I/(c^2*d+e)/d^3*e*sum((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^
2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R
1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z
^2+c^2*d))+I/(c^2*d+e)/d^2*c^2*arcsec(c*x)^2+I/(c^2*d+e)/d^3*e*arcsec(c*x)^
2+1/2*I/(c^2*d+e)/d^2*c^2*sum((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+
2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/
c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c
^2*d))-1/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/(c^2*d+e)/d^3*ln(1-d*c^2*(1/c/
x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-
1/2*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*arcsec(c*x)^2+3/4*I*(e*(c^2*d+e
))^(1/2)/(c^2*d+e)^2/d^3*e*arctanh(1/4*(2*c^2*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2
))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))+1/2*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/
d^3*e*arcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(
c^2*d+e))^(1/2)-2*e))+7/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*arctanh(1/4
*(2*c^2*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^(1/2))
*c^2-3/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*polylog(2,d*c^2*(1/c/x+I*(1-
1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))*c^2-1/4*I*(e*(c^2*d
+e))^(1/2)/(c^2*d+e)^2/d^3*e*polylog(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^
2/(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e))+3/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d
^2*c^2*arcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*
(c^2*d+e))^(1/2)-2*e))-3/4*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*arcsec(c*x
)^2*c^2-3/2*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c^2/d^4/(c^2*d+e)*e*ln(1-d*c^
2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcse
c(c*x)-1/4*I*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+
2*e^2)*c^2*arcsec(c*x)^2/d^2/e/(c^4*d^2+2*c^2*d*e+e^2)-1/4*I*(e*(c^2*d+e))^
(1/2)/(c^2*d+e)^2/d/e*arcsec(c*x)^2*c^4-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)/c
^4/d^5/(c^2*d+e)*e^2*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(
e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)+1/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d/
e*c^4*arcsec(c*x)*ln(1-d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d+2*(e*(
c^2*d+e))^(1/2)-2*e))+1/2*I*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*polylog(2,d*c
^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*e^2/
c^4/d^5/(c^2*d+e)+(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/
2)*e+2*e^2)/(c^4*d^2+2*c^2*d*e+e^2)/d^5*e^2/c^4*ln(1-d*c^2*(1/c/x+I*(1-1/c^
2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e))*arcsec(c*x)-1/8*I*(-(e*
(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c^2*polylog
(2,d*c^2*(1/c/x+I*(1-1/c^2/x^2)^(1/2))^2/(-c^2*d-2*(e*(c^2*d+e))^(1/2)-2*e)
)/d^2/e/(c^4*d^2+2*c^2*d*e+e^2)-1/2*I*arcsec(c*x)^2/d^3)
```

Fricas [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \int \frac{b\,\mathrm{arcsec}\left(cx\right) + a}{\left(ex^2 + d\right)^3 x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*asec(c*x))/x/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \int \frac{b\,\mathrm{arcsec}\left(cx\right) + a}{\left(ex^2 + d\right)^3 x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)
/d^3 + 4*log(x)/d^3) + b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3
*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x\,(d+ex^2)^3}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2+d)^3}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^3),x)
[Out] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

3.108 $\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 1124

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} + \frac{3b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} + \frac{3b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

```
[Out] 3/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arcsec(c*x))*ln(1+c*(1/
c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-
d)^(1/2)+3/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arcsec(c*x))*l
n(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
^(5/2)/(-d)^(1/2)-3/16*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*I*b*polylog(2,-c*(1
/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(
-d)^(1/2)-3/16*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*I*b*polylog(2,-c*(1/c/x+I*(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2
)+1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-
1/c^2/x^2)^(1/2))/e/(c^2*d+e)^(3/2)/d^(1/2)+1/16*b*arctanh((c^2*d+(-d)^(1/2
)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/(c^2*d+e)^(3/
2)/d^(1/2)+1/16*(a+b*arcsec(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(1/
2))^2+3/16*(a+b*arcsec(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arcsec
(c*x))*(-d)^(1/2)/e^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2-3/16*(a+b*arcsec(c*x))
/e^2/(d/x+(-d)^(1/2)*e^(1/2))+3/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c
/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d+e)^(1/2)+3
/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c
^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d+e)^(1/2)+1/16*b*c*(-d)^(1/2)*(1-1/c^2/x^2
)^(1/2)/e^(3/2)/(c^2*d+e)/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*b*c*(-d)^(1/2)*(1-
1/c^2/x^2)^(1/2)/e^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 1124, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules

used = {5348, 4758, 4828, 745, 739, 212, 4826, 4616, 2221, 2317, 2438}

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2} + \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{dc^2+e}} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e(dc^2+e)^{3/2}} + \frac{3b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{dc^2+e}} + \frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e(dc^2+e)^{3/2}} + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\sec^{-1}(cx))\log\left(\frac{\sqrt{-d}e^{i\sec^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}} + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\sec^{-1}(cx))\log\left(\frac{\sqrt{-d}e^{i\sec^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}} + \frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

```
[In] Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt
[e] - d/x)) + (b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)*
(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcSec[c*x]))/(16*e^(3/2)*(Sqr
t[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcSec[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e]
 - d/x)) - (Sqrt[-d]*(a + b*ArcSec[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] + d
/x)^2) - (3*(a + b*ArcSec[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*Arc
Tanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(
c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) + (3*b*ArcTanh[(c^2*d - (Sqrt
[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sq
rt[d]*e^2*Sqrt[c^2*d + e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*S
qrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(
3/2)) + (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d +
 e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) + (3*(a + b*A
rcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^
(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(
a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c
^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt
[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2))
 + (((3*I)/16)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqr
t[c^2*d + e]))])/(Sqrt[-d]*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, (c*Sqrt[-d]*
E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*
I)/16)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d
+ e]))])/(Sqrt[-d]*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*Arc
Sec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

```
&& NeQ[m, -1]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3}-\frac{3d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\right.\right.\\
&\qquad -\frac{d^3\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3}-\frac{3d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\\
&\qquad \left.\left.-\frac{3d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{8e^2\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)\\
&= \frac{(3d)\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2}+\frac{(3d)\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2}\\
&\quad +\frac{(3d)\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{8e^2}-\frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8e^{3/2}}\\
&\quad -\frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8e^{3/2}}
\end{aligned}$$

$$= \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{(3b)\text{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}-dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^2} - \frac{(3b)\text{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^2} + \frac{(3d)\text{Subst}\left(\int \left(-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}-\sqrt{-d}x)} - \frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}+\sqrt{-d}x)}\right)dx, x, \frac{1}{x}\right)}{8e^2} + \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}-dx)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^{3/2}} - \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}+dx)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^{3/2}}$$

$$= \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16e^{5/2}} - \frac{3\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16e^{5/2}} - \frac{(3b)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16ce^2} + \frac{(3b)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16ce^2} + \frac{b\text{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}-dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce(c^2d+e)} - \frac{b\text{Subst}\left(\int \frac{1}{(\sqrt{-d}\sqrt{e}+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce(c^2d+e)}$$

$$= \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$- \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}}$$

$$+ \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} + \frac{3\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{16e^{5/2}}$$

$$+ \frac{3\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{16e^{5/2}}$$

$$- \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16ce\left(c^2d+e\right)} + \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16ce\left(c^2d+e\right)}$$

$$= \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+ \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+ \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} + \frac{3b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}}$$
$$+ \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} + \frac{3b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}}$$
$$- \frac{(3i)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{16e^{5/2}}$$
$$- \frac{(3i)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{16e^{5/2}}$$
$$- \frac{(3i)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{16e^{5/2}}$$
$$- \frac{(3i)\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{16e^{5/2}}$$

$$= \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$- \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}}$$

$$+ \frac{3b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} + \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}}$$

$$+ \frac{3b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{(3b)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \sec^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{(3b)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \sec^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{(3b)\text{Subst}\left(\int\log\left(1-\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \sec^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{(3b)\text{Subst}\left(\int\log\left(1+\frac{\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \sec^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$= \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$- \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}}$$

$$+ \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}}$$

$$+ \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{(3ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{(3ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{(3ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{(3ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\sec^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}}$$

$$
\begin{aligned}
&= \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+ \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\sec^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&- \frac{3(a+b\sec^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} \\
&+ \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} \\
&+ \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&- \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+ \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&- \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+ \frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+ \frac{3ib\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 6.07 (sec) , antiderivative size = 1819, normalized size of antiderivative = 1.62

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{adx}{4e^2(d+ex^2)^2}-\frac{5ax}{8e^2(d+ex^2)}+\frac{3a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{5/2}}$$

$$+b\left(\frac{5\left(-\frac{\sec^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex}+\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}\right)}{16e^2}\right.$$

$$+\frac{5\left(-\frac{\sec^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex}-\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(-\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}\right)}{16e^2}$$

$$+\frac{i\sqrt{d}\left(-\frac{\sec^{-1}(cx)}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}x\right)^2}+\frac{\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-i\left(\frac{c\sqrt{d}\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}{\left(c^2d+e\right)\left(-i\sqrt{d}+\sqrt{e}x\right)}+\frac{\left(2c^2d+e\right)\log\left(-\frac{4d\sqrt{e}\sqrt{c^2d+e}\left(i\sqrt{e}+c\left(c\sqrt{d}-\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\left(2c^2d+e\right)\left(-i\sqrt{d}+\sqrt{e}x\right)}\right)}{\left(c^2d+e\right)^{3/2}}\right)}{d}\right)}{16e^2}$$

$$-\frac{i\sqrt{d}\left(\frac{ic\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}{\sqrt{d}(c^2d+e)\left(i\sqrt{d}+\sqrt{e}x\right)}-\frac{\sec^{-1}(cx)}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}x\right)^2}+\frac{\arcsin\left(\frac{1}{cx}\right)}{d\sqrt{e}}-\frac{i(2c^2d+e)\log\left(\frac{4d\sqrt{e}\sqrt{c^2d+e}\left(-i\sqrt{e}+c\left(c\sqrt{d}+\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\left(2c^2d+e\right)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d(c^2d+e)^{3/2}}\right)}{16e^2}$$

$$+\frac{3\left(8\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-2i\sec^{-1}(cx)\log\left(1+\frac{i\left(\sqrt{e}-\sqrt{c^2d+e}\right)e^{i\sec^{-1}(cx)}}{c\sqrt{d}}\right)\right.}{}$$

```
[In] Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]

[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((5*(-(ArcSec[c*x]/(I*Sqrt[d]*
Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqr
t[e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt
[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16
*e^2) + (5*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x
)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2
*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e
]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*e^2) + ((I/16)*Sqrt[d]*(-(ArcSec[
c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] - I
*((c*Sqrt[d]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d] +
Sqrt[e]*x)) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] +
 c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(
(-I)*Sqrt[d] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/e^2 - ((I/16)*Sqrt[d]*
((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sq
rt[e]*x)) - ArcSec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/(c*x
)]/(d*Sqrt[e]) - (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sq
rt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d
 + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 + (3*(8*ArcSin
[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])
*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[
e] - Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1
 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])
*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] +
Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I
*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x
])] - 2*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sq
rt[d])] - 2*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))
/(c*Sqrt[d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/(32*Sqrt[d]*e^(5/2)) -
 (3*(8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sq
rt[d] + Sqrt[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*L
og[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4
*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e]
+ Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[
1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*
ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqr
t[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E
^((2*I)*ArcSec[c*x])] - 2*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*
E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/(32*
Sqrt[d]*e^(5/2)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 66.80 (sec) , antiderivative size = 1822, normalized size of antiderivative = 1.62

method	result	size
parts	Expression too large to display	1822
derivativedivides	Expression too large to display	1845
default	Expression too large to display	1845

```
[In] int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*((-5/8/e*x^3-3/8*d/e^2*x)/(e*x^2+d)^2+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*e
)^(1/2)))+b/c^5*(-1/8*x*c^7*(3*d^2*c^4*arcsec(c*x)+5*c^4*d*e*arcsec(c*x)*x^
2-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*c^3
*x^3+3*c^2*d*e*arcsec(c*x)+5*e^2*arcsec(c*x)*c^2*x^2)/e^2/(c^2*d+e)/(c^2*e*
x^2+c^2*d)^2-1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e
*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d
+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d^3+3/8*I*(-(c^2*d-2*(e*(
c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^
2*d+e))^(1/2)*e+2*e^2)*c^3*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2
*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e^2/d^2+3/16*I/(c^2*d+e
)/e*c^6*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1
/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/
2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c*d*(1/c/x+I*(1
-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/e
/d^3-3/16*I/(c^2*d+e)/e*c^6*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x
)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2
)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*I*((c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*
e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))
/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3-3/16*I/(c^2
*d+e)/e^2*c^8*d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/
c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1
)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3/8*I*((c^2*d+2*(e*(c^2
*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*
d+e))^(1/2)*e+2*e^2)*c^3*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2
*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e^2/d^2+1/2*I*(-(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(
c^2*d+e))^(1/2)*e+2*e^2)*c*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2
*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3-3/8*I*((c^2*d+2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c^3*arct
an(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^
(1/2))/(c^2*d+e)/e^2/d^2-3/8*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)
```

```
*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c^3*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(
1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e^2/d^2+3/16*
I/(c^2*d+e)/e^2*c^8*d*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R
1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))
/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\operatorname{arcsec}(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsec(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

```
[In] integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\sec^{-1}(cx))}{(d + ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\sec^{-1}(cx))}{(d + ex^2)^3}\,dx = \int \frac{x^4\,\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^3}\,dx$$

```
[In] int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

3.109 $\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 1124

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}(c^2d+e)^{3/2}} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} - \frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}(c^2d+e)^{3/2}} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

```
[Out] -1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1
/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(3/2)-1/16*b*arctanh((c^2*d+(-d)^(1/2)*e
^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^
(3/2)-1/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsec(c*x))*ln(1
+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)
^(3/2)/e^(3/2)-1/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*
(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsec(c
*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/
2)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*
(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*polylog(2
,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d
)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*polylog(2,-c*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/
e^(3/2)+1/16*(a+b*arcsec(c*x))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))
^2+1/16*(a+b*arcsec(c*x))/d/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*(-a-b*arcsec(c
*x))/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(-a-b*arcsec(c*x))/
d/e/(d/x+(-d)^(1/2)*e^(1/2))+1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/
d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/e/(c^2*d+e)^(1/2)+1/16
*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/
x^2)^(1/2))/d^(3/2)/e/(c^2*d+e)^(1/2)+1/16*b*c*(1-1/c^2/x^2)^(1/2)/(c^2*d+e
)/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*b*c*(1-1/c^2/x^2)^(1/2)
/(c^2*d+e)/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 1124, normalized size of antiderivative = 1.00, number of steps used = 63, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules

used = {5348, 4818, 4758, 4828, 745, 739, 212, 4826, 4616, 2221, 2317, 2438}

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}\,(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}\,(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{dc^2+e}} - \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}(dc^2+e)^{3/2}} + \frac{b\,\mathrm{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{dc^2+e}} - \frac{b\,\mathrm{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}(dc^2+e)^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\sec^{-1}(cx))\log\left(\frac{\sqrt{-d}e^{i\sec^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}} - \frac{(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\sec^{-1}(cx))\log\left(\frac{\sqrt{-d}e^{i\sec^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

```
[In] Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqrt
[e] - d/x)) + (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*
(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcSec[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[
-d]*Sqrt[e] - d/x)^2) + (a + b*ArcSec[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] - d/x
)) - (a + b*ArcSec[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2) -
 (a + b*ArcSec[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d
- (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])
/(16*d^(3/2)*(c^2*d + e)^(3/2)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)
/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[c^2
*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d
+ e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) + (b*ArcTanh[(
c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^
2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) - ((a + b*ArcSec[c*x])*Log[1 - (c*Sqr
t[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/
2)) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcSec[c*x])*Log[1 -
 (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2
)*e^(3/2)) + ((a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(S
qrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2,
-((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]))])/((-d)^(3/2)
*e^(3/2)) + ((I/16)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] -
Sqrt[c^2*d + e])])/((-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, -((c*Sqrt[-d
]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]))])/((-d)^(3/2)*e^(3/2)) +
((I/16)*b*PolyLog[2, (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d +
 e])])/((-d)^(3/2)*e^(3/2))
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 739</u>

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

<u>Rule 745</u>

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
```

```
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5348

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^3}+\frac{a+b\arccos\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)^2}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{d}$$

$$+\frac{e\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3}-\frac{3d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d^3\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3}-\frac{3d\left(a+b\arccos\left(\frac{x}{c}\right.\right.}{16e^2\left(\sqrt{-d}\sqrt{e}+dx\right.}\right.\right.}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e}-\frac{3\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e}$$

$$+\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e}+\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e}$$

$$-\frac{3\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{8e}+\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2e}$$

$$-\frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{e}}-\frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{e}}$$

$$= \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}+\frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$
$$-\frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{3\text{Subst}\left(\int\left(-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}-\sqrt{-d}x)}-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}+\sqrt{-d}x)}\right)dx,x,\frac{1}{x}\right)}{8e}$$
$$+\frac{\text{Subst}\left(\int\left(-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}-\sqrt{-d}x)}-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}(\sqrt{e}+\sqrt{-d}x)}\right)dx,x,\frac{1}{x}\right)}{2e}$$
$$-\frac{(3b)\text{Subst}\left(\int\frac{1}{(\sqrt{-d}\sqrt{e}-dx)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16cde}$$
$$+\frac{(3b)\text{Subst}\left(\int\frac{1}{(\sqrt{-d}\sqrt{e}+dx)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16cde}$$
$$+\frac{b\text{Subst}\left(\int\frac{1}{(\sqrt{-d}\sqrt{e}-dx)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{4cde}-\frac{b\text{Subst}\left(\int\frac{1}{(\sqrt{-d}\sqrt{e}+dx)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{4cde}$$
$$+\frac{b\text{Subst}\left(\int\frac{1}{(\sqrt{-d}\sqrt{e}-dx)^2\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16c\sqrt{-d}\sqrt{e}}-\frac{b\text{Subst}\left(\int\frac{1}{(\sqrt{-d}\sqrt{e}+dx)^2\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16c\sqrt{-d}\sqrt{e}}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+\frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}+\frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$
$$-\frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{3\text{Subst}\left(\int\frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{16de^{3/2}}$$
$$+\frac{3\text{Subst}\left(\int\frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{16de^{3/2}}-\frac{\text{Subst}\left(\int\frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{4de^{3/2}}$$
$$-\frac{\text{Subst}\left(\int\frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{4de^{3/2}}+\frac{(3b)\text{Subst}\left(\int\frac{1}{d^2+\frac{de}{c^2}-x^2}dx,x,\frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cde}$$
$$-\frac{(3b)\text{Subst}\left(\int\frac{1}{d^2+\frac{de}{c^2}-x^2}dx,x,\frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cde}$$
$$-\frac{b\text{Subst}\left(\int\frac{1}{d^2+\frac{de}{c^2}-x^2}dx,x,\frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cde}+\frac{b\text{Subst}\left(\int\frac{1}{d^2+\frac{de}{c^2}-x^2}dx,x,\frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cde}$$
$$-\frac{b\text{Subst}\left(\int\frac{1}{(\sqrt{-d}\sqrt{e}-dx)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16cd\left(c^2d+e\right)}+\frac{b\text{Subst}\left(\int\frac{1}{(\sqrt{-d}\sqrt{e}+dx)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16cd\left(c^2d+e\right)}$$

$$= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$- \frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}}$$

$$+ \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} - \frac{3\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{16de^{3/2}}$$

$$- \frac{3\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{16de^{3/2}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{4de^{3/2}}$$

$$+ \frac{\text{Subst}\left(\int \frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx, x, \sec^{-1}(cx)\right)}{4de^{3/2}} + \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cd\,(c^2d+e)}$$

$$- \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cd\,(c^2d+e)}$$

$$
\begin{aligned}
&= \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+ \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&- \frac{a+b\sec^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\sec^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&- \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d+e\right)^{3/2}} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} \\
&- \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d+e\right)^{3/2}} + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} \\
&+ \frac{(3i)\operatorname{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{16de^{3/2}} \\
&+ \frac{(3i)\operatorname{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{16de^{3/2}} \\
&+ \frac{(3i)\operatorname{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{16de^{3/2}} \\
&+ \frac{(3i)\operatorname{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{16de^{3/2}} \\
&- \frac{i\operatorname{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4de^{3/2}} \\
&- \frac{i\operatorname{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4de^{3/2}} \\
&- \frac{i\operatorname{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4de^{3/2}} \\
&- \frac{i\operatorname{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{4de^{3/2}}
\end{aligned}
$$

$= \text{Too large to display}$

Mathematica [A] (warning: unable to verify)

Time = 6.07 (sec) , antiderivative size = 1827, normalized size of antiderivative = 1.63

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{ax}{4e\,(d+ex^2)^2}+\frac{ax}{8de\,(d+ex^2)}+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}$$

$$+b\left(-\frac{-\frac{\sec^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex}+\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}}{16de}-\frac{-\frac{\sec^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex}-\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{\cdots}}{\cdots}\right)}{\cdots}\right)}{\cdots}}{16\cdots}\cdots\right.$$

```
[In] Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(-1/16*(-(ArcSec[c*x]/(I*Sqrt[d]*Sq
rt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[
e] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-
(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(d*e)
- (-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/Sqrt[e
] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*
Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sq
rt[-(c^2*d) - e]))/Sqrt[d])/(16*d*e) - ((I/16)*(-(ArcSec[c*x]/(Sqrt[e]*((-I
)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] - I*((c*Sqrt[d]*Sqrt[
e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) + ((2*
c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*Sqrt[d] - Sq
rt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqr
t[e]*x))])/(c^2*d + e)^(3/2)))/d))/(Sqrt[d]*e) + ((I/16)*((I*c*Sqrt[e]*Sqrt
[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSec
[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/(c*x)]/(d*Sqrt[e]) - (
I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c*(c*Sqrt[
d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I*Sqrt[d] +
 Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + (8*ArcSin[Sqrt[1 + (I*
```

```
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec[c
*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2
*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSec[
c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d +
e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x])
)/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*PolyL
og[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2*P
olyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])]
 + PolyLog[2, -E^((2*I)*ArcSec[c*x])])/(32*d^(3/2)*e^(3/2)) - (8*ArcSin[Sqr
t[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*
Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(-Sqrt[
e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1
 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]
)*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 - (I*(Sqrt[e] +
 Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 - (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(
I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*
x])] - 2*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(
c*Sqrt[d])] - 2*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x])
)/(c*Sqrt[d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])])/(32*d^(3/2)*e^(3/2)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 33.26 (sec) , antiderivative size = 1288, normalized size of antiderivative = 1.15

method	result	size
parts	Expression too large to display	1288
derivativedivides	Expression too large to display	1307
default	Expression too large to display	1307

```
[In] int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/
2)))+b/c^3*(1/8*x*c^5*(c^4*d*e*arcsec(c*x)*x^2-d^2*c^4*arcsec(c*x)-((c^2*x^
2-1)/c^2/x^2)^(1/2)*e^2*c^3*x^3-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x+e^2*a
rcsec(c*x)*c^2*x^2-c^2*d*e*arcsec(c*x))/d/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-1
/8*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/
2)+2*e)*c*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/e/d^3+1/8*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*
e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2
*e^2)*c*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1
```

```
/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3-1/8*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d
^3+1/8*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*
c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctanh(c*d*(1/c/x+I*(1-1/
c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e
/d^3+1/16*I/(c^2*d+e)/d*c^4*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*
ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^
(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/16*I/(c^2*d
+e)/e*c^6*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1
-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=R
ootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/16*I/(c^2*d+e)/d*c^4*sum(1/_R
1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2
))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4
+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/16*I/(c^2*d+e)/e*c^6*sum(1/_R1/(_R1^2*c^2*d+c
^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_
R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_
Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*x^2*arcsec(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

```
[In] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^3, x)
```

3.110 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1114

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}(a+b\sec^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}(a+b\sec^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} + \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{be\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(c^2d+e)^{3/2}} - \frac{5b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d+e}} + \frac{be\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(c^2d+e)^{3/2}} - \frac{5b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d+e}} + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3(a+b\sec^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

```
[Out] 1/16*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-
1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(3/2)+1/16*b*e*arctanh((c^2*d+(-d)^(1/2
)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+
e)^(3/2)+3/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arcsec(c*x))*l
```

```
n(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(
-d)^(5/2)/e^(1/2)+3/16*(a+b*arcsec(c*x))*ln(1-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2
))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arcse
c(c*x))*ln(1+c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^
(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b*polylog(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2
))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b*polylo
g(2,c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(
-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,-c*(1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,-c*(
1/c/x+I*(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/
2)/e^(1/2)+1/16*(a+b*arcsec(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+(-d)^(1/2)*e^(1/
2))^2-5/16*(a+b*arcsec(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arcsec
(c*x))*e^(1/2)/(-d)^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+5/16*(a+b*arcsec(c*x))
/d^2/(d/x+(-d)^(1/2)*e^(1/2))-5/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c
/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)-5/16*
b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x
^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+1/16*b*c*e^(1/2)*(1-1/c^2/x^2)^(1/2)/(-d
)^(3/2)/(c^2*d+e)/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*b*c*e^(1/2)*(1-1/c^2/x^2)^
(1/2)/(-d)^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 2.68 (sec) , antiderivative size = 1114, normalized size of antiderivative = 1.00, number of steps used = 81, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {5338, 4818, 4758, 4828, 745, 739, 212, 4826, 4616, 2221, 2317, 2438}

$$\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = & \frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
& + \frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} \\
& - \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} \\
& + \frac{\sqrt{e}(a+b\sec^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{\sqrt{e}(a+b\sec^{-1}(cx))}{16(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2} \\
& - \frac{5b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}} + \frac{be\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(dc^2+e)^{3/2}} \\
& - \frac{5b\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}} + \frac{be\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(dc^2+e)^{3/2}} \\
& + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
& - \frac{3(a+b\sec^{-1}(cx))\log\left(\frac{\sqrt{-d}e^{i\sec^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}} \\
& + \frac{3(a+b\sec^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
& - \frac{3(a+b\sec^{-1}(cx))\log\left(\frac{\sqrt{-d}e^{i\sec^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}} \\
& + \frac{3ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} \\
& + \frac{3ib\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{i\sec^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}
\end{aligned}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sq
rt[e] - d/x)) + (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d +
 e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcSec[c*x]))/(16*(-d)^(3/2
)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcSec[c*x]))/(16*d^2*(Sqrt[-d]*S
qrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcSec[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqr
t[e] + d/x)^2) + (5*(a + b*ArcSec[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x))
```

```
+ (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sq
rt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) - (5*b*ArcTanh[(c^2*d
 - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])]
)/(16*d^(5/2)*Sqrt[c^2*d + e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x
)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d +
e)^(3/2)) - (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2
*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) + (3*(a + b*A
rcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSec[c*x])*Log[1 + (c*Sqrt[-d]*
E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) +
(3*(a + b*ArcSec[c*x])*Log[1 - (c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSec[c*x])*Log[1 + (
c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*
Sqrt[e]) + (((3*I)/16)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[
e] - Sqrt[c^2*d + e]))])/((-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, (c
*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(5/2)*Sqrt
[e]) + (((3*I)/16)*b*PolyLog[2, -((c*Sqrt[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] +
 Sqrt[c^2*d + e]))])/((-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, (c*Sqr
t[-d]*E^(I*ArcSec[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(5/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4758

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4818

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

```
&& NeQ[m, -1]
```

Rule 5338

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcCos[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^4\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{e^2\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2e\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{a+b\arccos\left(\frac{x}{c}\right)}{d^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d^2}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\arccos\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{d^2}$$
$$+ \frac{(2e)\text{Subst}\left(\int \left(-\frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{d^2}$$
$$- \frac{e^2\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d^3\left(a+b\arccos\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3} - \frac{3d(a+b\arccos(}{16e^2(\sqrt{-d}\sqrt{e}+}\right.}{d^2}$$

$$= \frac{3\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d}$$
$$+ \frac{3\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{8d} - \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{2d}$$
$$- \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{d}$$
$$- \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}}$$
$$- \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{-d}} - \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{-d}}$$

$$
\begin{aligned}
&= \frac{\sqrt{e}(a+b\sec^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{\sqrt{e}(a+b\sec^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&+\frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{(3b)\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{16cd^2} \\
&-\frac{(3b)\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{16cd^2} \\
&-\frac{b\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2cd^2}+\frac{b\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2cd^2} \\
&+\frac{3\text{Subst}\left(\int\left(-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)}{8d} \\
&-\frac{\text{Subst}\left(\int\left(-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\arccos\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)}{d} \\
&+\frac{\text{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\cos(x)}\,dx,x,\sec^{-1}(cx)\right)}{2d^2\sqrt{e}} \\
&+\frac{\text{Subst}\left(\int\frac{(a+bx)\sin(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\cos(x)}\,dx,x,\sec^{-1}(cx)\right)}{2d^2\sqrt{e}} \\
&+\frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{16c(-d)^{3/2}} \\
&-\frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{16c(-d)^{3/2}}
\end{aligned}
$$

$$= \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+ \frac{\sqrt{e}(a+b\sec^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}(a+b\sec^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$
$$+ \frac{5(a+b\sec^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{(3b)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cd^2}$$
$$+ \frac{(3b)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cd^2}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2cd^2} - \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2cd^2}$$
$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2d^2\sqrt{e}}$$
$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2d^2\sqrt{e}}$$
$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2d^2\sqrt{e}}$$
$$- \frac{i\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+\sqrt{-d}e^{ix}}\,dx, x, \sec^{-1}(cx)\right)}{2d^2\sqrt{e}} - \frac{3\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16d^2\sqrt{e}}$$
$$- \frac{3\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16d^2\sqrt{e}} + \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}}$$
$$+ \frac{\text{Subst}\left(\int \frac{a+b\arccos\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}} + \frac{(be)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16cd^2\left(c^2d+e\right)}$$
$$- \frac{(be)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16cd^2\left(c^2d+e\right)}$$

= Too large to display

Mathematica [A] (warning: unable to verify)

Time = 6.04 (sec) , antiderivative size = 1812, normalized size of antiderivative = 1.63

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \frac{ax}{4d\left(d+ex^2\right)^2}+\frac{3ax}{8d^2\left(d+ex^2\right)}+\frac{3a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}}$$

$$+b\left(-\frac{3\left(-\frac{\sec^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex}+\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}\right)}{16d^2}\right.$$

$$-\frac{3\left(-\frac{\sec^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex}-\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(-\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}\right)}{16d^2}$$

$$+\frac{i\left(-\frac{\sec^{-1}(cx)}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}x\right)^2}+\frac{\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-i\left(\frac{c\sqrt{d}\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}{\left(c^2d+e\right)\left(-i\sqrt{d}+\sqrt{e}x\right)}+\frac{\left(2c^2d+e\right)\log\left(-\frac{4d\sqrt{e}\sqrt{c^2d+e}\left(i\sqrt{e}+c\left(c\sqrt{d}-\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\left(2c^2d+e\right)\left(-i\sqrt{d}+\sqrt{e}x\right)}\right)}{\left(c^2d+e\right)^{3/2}}\right)}{d}\right)}{16d^{3/2}}$$

$$-\frac{i\left(\frac{ic\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}{\sqrt{d}(c^2d+e)\left(i\sqrt{d}+\sqrt{e}x\right)}-\frac{\sec^{-1}(cx)}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}x\right)^2}+\frac{\arcsin\left(\frac{1}{cx}\right)}{d\sqrt{e}}-\frac{i(2c^2d+e)\log\left(\frac{4d\sqrt{e}\sqrt{c^2d+e}\left(-i\sqrt{e}+c\left(c\sqrt{d}+\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\left(2c^2d+e\right)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d(c^2d+e)^{3/2}}\right)}{16d^{3/2}}$$

$$+\frac{3\left(8\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\tan\left(\frac{1}{2}\sec^{-1}(cx)\right)}{\sqrt{c^2d+e}}\right)-2i\sec^{-1}(cx)\log\left(1+\frac{i\left(\sqrt{e}-\sqrt{c^2d+e}\right)e^{i\sec^{-1}(cx)}}{c\sqrt{d}}\right)\right.}{}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^3,x]

[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*((-3*(-(ArcSec[c*x]/(I*Sqrt[d]*Sqr
t[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e
] + c*(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(
c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*d^
2) - (3*(-(ArcSec[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/
Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x
))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*d^2) + ((I/16)*(-(ArcSec[c*x]/(Sqrt[
e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)) + (ArcSin[1/(c*x)]/Sqrt[e] - I*((c*Sqrt[d
]*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))
 + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*Sqrt[
d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*Sqrt[d
] + Sqrt[e]*x))])/(c^2*d + e)^(3/2)))/d))/d^(3/2) - ((I/16)*((I*c*Sqrt[e]*S
qrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - Arc
Sec[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + ArcSin[1/(c*x)]/(d*Sqrt[e])
- (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c*(c*Sq
rt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I*Sqrt[d
] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) + (3*(8*ArcSin[Sqrt[1 + (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Tan[ArcSec
[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c
^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e])*E^(I*ArcSe
c[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d
+ e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x
]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*ArcSec[c*x])] - 2*Pol
yLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - 2
*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d]
)] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/(32*d^(5/2)*Sqrt[e]) - (3*(8*ArcS
in[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqr
t[e])*Tan[ArcSec[c*x]/2])/Sqrt[c^2*d + e]] - (2*I)*ArcSec[c*x]*Log[1 + (I*(
-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (4*I)*ArcSin[
Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*
d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] - (2*I)*ArcSec[c*x]*Log[1 - (I*(Sqr
t[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (4*I)*ArcSin[Sqrt
[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e
])*E^(I*ArcSec[c*x]))/(c*Sqrt[d])] + (2*I)*ArcSec[c*x]*Log[1 + E^((2*I)*Arc
Sec[c*x])] - 2*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec[c*
x]))/(c*Sqrt[d])] - 2*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e])*E^(I*ArcSec
[c*x]))/(c*Sqrt[d])] + PolyLog[2, -E^((2*I)*ArcSec[c*x])]))/(32*d^(5/2)*Sqr
t[e]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 94.04 (sec) , antiderivative size = 1812, normalized size of antiderivative = 1.63

method	result	size
parts	Expression too large to display	1812
derivativedivides	Expression too large to display	1837
default	Expression too large to display	1837

```
[In] int((a+b*arcsec(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x/d/(e*x^2+d)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(d*e)^(1/2)*arctan(e*
x/(d*e)^(1/2))+b/c*(1/8*x*c^3*(5*d^2*c^4*arcsec(c*x)+3*c^4*d*e*arcsec(c*x)*
x^2+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*c
^3*x^3+5*c^2*d*e*arcsec(c*x)+3*e^2*arcsec(c*x)*c^2*x^2)/d^2/(c^2*d+e)/(c^2*
e*x^2+c^2*d)^2-5/8*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*
d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c*d*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^
2*d+e)^2/d^4/c-3/16*I/(c^2*d+e)/d^2*c^2*e*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)
*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-
I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d
))+1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e)
)^(1/2)+2*e)*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d
+e))^(1/2)-2*e)*d)^(1/2))*e/c^3/d^5/(c^2*d+e)-1/2*I*((c^2*d+2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^
(1/2)*e+2*e^2)*e*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2
*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^5/c^3+5/8*I*((c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+
I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/c/d^4/(
c^2*d+e)+5/8*I*arctanh(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2
*d+e))^(1/2)-2*e)*d)^(1/2))*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)/c/d^4/(c^2*d+e)+3/16*I/(c^2*d+e)/d^2*c^2*e*
sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2
)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*
d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+3/16*I/(c^2*d+e)/d*c^4*sum(_R1/(_R1^2*c^2
*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)+dilo
g((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*
e)*_Z^2+c^2*d))-5/8*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2
*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctan(c*d*(1/c
/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2
*d+e)^2/d^4/c-1/2*I*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d
+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctanh(c*d*(1/
c/x+I*(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c
^2*d+e)^2/d^5/c^3+1/2*I*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-
```

```
2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(1/c/x+I*(1-1/c^2/x^2)^(1/2))/((c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))*e/c^3/d^5/(c^2*d+e)-3/16*I/(c^2*d+e)/
d*c^4*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(I*arcsec(c*x)*ln((_R1-1/c/x-I*(1-1
/c^2/x^2)^(1/2))/_R1)+dilog((_R1-1/c/x-I*(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{a+b\sec^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asec(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)}{\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x^2)^3, x)
```

3.111 $\int x^5\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 403

$$\begin{aligned}\int x^5\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx &= \frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}}\\ &+\frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}}-\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}}\\ &+\frac{d^2(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3}-\frac{2d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3}\\ &+\frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^3}+\frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}}\\ &-\frac{b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

```
[Out] 1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*ar
csec(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arcsec(c*x))/e^3+8/105*b*c*d^(7/2)*
x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)-1/1
680*b*(105*c^6*d^3-35*c^4*d^2*e+63*c^2*d*e^2+75*e^3)*x*arctanh(e^(1/2)*(c^2
*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(5/2)/(c^2*x^2)^(1/2)+1/840*b*(29*c^
2*d-25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)-1/42*
b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)+1/1680*b*(23*c^
4*d^2+12*c^2*d*e-75*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^5/e^2/(c^2*x
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 5346, 12, 1629, 159, 163, 65, 223, 212, 95, 210}

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{d^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3}-\frac{2d(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3}+\frac{(d+ex^2)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3}+\frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{105e^3\sqrt{c^2x^2}}-\frac{bx(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(29c^2d-25e)\,(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(23c^4d^2+12c^2de-75e^2)\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}}$$

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

```
[Out] (b*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
/(1680*c^5*e^2*Sqrt[c^2*x^2]) + (b*(29*c^2*d - 25*e)*x*Sqrt[-1 + c^2*x^2]*(
d + e*x^2)^(3/2))/(840*c^3*e^2*Sqrt[c^2*x^2]) - (b*x*Sqrt[-1 + c^2*x^2]*(d
+ e*x^2)^(5/2))/(42*c*e^2*Sqrt[c^2*x^2]) + (d^2*(d + e*x^2)^(3/2)*(a + b*Ar
cSec[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(5*e^3) +
 ((d + e*x^2)^(7/2)*(a + b*ArcSec[c*x]))/(7*e^3) + (8*b*c*d^(7/2)*x*ArcTan[
Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(105*e^3*Sqrt[c^2*x^2]) - (b
*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*x*ArcTanh[(Sqrt[e]*Sq
rt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^6*e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d^2(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} \\ &\quad + \frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^3} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(8d^2-12dex^2+15e^2x^4)}{105e^3x\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\ &= \frac{d^2(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} \\ &\quad + \frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^3} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(8d^2-12dex^2+15e^2x^4)}{x\sqrt{-1+c^2x^2}}\,dx}{105e^3\sqrt{c^2x^2}}\end{aligned}$$

$$= \frac{d^2(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^3} - \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^{3/2}(8d^2-12dex+15e^2x^2)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{210e^3\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^3} - \frac{(bx)\text{Subst}\left(\int \frac{(d+ex)^{3/2}\left(24c^2d^2e-\frac{3}{2}(29c^2d-25e)e^2x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{630ce^4\sqrt{c^2x^2}}$$

$$= \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^3} - \frac{(bx)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(48c^4d^3e-\frac{3}{4}e^2(23c^4d^2+12c^2de-75e^2)x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{1260c^3e^4\sqrt{c^2x^2}}$$

$$= \frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} + \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^3} - \frac{(bx)\text{Subst}\left(\int \frac{48c^6d^4e+\frac{3}{8}e^2(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{1260c^5e^4\sqrt{c^2x^2}}$$

$$= \frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} + \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^3} - \frac{(4bcd^4x)\,\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{105e^3\sqrt{c^2x^2}} - \frac{(b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x)\,\text{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3360c^5e^2\sqrt{c^2x^2}}$$

$$= \frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} + \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^3} - \frac{(8bcd^4x)\,\text{Subst}\left(\int\frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}} - \frac{(b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x)\,\text{Subst}\left(\int\frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{1680c^7e^2\sqrt{c^2x^2}}$$

$$= \frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} + \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^3} + \frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}} - \frac{(b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x)\,\text{Subst}\left(\int\frac{1}{1-\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{1680c^7e^2\sqrt{c^2x^2}}$$

$$= \frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} + \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^3} + \frac{(d+ex^2)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^3} + \frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}} - \frac{b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.62 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.85

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= \frac{32a(d+ex^2)\left(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6\right) - \frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)\left(75e^2+2c^2e(19d+25ex^2)+c^4(-41d^2+22dex^2+40e\right.}{c^5}}{}$$

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

```
[Out] (32*a*(d + e*x^2)*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6) - (2*b*e
*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)*(75*e^2 + 2*c^2*e*(19*d + 25*e*x^2) +
c^4*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)))/c^5 + (128*b*d^4*Sqrt[1 + d/(e*x^
2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/(c*x) + (b*e*(105*
c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*Sqrt[1 - 1/(c^2*x^2)]*x^3*S
qrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/(c^5*Sq
rt[1 - c^2*x^2]) + 32*b*(d + e*x^2)*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15
*e^3*x^6)*ArcSec[c*x])/(3360*e^3*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^5\left(a+b\,\text{arcsec}\,(cx)\right)\sqrt{e\,x^2+d}dx$$

```
[In] int(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 2.37 (sec) , antiderivative size = 1701, normalized size of antiderivative = 4.22

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6720*(128*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*
e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*
e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c
^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4
*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arcsec(c*x) - (40*b*c^5*e^3*x^4 - 41*b*c^5*
d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2
)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^3), 1/6720*(256*b*c^7*d^(7/2)*
arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(
d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (105*b*c^6*d^3 - 35*b*c^4*d
^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c
^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2
*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*
d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*b
*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arcsec(c*x) - (40*b*c^5*e
^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 +
 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^3), 1/3360*(
64*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d
*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(
-d) + 8*d^2)/x^4) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b
*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240
*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 +
16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)
*arcsec(c*x) - (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c
*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
 + d))/(c^7*e^3), 1/3360*(128*b*c^7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*(
(c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*
e)*x^2 - d^2)) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^
3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240*a*
c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*
(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*ar
```

```
csec(c*x) - (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^
3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 +
d))/(c^7*e^3)]
```

Sympy [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^5(a+b\operatorname{asec}(cx))\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**5*(a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)x^5\,dx$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^5*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
[Out] int(x^5*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.112 $\int x^3\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 294

$$\begin{aligned}\int x^3\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx &= -\frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} \\ &\quad -\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} \\ &\quad -\frac{d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^2} \\ &\quad +\frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^2} \\ &\quad -\frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}} \\ &\quad +\frac{b(15c^4d^2-10c^2de-9e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{c^2x^2}}\end{aligned}$$

```
[Out] -1/3*d*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arcse
c(c*x))/e^2-2/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^2/(c^2*x^2)^(1/2)+1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*x*arctanh(e
^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(3/2)/(c^2*x^2)^(1/2)-1/2
0*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e/(c^2*x^2)^(1/2)-1/120*b*(c^2*d+
9*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 5346, 12, 587, 159, 163, 65, 223, 212, 95, 210}

$$\begin{aligned}\int x^3\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = &-\frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2}\\&+\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2}\\&-\frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{15e^2\sqrt{c^2x^2}}\\&+\frac{bx(15c^4d^2-10c^2de-9e^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{c^2x^2}}\\&-\frac{bx\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}\\&-\frac{bx\sqrt{c^2x^2-1}(c^2d+9e)\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}\end{aligned}$$

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/120*(b*(c^2*d + 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(c^3*e*Sqrt[c
^2*x^2]) - (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e*Sqrt[c^2*x^2]
) - (d*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(3*e^2) + ((d + e*x^2)^(5/2)*
(a + b*ArcSec[c*x]))/(5*e^2) - (2*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqr
t[d]*Sqrt[-1 + c^2*x^2])])/(15*e^2*Sqrt[c^2*x^2]) + (b*(15*c^4*d^2 - 10*c^2
*d*e - 9*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/
(120*c^4*e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} \\
&\quad + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(-2d+3ex^2)}{15e^2x\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} \\
&\quad + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(-2d+3ex^2)}{x\sqrt{-1+c^2x^2}}\,dx}{15e^2\sqrt{c^2x^2}} \\
&= -\frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} \\
&\quad - \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^{3/2}(-2d+3ex)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{30e^2\sqrt{c^2x^2}} \\
&= -\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} \\
&\quad + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} - \frac{(bx)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(-4c^2d^2+\frac{1}{2}e(c^2d+9e)x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{60ce^2\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} - \frac{(bx)\text{Subst}\left(\int \frac{-4c^4d^3-\frac{1}{4}e(15c^4d^2-10c^2de-9e^2)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{60c^3e^2\sqrt{c^2x^2}}$$

$$= -\frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} + \frac{(bcd^3x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{15e^2\sqrt{c^2x^2}} + \frac{(b(15c^4d^2-10c^2de-9e^2)x)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{240c^3e\sqrt{c^2x^2}}$$

$$= -\frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} + \frac{(2bcd^3x)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}} + \frac{(b(15c^4d^2-10c^2de-9e^2)x)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{120c^5e\sqrt{c^2x^2}}$$

$$= -\frac{b(c^2d+9e)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} - \frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}} + \frac{(b(15c^4d^2-10c^2de-9e^2)x)\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{120c^5e\sqrt{c^2x^2}}$$

$$= -\frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^2} - \frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}} + \frac{b(15c^4d^2-10c^2de-9e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.70 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.89

$$\int x^3\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\ dx$$

$$= \frac{16a(d+ex^2)^2\,(-2d+3ex^2) - \frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(9e+c^2(7d+6ex^2))}{c^3} + \frac{b\left(-16c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+\right.}{}}{240e^2\sqrt{d+e}}$$

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

```
[Out] (16*a*(d + e*x^2)^2*(-2*d + 3*e*x^2) - (2*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
e*x^2)*(9*e + c^2*(7*d + 6*e*x^2)))/c^3 + (b*(-16*c^2*d^3*Sqrt[1 + d/(e*x^2
)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (e*(-15*c^4*d^2 +
10*c^2*d*e + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[
1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^3*x) + 16*b*
(d + e*x^2)^2*(-2*d + 3*e*x^2)*ArcSec[c*x])/(240*e^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^3(a+b\ \text{arcsec}\,(cx))\,\sqrt{e\,x^2+d}dx$$

```
[In] int(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 1383, normalized size of antiderivative = 4.70

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(16*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2
*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)
*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2
*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2)
+ 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4
 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 + 7*b*c^3*d*
e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2), -1/480*(32*b*
c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^
2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 -
 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e +
8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)
*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 1
6*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arcsec(c*x)
 - (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^
2 + d))/(c^5*e^2), 1/240*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e +
e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2
*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e -
 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*s
qrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*
(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b
*c^5*d*e*x^2 - 2*b*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e +
9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2), -1/240*(16*b*c^5*
d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 - 10*
b*c^2*d*e - 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2
*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)
*x^2)) - 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*
e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 + 7*b
*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2)]
```

Sympy [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^3(a+b\operatorname{asec}(cx))\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**3*(a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x**3*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^3\,\sqrt{e\,x^2+d}\,\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.113 $\int x\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 195

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}+\frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}}-\frac{b(3c^2d+e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e+1/3*b*c*d^(3/2)*x*arctan((e*x^2+d)^
(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-1/6*b*(3*c^2*d+e)*x*arct
anh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(1/2)/(c^2*x^2)^(1/2
)-1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{5344, 457, 104, 163, 65, 223, 212, 95, 210\}$

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e}+\frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e\sqrt{c^2x^2}}-\frac{bx(3c^2d+e)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}$$

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/6*(b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(c*Sqrt[c^2*x^2]) + ((d + e*x
^2)^(3/2)*(a + b*ArcSec[c*x]))/(3*e) + (b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2
]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*e*Sqrt[c^2*x^2]) - (b*(3*c^2*d + e)*x*A
rcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*Sqrt[e]*Sq
rt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5344

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{(bcx)\int \frac{\left(d+ex^2\right)^{3/2}}{x\sqrt{-1+c^2x^2}}\,dx}{3e\sqrt{c^2x^2}}$$

$$= \frac{\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e} - \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{6e\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e} - \frac{(bx)\text{Subst}\left(\int \frac{c^2d^2+\frac{1}{2}e(3c^2d+e)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6ce\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e} - \frac{(bcd^2x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6e\sqrt{c^2x^2}} - \frac{(b(3c^2d+e)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{12c\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e} - \frac{(bcd^2x)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}} - \frac{(b(3c^2d+e)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{6c^3\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e} + \frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}} - \frac{(b(3c^2d+e)\,x)\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^3\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e} + \frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}} - \frac{b(3c^2d+e)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.59 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$
$$=\frac{\frac{2bd^2\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cex}+\frac{\frac{b\left(3c^2d+e\right)\sqrt{1-\frac{1}{c^2x^2}}x^3\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{\sqrt{1-c^2x^2}}+\frac{2\left(d+ex^2\right)\left(-be\sqrt{1-\frac{1}{c^2x^2}}x+2ac\left(\cdots\right.\right.}{\cdots}}{c}}{12\sqrt{d+ex^2}}$$

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

```
[Out] ((2*b*d^2*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*
x^2))])/(c*e*x) + ((b*(3*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 + (e*x
^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2] +
 (2*(d + e*x^2)*(-(b*e*Sqrt[1 - 1/(c^2*x^2)]*x) + 2*a*c*(d + e*x^2) + 2*b*c
*(d + e*x^2)*ArcSec[c*x]))/e)/c)/(12*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x\left(a+b\,\text{arcsec}\left(cx\right)\right)\sqrt{e\,x^2+d}\,dx$$

```
[In] int(x*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 1100, normalized size of antiderivative = 5.64

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(2*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*s
qrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d
^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 + 2*a*c
^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arcsec(c*x))*sqr
t(e*x^2 + d))/(c^3*e), 1/24*(4*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*
```

```
((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d
*e)*x^2 - d^2)) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6
*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c
^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 + 2*a*c^3*d -
 sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arcsec(c*x))*sqrt(e*x^
2 + d))/(c^3*e), 1/12*(b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^
4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqr
t(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(-e)*arctan(1/2
*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*
e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x^2 + 2*a*c^3*d -
sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arcsec(c*x))*sqrt(e*x^2
 + d))/(c^3*e), 1/12*(2*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d
 - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2
 - d^2)) + (3*b*c^2*d + b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e
- c*e^2)*x^2)) + 2*(2*a*c^3*e*x^2 + 2*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2
*(b*c^3*e*x^2 + b*c^3*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c^3*e)]
```

Sympy [F]

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x(a+b\operatorname{asec}(cx))\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x*(a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\operatorname{arcsec}(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/3*(e*x^2 + d)^(3/2)*a/e + 1/3*((e*x^2 + d)^(3/2)*arctan(sqrt(c*x + 1)*sqr
t(c*x - 1)) - 3*e*integrate((3*(c^2*e*x^3 - e*x + (c^2*e*x^3 - e*x)*e^(log(
c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)*log(x) + (3*c^2*e*x^3*log(c) - 3*
e*x*log(c) + ((3*c^2*log(c) + c^2)*e*x^3 + (c^2*d - 3*e*log(c))*x)*e^(log(c
*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(l
og(c*x + 1) + log(c*x - 1)) - e), x))*b/e
```

Giac [F]

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.114 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x} dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}, x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x} dx = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x} dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 7.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\mathrm{arcsec}\,(cx))\,\sqrt{e\,x^2+d}}{x}dx$$

```
[In] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x)
[Out] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 10.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}}{x}\,dx$$

```
[In] integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x,x)
[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x, x)
```

3.115 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx=\text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx=\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^3,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^3, x]
```

Rubi steps

$$\text{integral}=\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx=\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^3,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.74 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\mathrm{arcsec}\,(cx))\,\sqrt{e\,x^2+d}}{x^3}dx$$

```
[In] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x)
[Out] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx=\int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 14.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx=\int \frac{(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}}{x^3}\,dx$$

```
[In] integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**3,x)
[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^3, x)
```

3.116 $\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx=\operatorname{Int}\left(x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx)),x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx=\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$$

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
```

```
[Out] Defer[Int][x^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

Rubi steps

$$\text{integral}=\int x^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$$

Mathematica [N/A]

Not integrable

Time = 12.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+b\ \mathrm{arcsec}\,(cx))\,\sqrt{e\,x^2+d}dx$$

```
[In] int(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
[Out] int(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arcsec(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 116.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**2*(a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x**2*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^2\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.117 $\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\sec^{-1}(cx)),x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx = \int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$$

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \sqrt{d+ex^2}(a+b\sec^{-1}(cx))\,dx$$

Mathematica [N/A]

Not integrable

Time = 16.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\,\operatorname{arcsec}\left(cx\right)\right)\sqrt{e\,x^{2}+d}dx$$

```
[In] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\operatorname{arcsec}\left(cx\right)+a)\,dx$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 47.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(a+b\operatorname{asec}\left(cx\right)\right)\sqrt{d+ex^2}\,dx$$

```
[In] integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}\,(b\,\mathrm{arcsec}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.118 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^2,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\mathrm{arcsec}\,(cx))\,\sqrt{e\,x^2+d}}{x^2}dx$$

```
[In] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x)
[Out] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 7.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}}{x^2}\,dx$$

```
[In] integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**2,x)
[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^2,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^2, x)
```

3.119 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 328

$$
\begin{aligned}
\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx
&= \frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} \\
&+ \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3} \\
&- \frac{2bc^2(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{b(c^2d+e)(2c^2d+3e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[Out] -1/3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/d/x^3+2/9*b*c*(c^2*d+2*e)*(c^2*x^2-1
)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)+1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+
d)^(1/2)/x^2/(c^2*x^2)^(1/2)-2/9*b*c^2*(c^2*d+2*e)*x*EllipticE(c*x,(-e/c^2/
d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^
(1/2)/(1+e*x^2/d)^(1/2)+1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*x*EllipticF(c*x,(-e/c
^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^
2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {270, 5346, 12, 485, 597, 538, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx$$
$$=-\frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3dx^3}+\frac{bx\sqrt{1-c^2x^2}(c^2d+e)\,(2c^2d+3e)\,\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2bc^2x\sqrt{1-c^2x^2}(c^2d+2e)\,\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}+\frac{2bc\sqrt{c^2x^2-1}(c^2d+2e)\,\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}}+\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^4,x]
```

```
[Out] (2*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*Sqrt[c^2*x^2]
) + (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*x^2*Sqrt[c^2*x^2]) - ((d +
e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(3*d*x^3) - (2*b*c^2*(c^2*d + 2*e)*x*Sqrt
[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d*Sq
rt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (b*(c^2*d + e)*(2*c^2
*d + 3*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(
e/(c^2*d))])/(9*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3dx^3} - \frac{(bcx)\int -\frac{(d+ex^2)^{3/2}}{3dx^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3dx^3} + \frac{(bcx)\int \frac{(d+ex^2)^{3/2}}{x^4\sqrt{-1+c^2x^2}}\,dx}{3d\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3dx^3} - \frac{(bcx)\int \frac{-2d(c^2d+2e)-e(c^2d+3e)x^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d\sqrt{c^2x^2}} \\
&= \frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3dx^3} - \frac{(bcx)\int \frac{-de(c^2d+3e)+2c^2de(c^2d+2e)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{c^2x^2}} \\
&= \frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3dx^3} - \frac{(2bc^3(c^2d+2e)\,x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{9d\sqrt{c^2x^2}} \\
&\quad + \frac{(bc(c^2d+e)\,(2c^2d+3e)\,x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3} - \frac{\left(2bc^3(c^2d+2e)x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}dx}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} + \frac{\left(bc(c^2d+e)(2c^2d+3e)x\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}dx}{9d\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3} - \frac{\left(2bc^3(c^2d+2e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}dx}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bc(c^2d+e)(2c^2d+3e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}dx}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3dx^3} - \frac{2bc^2(c^2d+2e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,|-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{b(c^2d+e)(2c^2d+3e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}dx$$

$$= \frac{\sqrt{d+ex^2}\left(-3a(d+ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+4ex^2)-3b(d+ex^2)\sec^{-1}(cx)\right)}{9dx^3} - \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d+2e)E\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-(2c^4d^2+5c^2de+3e^2)\operatorname{EllipticF}\right.}{9\sqrt{-c^2}d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^4,x]
```

```
[Out] (Sqrt[d + e*x^2]*(-3*a*(d + e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2
*d*x^2 + 4*e*x^2) - 3*b*(d + e*x^2)*ArcSec[c*x]))/(9*d*x^3) - ((I/9)*b*c*Sq
rt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d + 2*e)*EllipticE[
I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (2*c^4*d^2 + 5*c^2*d*e + 3*e^2)*El
lipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d*Sqrt[1 - c^2
*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsec}\,(cx))\,\sqrt{e\,x^2+d}}{x^4}dx$$

```
[In] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

```
[Out] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = -\frac{\left(3\,acdex^2+3\,acd^2+3\,(bcdex^2+bcd^2)\,\mathrm{arcsec}\,(cx)-(bcd^2+2\,(bc^3d^2+2\,bcde)x^2)\sqrt{c^2x^2-1}\right)\sqrt{ex^2+d}-}{9}$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*((3*a*c*d*e*x^2 + 3*a*c*d^2 + 3*(b*c*d*e*x^2 + b*c*d^2)*arcsec(c*x) -
(b*c*d^2 + 2*(b*c^3*d^2 + 2*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d
) - (2*(b*c^6*d^2 + 2*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d)) -
(2*b*c^6*d^2 + (4*b*c^4 + b*c^2)*d*e + 3*b*e^2)*x^3*elliptic_f(arcsin(c*x),
 -e/(c^2*d)))*sqrt(-d))/(c*d^2*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}}{x^4}\,dx$$

```
[In] integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^4}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^4, x)
```

3.120 $\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 453

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx \\
&= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} \\
&+ \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} \\
&- \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} \\
&- \frac{bc^2(24c^4d^2+19c^2de-31e^2)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{b(c^2d+e)(24c^4d^2+7c^2de-30e^2)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[Out] -1/5*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*ar
csec(c*x))/d^2/x^3-2/15*b*c*e^2*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*
x^2)^(1/2)+1/45*b*c*e*(2*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^
2*x^2)^(1/2)+1/75*b*c*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+
d)^(1/2)/d^2/(c^2*x^2)^(1/2)+1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^4
/(c^2*x^2)^(1/2)+1/45*b*c*e*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^
2)^(1/2)+1/75*b*c*(4*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*
x^2)^(1/2)+2/15*b*c^2*e^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)
-1/45*b*c^2*e*(2*c^2*d+e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)
```

```
-1/75*b*c^2*(8*c^4*d^2+3*c^2*d*e-2*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(
-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+
e*x^2/d)^(1/2)+1/75*b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^
(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(
1/2)/(e*x^2+d)^(1/2)+2/45*b*c^2*e*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2
))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
/(e*x^2+d)^(1/2)-2/15*b*e^2*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c
^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {277, 270, 5346, 12, 594, 597, 538, 438, 437, 435, 432, 430}

$$\begin{aligned}&\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx\\&=\frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5}\\&\quad+\frac{bx\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\\&\quad-\frac{bc^2x\sqrt{1-c^2x^2}(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{225d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\\&\quad+\frac{bc\sqrt{c^2x^2-1}(12c^2d-e)\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}}+\frac{bc\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}}\\&\quad+\frac{bc\sqrt{c^2x^2-1}(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}}\end{aligned}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^6,x]
```

```
[Out] (b*c*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
/(225*d^2*Sqrt[c^2*x^2]) + (b*c*(12*c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
e*x^2])/(225*d*x^2*Sqrt[c^2*x^2]) + (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/
2))/(25*d*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(5*d
*x^5) + (2*e*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(15*d^2*x^3) - (b*c^2*(
24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*Ellip
ticE[ArcSin[c*x], -(e/(c^2*d))])/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*
Sqrt[1 + (e*x^2)/d]) + (b*(c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*x*S
qrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/
(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(-3d+2ex^2)}{15d^2x^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(-3d+2ex^2)}{x^6\sqrt{-1+c^2x^2}}\,dx}{15d^2\sqrt{c^2x^2}}$$

$$= \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}(d(12c^2d-e)+(3c^2d-10e)ex^2)}{x^4\sqrt{-1+c^2x^2}}\,dx}{75d^2\sqrt{c^2x^2}}$$

$$= \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)\int \frac{-d(24c^4d^2+19c^2de-31e^2)-2e(6c^4d^2+4c^2de-15e^2)x^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^2\sqrt{c^2x^2}}$$

$$= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{(bcx)\int \frac{-2de(6c^4d^2+4c^2de-15e^2)+c^2de(24c^4d^2+19c^2de-31e^2)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^3\sqrt{c^2x^2}}$$

$$= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{(bc^3(24c^4d^2+19c^2de-31e^2)x)\int\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{225d^2\sqrt{c^2x^2}} + \frac{(bc(c^2d+e)(24c^4d^2+7c^2de-30e^2)x)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^2\sqrt{c^2x^2}}$$

$$= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{\left(bc^3(24c^4d^2+19c^2de-31e^2)x\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} + \frac{\left(bc(c^2d+e)(24c^4d^2+7c^2de-30e^2)x\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{225d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{\left(bc^3(24c^4d^2+19c^2de-31e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bc(c^2d+e)(24c^4d^2+7c^2de-30e^2)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} + \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{bc^2(24c^4d^2+19c^2de-31e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{b(c^2d+e)(24c^4d^2+7c^2de-30e^2)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.83 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx$$

$$= \frac{\sqrt{d+ex^2}\Big(-15a(3d^2+dex^2-2e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(-31e^2x^4+dex^2(8+19c^2x^2)+3d^2(3+4c^2x^2+8c^4}{225d^2x^5} - \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(24c^4d^2+19c^2de-31e^2)E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)+(-24c^6d^3-31c^4d^2e\right.}{225\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/x^6,x]
```

```
[Out] (Sqrt[d + e*x^2]*(-15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2
*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8
*c^4*x^4)) - 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcSec[c*x]))/(225*d^2*x^5)
 - ((I/225)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24*c^4*
d^2 + 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]
 + (-24*c^6*d^3 - 31*c^4*d^2*e + 23*c^2*d*e^2 + 30*e^3)*EllipticF[I*ArcSinh
[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d +
e*x^2])
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arcsec}\,(cx))\,\sqrt{e\,x^{2}+d}}{x^{6}}dx$$

[In] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^6,x)

[Out] int((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^6,x)

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx$$
$$= \frac{(30\,acde^2x^4 - 15\,acd^2ex^2 - 45\,acd^3 + 15\,(2\,bcde^2x^4 - bcd^2ex^2 - 3\,bcd^3)\,\mathrm{arcsec}\,(cx) + (9\,bcd^3 + (24\,bc^5d^3 \cdots}{\cdots}$$

[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")

[Out] 1/225*((30*a*c*d*e^2*x^4 - 15*a*c*d^2*e*x^2 - 45*a*c*d^3 + 15*(2*b*c*d*e^2*x^4 - b*c*d^2*e*x^2 - 3*b*c*d^3)*arcsec(c*x) + (9*b*c*d^3 + (24*b*c^5*d^3 + 19*b*c^3*d^2*e - 31*b*c*d*e^2)*x^4 + 4*(3*b*c^3*d^3 + 2*b*c*d^2*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) + ((24*b*c^8*d^3 + 19*b*c^6*d^2*e - 31*b*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (24*b*c^8*d^3 + (19*b*c^6 + 12*b*c^4)*d^2*e - (31*b*c^4 - 8*b*c^2)*d*e^2 - 30*b*e^3)*x^5*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^3*x^5)

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}}{x^6}\,dx$$

[In] integrate((a+b*asec(c*x))*(e*x**2+d)**(1/2)/x**6,x)

[Out] Integral((a + b*asec(c*x))*sqrt(d + e*x**2)/x**6, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)}{x^6}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))))/x^6, x)
```

3.121 $\int x^3(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 374

$$\int x^3(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2} - \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} + \frac{b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{c^2x^2}}$$

```
[Out] -1/5*d*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arcse
c(c*x))/e^2-2/35*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^2/(c^2*x^2)^(1/2)+1/560*b*(35*c^6*d^3-35*c^4*d^2*e-63*c^2*d*e^2-25*
e^3)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(3/2)/(c^
2*x^2)^(1/2)-1/840*b*(13*c^2*d+25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c^
3/e/(c^2*x^2)^(1/2)-1/42*b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e/(c^2*x^2
)^(1/2)+1/560*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)
^(1/2)/c^5/e/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 5346, 12, 587, 159, 163, 65, 223, 212, 95, 210}

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2}-\frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{35e^2\sqrt{c^2x^2}}+\frac{bx(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}(13c^2d+25e)\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(3c^4d^2-38c^2de-25e^2)\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}}$$

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/
(560*c^5*e*Sqrt[c^2*x^2]) - (b*(13*c^2*d + 25*e)*x*Sqrt[-1 + c^2*x^2]*(d +
e*x^2)^(3/2))/(840*c^3*e*Sqrt[c^2*x^2]) - (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^
2)^(5/2))/(42*c*e*Sqrt[c^2*x^2]) - (d*(d + e*x^2)^(5/2)*(a + b*ArcSec[c*x])
)/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcSec[c*x]))/(7*e^2) - (2*b*c*d^(7/2
)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(35*e^2*Sqrt[c^2*
x^2]) + (b*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 - 25*e^3)*x*ArcTanh[(S
qrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^6*e^(3/2)*Sqrt[c^2*
x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2} - \frac{(bcx)\int \frac{(d+ex^2)^{5/2}(-2d+5ex^2)}{35e^2x\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2} - \frac{(bcx)\int \frac{(d+ex^2)^{5/2}(-2d+5ex^2)}{x\sqrt{-1+c^2x^2}}\,dx}{35e^2\sqrt{c^2x^2}} \\
&= -\frac{d(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\sec^{-1}(cx))}{7e^2} - \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^{5/2}(-2d+5ex)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{70e^2\sqrt{c^2x^2}}
\end{aligned}
$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2} - \frac{(bx)\text{Subst}\left(\int \frac{(d+ex)^{3/2}\left(-6c^2d^2+\frac{1}{2}e\left(13c^2d+25e\right)x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{210ce^2\sqrt{c^2x^2}}$$

$$= -\frac{b\left(13c^2d+25e\right)x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2} - \frac{(bx)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(-12c^4d^3-\frac{3}{4}e\left(3c^4d^2-38c^2de-25e^2\right)x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{420c^3e^2\sqrt{c^2x^2}}$$

$$= \frac{b\left(3c^4d^2-38c^2de-25e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b\left(13c^2d+25e\right)x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2} - \frac{(bx)\text{Subst}\left(\int \frac{-12c^6d^4-\frac{3}{8}e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{420c^5e^2\sqrt{c^2x^2}}$$

$$= \frac{b\left(3c^4d^2-38c^2de-25e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b\left(13c^2d+25e\right)x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\sec^{-1}(cx)\right)}{7e^2} + \frac{\left(bcd^4x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{35e^2\sqrt{c^2x^2}} + \frac{\left(b\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)x\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{1120c^5e\sqrt{c^2x^2}}$$

$$= \frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^2} + \frac{(2bcd^4x)\,\mathrm{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} + \frac{(b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\,x)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{560c^7e\sqrt{c^2x^2}}$$

$$= \frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^2} - \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} + \frac{(b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\,x)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{560c^7e\sqrt{c^2x^2}}$$

$$= \frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} - \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}\,(a+b\sec^{-1}(cx))}{7e^2} - \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} + \frac{b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\,x\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.58 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.82

$$\int x^3(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))\,dx = \frac{96a(d+ex^2)^3(-2d+5ex^2)-\frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(75e^2+2c^2e(82d+25ex^2)+c^4(57d^2+106dex^2+40e^2x^4}{c^5}}{}$$

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] (96*a*(d + e*x^2)^3*(-2*d + 5*e*x^2) - (2*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
e*x^2)*(75*e^2 + 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 + 40
*e^2*x^4)))/c^5 + (3*b*(-32*c^4*d^4*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/
2, 2, 1/(c^2*x^2), -(d/(e*x^2))] - (e*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d
*e^2 - 25*e^3)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/
2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^5*x) + 96*b*(d +
e*x^2)^3*(-2*d + 5*e*x^2)*ArcSec[c*x])/(3360*e^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^3\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\operatorname{arcsec}\left(cx\right)\right)dx$$

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Fricas [A] (verification not implemented)

none

Time = 2.31 (sec) , antiderivative size = 1701, normalized size of antiderivative = 4.55

$$\int x^3(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] [1/6720*(96*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^
2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
 d)*sqrt(-d) + 8*d^2)/x^4) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*
e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*
```

```
e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*
c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*
c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcsec(c*x) - (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2
*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*
sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^2), -1/6720*(192*b*c^7*d^(7/2)*a
rctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d
)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(35*b*c^6*d^3 - 35*b*c^4*d
^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c
^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2
*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(240*a*c^7*e^3*x^6 + 384*a*c^7
*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7*e^3*x^6 + 8*b*
c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcsec(c*x) - (40*b*c^5*e^3*
x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(53*b*c^5*d*e^2 + 2
5*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^2), 1/3360*(48
*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e
)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d
) + 8*d^2)/x^4) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*
e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e
*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240*
a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 4
8*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arc
sec(c*x) - (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^
3 + 2*(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 +
d))/(c^7*e^2), -1/3360*(96*b*c^7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^
2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*
x^2 - d^2)) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)
*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2
 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(240*a*c^
7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5
*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcsec(
c*x) - (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 +
2*(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/
(c^7*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^3(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^3(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int (ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)x^3\,dx$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int x^3\,\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

3.122 $\int x(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 262

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=-\frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}}-\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}+\frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}}-\frac{b(15c^4d^2+10c^2de+3e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}}$$

```
[Out] 1/5*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/e+1/5*b*c*d^(5/2)*x*arctan((e*x^2+d)^
(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)-1/40*b*(15*c^4*d^2+10*c^
2*d*e+3*e^2)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(
1/2)/(c^2*x^2)^(1/2)-1/20*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)
^(1/2)-1/40*b*(7*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/(c^2*x^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.476$, Rules

used = {5344, 457, 104, 159, 163, 65, 223, 212, 95, 210}

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e}+\frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{5e\sqrt{c^2x^2}}-\frac{bx(15c^4d^2+10c^2de+3e^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{20c\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}(7c^2d+3e)\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}}$$

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] -1/40*(b*(7*c^2*d + 3*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(c^3*Sqrt[c^
2*x^2]) - (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*Sqrt[c^2*x^2]) +
 ((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(5*e) + (b*c*d^(5/2)*x*ArcTan[Sqrt
[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(5*e*Sqrt[c^2*x^2]) - (b*(15*c^4
*d^2 + 10*c^2*d*e + 3*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
 + e*x^2])])/(40*c^4*Sqrt[e]*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5344

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e} - \frac{(bcx)\int \frac{(d+ex^2)^{5/2}}{x\sqrt{-1+c^2x^2}}\,dx}{5e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e} - \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{10e\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e} - \frac{(bx)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(2c^2d^2+\frac{1}{2}e(7c^2d+3e)x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{20ce\sqrt{c^2x^2}}$$

$$= -\frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e} - \frac{(bx)\text{Subst}\left(\int \frac{2c^4d^3+\frac{1}{4}e(15c^4d^2+10c^2de+3e^2)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{20c^3e\sqrt{c^2x^2}}$$

$$= -\frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e} - \frac{(bcd^3x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{10e\sqrt{c^2x^2}} - \frac{(b(15c^4d^2+10c^2de+3e^2)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{80c^3\sqrt{c^2x^2}}$$

$$= -\frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e} - \frac{(bcd^3x)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}} - \frac{(b(15c^4d^2+10c^2de+3e^2)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{40c^5\sqrt{c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} \\
&\quad + \frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} + \frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}} \\
&\quad - \frac{\left(b(15c^4d^2+10c^2de+3e^2)\,x\right)\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{40c^5\sqrt{c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} \\
&\quad + \frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5e} + \frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}} \\
&\quad - \frac{b(15c^4d^2+10c^2de+3e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.63 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.94

$$
\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx = \frac{\frac{16a(d+ex^2)^3}{e} - \frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)\left(3e+c^2(9d+2ex^2)\right)}{c^3} + \frac{b\left(\frac{8c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{e} + \frac{(15c^4d^2 \cdots}{\cdots}\right.}{\cdots}}{80\sqrt{d+ex^2}}
$$

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] ((16*a*(d + e*x^2)^3)/e - (2*b*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)*(3*e + c
^2*(9*d + 2*e*x^2)))/c^3 + (b*((8*c^2*d^3*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1
/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/e + ((15*c^4*d^2 + 10*c^2*d*e + 3*e
^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2,
c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^3*x) + (16*b*(d + e*x^2)^3*A
rcSec[c*x])/e)/(80*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsec}\left(cx\right)\right)dx$$

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 1377, normalized size of antiderivative = 5.26

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
```

```
[Out] [1/160*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*
d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d
)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*
log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*
c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) +
 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2
*b*c^5*d*e*x^2 + b*c^5*d^2)*arcsec(c*x) - (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e +
3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), 1/160*(16*b*c^5*d^(
5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*
sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c
^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*
d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e
*x^2 + d)*sqrt(e) + e^2) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*
d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arcsec(c*x) - (2*b*c^
3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c
^5*e), 1/80*(4*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*
(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^
2 + d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqr
t(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d
)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(8*a*c^5*e^2*
x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 +
 b*c^5*d^2)*arcsec(c*x) - (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(
c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), 1/80*(8*b*c^5*d^(5/2)*arctan(-1/2*s
qrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x
^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*
sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2
+ d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(8*a*c^5*e
```

```
^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^
2 + b*c^5*d^2)*arcsec(c*x) - (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sq
rt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e)]
```

Sympy [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x(a+b\operatorname{asec}(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
[Out] Integral(x*(a + b*asec(c*x))*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int\left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] 1/5*(e*x^2 + d)^(5/2)*a/e + 1/5*((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d
)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - 5*e*integrate((5*(c^2*e^2*x^5 + (c^
2*d*e - e^2)*x^3 - d*e*x + (c^2*e^2*x^5 + (c^2*d*e - e^2)*x^3 - d*e*x)*e^(l
og(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)*log(x) + (5*c^2*e^2*x^5*log(c)
 + 5*(c^2*d*e*log(c) - e^2*log(c))*x^3 - 5*d*e*x*log(c) + ((5*c^2*log(c) +
c^2)*e^2*x^5 + ((5*c^2*log(c) + 2*c^2)*d*e - 5*e^2*log(c))*x^3 + (c^2*d^2 -
 5*d*e*log(c))*x)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d))/(c^2*e*
x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x))*b/e
```

Giac [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int\left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x\,\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

3.123 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx = \mathrm{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx=\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsec}\,(cx))}{x}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x, x
)
```

Sympy [N/A]

Not integrable

Time = 71.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x,x)
[Out] Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x, x)
```

3.124 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3} dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3} dx = \int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3} dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^3,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3} dx$$

Mathematica [N/A]

Not integrable

Time = 6.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^3,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arcsec}(cx))}{x^3}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x^3,
 x)
```

Sympy [N/A]

Not integrable

Time = 61.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{asec}(cx))(d+ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**3,x)
```

```
[Out] Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arcsec}(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^3}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^3, x)
```

3.125 $\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Int}\left(x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
[Out] Defer[Int][x^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

Rubi steps

$$\text{integral}=\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 12.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^{2}\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\ \operatorname{arcsec}\left(c x\right)\right)dx$$

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arcsec}\left(cx\right)+a)x^2\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arcsec(c*x))*sqrt(e*x^2 +
 d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 (d + ex^2)^{3/2} (a + b\sec^{-1}(cx)) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 (d + ex^2)^{3/2} (a + b\sec^{-1}(cx)) \, dx = \int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsec}(cx) + a) x^2 \, dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2 (d + ex^2)^{3/2} (a + b\sec^{-1}(cx)) \, dx = \int x^2 (e x^2 + d)^{3/2} \left(a + b \operatorname{acos}\left(\frac{1}{c x}\right)\right) \, dx$$

```
[In] int(x^2*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

3.126 $\int (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx = \text{Int}\Big((d+ex^2)^{3/2} (a+b\sec^{-1}(cx)), x\Big)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx = \int (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx$$

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

Rubi steps

$$\text{integral} = \int (d+ex^2)^{3/2} (a+b\sec^{-1}(cx)) \, dx$$

Mathematica [N/A]

Not integrable

Time = 17.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx=\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$$

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\ \operatorname{arcsec}\left(c x\right)\right)dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx=\int { {\left(e x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arcsec}\left(c x\right) + a\right)} \,d x}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}} \left(b\,\mathrm{arcsec}\left(cx\right)+a\right)dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int {\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \,dx$$

```
[In] int((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

3.127 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \mathrm{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^2,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 32.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx=\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^2,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\ \mathrm{arcsec}\left(c x\right)\right)}{x^{2}}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x^2,
 x)
```

Sympy [N/A]

Not integrable

Time = 99.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**2,x)
[Out] Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^2}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^2, x)
```

3.128 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4} dx = \operatorname{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}, x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4} dx = \int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4} dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^4,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^4, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4} dx$$

Mathematica [N/A]

Not integrable

Time = 8.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4}\,dx=\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^4,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arcsec}\,(cx))}{x^4}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^4}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)/x^4,
 x)
```

Sympy [N/A]

Not integrable

Time = 67.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{asec}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**4,x)
[Out] Integral((a + b*asec(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^4,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^4, x)
```

3.129 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6} dx$

Optimal result

Integrand size = 23, antiderivative size = 416

$$\begin{aligned}\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6}\,dx =&\ \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} \\ &+\frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} \\ &+\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5} \\ &-\frac{bc^2(8c^4d^2+23c^2de+23e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\ &+\frac{b(c^2d+e)(8c^4d^2+19c^2de+15e^2)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

```
[Out] -1/5*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/d/x^5+1/25*b*c*(e*x^2+d)^(3/2)*(c^2*
x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)+1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*(c^
2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)+4/75*b*c*(c^2*d+2*e)*(c^2*
x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^2/(c^2*x^2)^(1/2)-1/75*b*c^2*(8*c^4*d^2+23*c
^2*d*e+23*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+
d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/75*b*(c^2*
d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*
x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d
)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {270, 5346, 12, 485, 594, 597, 538, 438, 437, 435, 432, 430}

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6}\,dx = -\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{bx\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} + \frac{4bc\sqrt{c^2x^2-1}(c^2d+2e)\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} + \frac{bc\sqrt{c^2x^2-1}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^6,x]
```

```
[Out] (b*c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/
(75*d*Sqrt[c^2*x^2]) + (4*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x
^2])/(75*x^2*Sqrt[c^2*x^2]) + (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(2
5*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(5*d*x^5) -
(b*c^2*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2
]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x
^2]*Sqrt[1 + (e*x^2)/d]) + (b*(c^2*d + e)*(8*c^4*d^2 + 19*c^2*d*e + 15*e^2)
*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d)
)])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5dx^5} - \frac{(bcx)\int -\frac{(d+ex^2)^{5/2}}{5dx^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5dx^5} + \frac{(bcx)\int \frac{(d+ex^2)^{5/2}}{x^6\sqrt{-1+c^2x^2}}\,dx}{5d\sqrt{c^2x^2}}$$

$$= \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5dx^5} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}\left(-4d(c^2d+2e)-e(c^2d+5e)x^2\right)}{x^4\sqrt{-1+c^2x^2}}\,dx}{25d\sqrt{c^2x^2}}$$

$$= \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5} + \frac{(bcx)\int \frac{d(8c^4d^2+23c^2de+23e^2)+e(4c^4d^2+11c^2de+15e^2)x^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{75d\sqrt{c^2x^2}}$$

$$= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5} + \frac{(bcx)\int \frac{de(4c^4d^2+11c^2de+15e^2)-c^2de(8c^4d^2+23c^2de+23e^2)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{75d^2\sqrt{c^2x^2}}$$

$$= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5} + \frac{(bc(c^2d+e)(8c^4d^2+19c^2de+15e^2)x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{75d\sqrt{c^2x^2}} - \frac{(bc^3(8c^4d^2+23c^2de+23e^2)x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{75d\sqrt{c^2x^2}}$$

$$= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\sec^{-1}(cx)\right)}{5dx^5} - \frac{\left(bc^3(8c^4d^2+23c^2de+23e^2)x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} + \frac{\left(bc(c^2d+e)(8c^4d^2+19c^2de+15e^2)x\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{75d\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5} - \frac{\left(bc^3(8c^4d^2+23c^2de+23e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}dx}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bc(c^2d+e)(8c^4d^2+19c^2de+15e^2)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}dx}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} + \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5dx^5} - \frac{bc^2(8c^4d^2+23c^2de+23e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{b(c^2d+e)(8c^4d^2+19c^2de+15e^2)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.89 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.73

$$\int\frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6}dx = \frac{\sqrt{d+ex^2}\left(-15a(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}x(23e^2x^4+dex^2(11+23c^2\right.}{75dx^5} - \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(8c^4d^2+23c^2de+23e^2)E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)-(8c^6d^3+27c^4d^2e+34\right.}{75\sqrt{-c^2}d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^6,x]
```

```
[Out] (Sqrt[d + e*x^2]*(-15*a*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(23*e^2
*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*
(d + e*x^2)^2*ArcSec[c*x]))/(75*d*x^5) - ((I/75)*b*c*Sqrt[1 - 1/(c^2*x^2)]*
```

```
x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*EllipticE[I*
ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (8*c^6*d^3 + 27*c^4*d^2*e + 34*c^2*d
*e^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^
2]*d*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsec}\left(c x\right)\right)}{x^{6}}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{(15\,acde^2x^4+30\,acd^2ex^2+15\,acd^3+15\,(bcde^2x^4+2\,bcd^2ex^2+bcd^3)\,\mathrm{arcsec}\,(cx)-(3\,bcd^3+(8\,bc^5d^3+23}{}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="fricas")
[Out] -1/75*((15*a*c*d*e^2*x^4 + 30*a*c*d^2*e*x^2 + 15*a*c*d^3 + 15*(b*c*d*e^2*x^
4 + 2*b*c*d^2*e*x^2 + b*c*d^3)*arcsec(c*x) - (3*b*c*d^3 + (8*b*c^5*d^3 + 23
*b*c^3*d^2*e + 23*b*c*d*e^2)*x^4 + (4*b*c^3*d^3 + 11*b*c*d^2*e)*x^2)*sqrt(c
^2*x^2 - 1))*sqrt(e*x^2 + d) - ((8*b*c^8*d^3 + 23*b*c^6*d^2*e + 23*b*c^4*d*
e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (8*b*c^8*d^3 + (23*b*c^6 + 4
*b*c^4)*d^2*e + (23*b*c^4 + 11*b*c^2)*d*e^2 + 15*b*e^3)*x^5*elliptic_f(arcs
in(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**6,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^6}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^6, x)
```

3.130 $\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 554

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}$$
$$+\frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}}$$
$$+\frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}+\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}}$$
$$-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}$$
$$-\frac{bc^2(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+\frac{2b(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] -1/7*(e*x^2+d)^(5/2)*(a+b*arcsec(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*ar
csec(c*x))/d^2/x^5+1/1225*b*c*(30*c^2*d+11*e)*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(
1/2)/d/x^4/(c^2*x^2)^(1/2)+1/49*b*c*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/d/x^6
/(c^2*x^2)^(1/2)+1/3675*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^
3)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)+1/3675*b*c*(120*c^
4*d^2+159*c^2*d*e-37*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^2)
^(1/2)-1/3675*b*c^2*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*x*Ell
ipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^
2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+2/3675*b*(c^2*d+e)*(120*c^6*d^
3+204*c^4*d^2*e+17*c^2*d*e^2-105*e^3)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c
```

```
^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {277, 270, 5346, 12, 594, 597, 538, 438, 437, 435, 432, 430}

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5}$$

$$-\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7}$$

$$+\frac{2bx\sqrt{1-c^2x^2}(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3675d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

$$-\frac{bc^2x\sqrt{1-c^2x^2}(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3675d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}$$

$$+\frac{bc\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}}+\frac{bc\sqrt{c^2x^2-1}(30c^2d+11e)(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}$$

$$+\frac{bc\sqrt{c^2x^2-1}(120c^4d^2+159c^2de-37e^2)\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}}$$

$$+\frac{bc\sqrt{c^2x^2-1}(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^8,x]
```

```
[Out] (b*c*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[-1 + c^2*
x^2]*Sqrt[d + e*x^2])/(3675*d^2*Sqrt[c^2*x^2]) + (b*c*(120*c^4*d^2 + 159*c^
2*d*e - 37*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^2*Sqrt[c^2*x^
2]) + (b*c*(30*c^2*d + 11*e)*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*
x^4*Sqrt[c^2*x^2]) + (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(5/2))/(49*d*x^6*S
qrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(7*d*x^7) + (2*e*(d
 + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/(35*d^2*x^5) - (b*c^2*(240*c^6*d^3 + 5
28*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]
*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^
2*x^2]*Sqrt[1 + (e*x^2)/d]) + (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e
 + 17*c^2*d*e^2 - 105*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Elliptic
F[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sq
rt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5} - \frac{(bcx)\int \frac{(d+ex^2)^{5/2}(-5d+2ex^2)}{35d^2x^8\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5} - \frac{(bcx)\int \frac{(d+ex^2)^{5/2}(-5d+2ex^2)}{x^8\sqrt{-1+c^2x^2}}\,dx}{35d^2\sqrt{c^2x^2}}$$

$$= \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5} + \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(d(30c^2d+11e)+(5c^2d-14e)ex^2)}{x^6\sqrt{-1+c^2x^2}}\,dx}{245d^2\sqrt{c^2x^2}}$$

$$= \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}(-d(120c^4d^2+159c^2de-37e^2)-2e(15c^4d^2+18c^2de-35e^2)x^2)}{x^4\sqrt{-1+c^2x^2}}\,dx}{1225d^2\sqrt{c^2x^2}}$$

$$= \frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} + \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5} + \frac{(bcx)\int \frac{d(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)+e(120c^6d^3+249c^4d^2e+71c^2de^2-210e^3)x^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^2\sqrt{c^2x^2}}$$

$$= \frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} + \frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} + \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5} + \frac{(bcx)\int \frac{de(120c^6d^3+249c^4d^2e+71c^2de^2-210e^3)-c^2de(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^3\sqrt{c^2x^2}}$$

$$= \frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} + \frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} + \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5} - \frac{(bc^3(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{3675d^2\sqrt{c^2x^2}} + \frac{(2bc(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^2\sqrt{c^2x^2}}$$

$$= \frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} + \frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} + \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5} - \frac{\left(bc^3(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} + \frac{\left(2bc(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3675d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$
\begin{aligned}
&= \frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} \\
&+ \frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} \\
&+ \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} \\
&- \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5} \\
&- \frac{\left(bc^3(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}dx}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{\left(2bc(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}} d}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&= \frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} \\
&+ \frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} \\
&+ \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} \\
&- \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{35d^2x^5} \\
&- \frac{bc^2(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{2b(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),\right.}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.82 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.69

$$
\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}dx = \frac{\sqrt{d+ex^2}\left(-105a(5d-2ex^2)(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}x(-247e^3x^6+\right.}{}
$$

$$
-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)-2(120\right.}{3675\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d}}
$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/x^8,x]
```

```
[Out] (Sqrt[d + e*x^2]*(-105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c^
2*x^2)]*x*(-247*e^3*x^6 + d*e^2*x^4*(71 + 193*c^2*x^2) + 3*d^2*e*x^2*(61 +
83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)
) - 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcSec[c*x]))/(3675*d^2*x^7) - ((I/
3675)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^6*d^3 +
 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x]
, -(e/(c^2*d))] - 2*(120*c^8*d^4 + 324*c^6*d^3*e + 221*c^4*d^2*e^2 - 88*c^2
*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[
-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsec}\left(cx\right)\right)}{x^{8}}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.15 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{x^8}\,dx = \frac{(210\,acde^3x^6 - 105\,acd^2e^2x^4 - 840\,acd^3ex^2 - 525\,acd^4 + 105\,(2\,bcde^3$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] 1/3675*((210*a*c*d*e^3*x^6 - 105*a*c*d^2*e^2*x^4 - 840*a*c*d^3*e*x^2 - 525*
a*c*d^4 + 105*(2*b*c*d*e^3*x^6 - b*c*d^2*e^2*x^4 - 8*b*c*d^3*e*x^2 - 5*b*c*
d^4)*arcsec(c*x) + ((240*b*c^7*d^4 + 528*b*c^5*d^3*e + 193*b*c^3*d^2*e^2 -
247*b*c*d*e^3)*x^6 + 75*b*c*d^4 + (120*b*c^5*d^4 + 249*b*c^3*d^3*e + 71*b*c
*d^2*e^2)*x^4 + 3*(30*b*c^3*d^4 + 61*b*c*d^3*e)*x^2)*sqrt(c^2*x^2 - 1))*sqr
t(e*x^2 + d) + ((240*b*c^10*d^4 + 528*b*c^8*d^3*e + 193*b*c^6*d^2*e^2 - 247
*b*c^4*d*e^3)*x^7*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (240*b*c^10*d^4 + 2
4*(22*b*c^8 + 5*b*c^6)*d^3*e + (193*b*c^6 + 249*b*c^4)*d^2*e^2 - (247*b*c^4
 - 71*b*c^2)*d*e^3 - 210*b*e^4)*x^7*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sq
rt(-d))/(c*d^3*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*asec(c*x))/x**8,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)}{x^8}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{x^8}\,dx=\int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))))/x^8, x)
```

3.131 $\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 321

$$\begin{aligned}\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx &= \frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} \\ &\quad - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} \\ &\quad - \frac{2d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} \\ &\quad + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} + \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}} \\ &\quad - \frac{b(45c^4d^2-10c^2de+9e^2)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

```
[Out] -2/3*d*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arcse
c(c*x))/e^3+8/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^3/(c^2*x^2)^(1/2)-1/120*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*x*arctanh(e
^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(5/2)/(c^2*x^2)^(1/2)-1/2
0*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)+d^2*(a+b*arcs
ec(c*x))*(e*x^2+d)^(1/2)/e^3+1/120*b*(19*c^2*d-9*e)*x*(c^2*x^2-1)^(1/2)*(e*
x^2+d)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 5346, 12, 1629, 159, 163, 65, 223, 212, 95, 210}

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} + \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{15e^3\sqrt{c^2x^2}} - \frac{bx(45c^4d^2-10c^2de+9e^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2-1}(19c^2d-9e)\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}}$$

```
[In] Int[(x^5*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (b*(19*c^2*d - 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e^2*Sqrt
[c^2*x^2]) - (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e^2*Sqrt[c^2*
x^2]) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/e^3 - (2*d*(d + e*x^2)^(3
/2)*(a + b*ArcSec[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcSec[c*x]))/
(5*e^3) + (8*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^
2])])/(15*e^3*Sqrt[c^2*x^2]) - (b*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*x*ArcTa
nh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(5/2)*Sqrt
[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}(8d^2-4dex^2+3e^2x^4)}{15e^3x\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}(8d^2-4dex^2+3e^2x^4)}{x\sqrt{-1+c^2x^2}}\,dx}{15e^3\sqrt{c^2x^2}}$$

$$= \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\sec^{-1}(cx))}{5e^3} - \frac{(bcx)\text{Subst}\left(\int \frac{\sqrt{d+ex}(8d^2-4dex+3e^2x^2)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{30e^3\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} - \frac{(bx)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(16c^2d^2e-\frac{1}{2}(19c^2d-9e)e^2x\right)}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{60ce^4\sqrt{c^2x^2}}$$

$$= \frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} - \frac{(bx)\text{Subst}\left(\int \frac{16c^4d^3e+\frac{1}{4}e^2(45c^4d^2-10c^2de+9e^2)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{60c^3e^4\sqrt{c^2x^2}}$$

$$= \frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} - \frac{(4bcd^3x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{15e^3\sqrt{c^2x^2}} - \frac{(b(45c^4d^2-10c^2de+9e^2)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{240c^3e^2\sqrt{c^2x^2}}$$

$$= \frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} - \frac{(8bcd^3x)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}} - \frac{(b(45c^4d^2-10c^2de+9e^2)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{120c^5e^2\sqrt{c^2x^2}}$$

$$= \frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} + \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}} - \frac{(b(45c^4d^2-10c^2de+9e^2)\,x)\,\text{Subst}\left(\int\frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{120c^5e^2\sqrt{c^2x^2}}$$

$$= \frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} - \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}\,(a+b\sec^{-1}(cx))}{5e^3} + \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}} - \frac{b(45c^4d^2-10c^2de+9e^2)\,x\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.00 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.88

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{16a(d+ex^2)\,(8d^2-4dex^2+3e^2x^4) - \frac{2be\sqrt{1-\frac{1}{c^2x^2}}\,(d+ex^2)\,(9ex+c^2(-13dx+6ex^3))}{c^3} + \frac{64bd^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},\right.}{cx}}{240}$$

```
[In] Integrate[(x^5*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (16*a*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) - (2*b*e*Sqrt[1 - 1/(c^2*
x^2)]*(d + e*x^2)*(9*e*x + c^2*(-13*d*x + 6*e*x^3)))/c^3 + (64*b*d^3*Sqrt[1
 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/(c*x) +
(b*e*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 + (
e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/(c^3*Sqrt[1 - c^
2*x^2]) + 16*b*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4)*ArcSec[c*x])/(24
0*e^3*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arcsec}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.12 (sec) , antiderivative size = 1385, normalized size of antiderivative = 4.31

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(64*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2
*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
d)*sqrt(-d) + 8*d^2)/x^4) + (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)
*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2
*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2)
+ 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^
4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 - 13*b*c^
3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), 1/480*(12
8*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(
e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b*c^4*d
^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*
e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2
- 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^
2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arcs
ec(c*x) - (6*b*c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*s
qrt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c
^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e
)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (45*b*c^4*d^2 - 10*b*
c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x
^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x
^2)) + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e
^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arcsec(c*x) - (6*b*c^3*e^2*x^2 - 13
*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), 1/24
0*(64*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*s
qrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b*c
^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d -
```

```
 e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*
d*e - c*e^2)*x^2)) + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2
+ 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arcsec(c*x) - (6*b*c^
3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(
c^5*e^3)]
```

Sympy [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^5(a+b\,\mathrm{asec}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**5*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^5}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^5\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.132 $\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 225

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{\left(d+ex^2\right)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} - \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} + \frac{b(3c^2d-e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^2-2/3*b*c*d^(3/2)*x*arctan((e*x^2+d
)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/(c^2*x^2)^(1/2)+1/6*b*(3*c^2*d-e)*x*
arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(3/2)/(c^2*x^2)^
(1/2)-d*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/e^2-1/6*b*x*(c^2*x^2-1)^(1/2)*(e*
x^2+d)^(1/2)/c/e/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules

used = {272, 45, 5346, 12, 587, 159, 163, 65, 223, 212, 95, 210}

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = -\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^2} - \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^2\sqrt{c^2x^2}} + \frac{bx(3c^2d-e)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}}$$

```
[In] Int[(x^3*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -1/6*(b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(c*e*Sqrt[c^2*x^2]) - (d*Sqrt
[d + e*x^2]*(a + b*ArcSec[c*x]))/e^2 + ((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x
]))/(3*e^2) - (2*b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^
2*x^2])])/(3*e^2*Sqrt[c^2*x^2]) + (b*(3*c^2*d - e)*x*ArcTanh[(Sqrt[e]*Sqrt[
-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
```

```
]]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} - \frac{(bcx)\int \frac{(-2d+ex^2)\sqrt{d+ex^2}}{3e^2x\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} - \frac{(bcx)\int \frac{(-2d+ex^2)\sqrt{d+ex^2}}{x\sqrt{-1+c^2x^2}}\,dx}{3e^2\sqrt{c^2x^2}}$$

$$= -\frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} - \frac{(bcx)\text{Subst}\left(\int \frac{(-2d+ex)\sqrt{d+ex}}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{6e^2\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} - \frac{(bx)\text{Subst}\left(\int \frac{-2c^2d^2-\frac{1}{2}(3c^2d-e)ex}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6ce^2\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} + \frac{(bcd^2x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{3e^2\sqrt{c^2x^2}} + \frac{(b(3c^2d-e)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{12ce\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} + \frac{(2bcd^2x)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} + \frac{(b(3c^2d-e)x)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{6c^3e\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} - \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} + \frac{(b(3c^2d-e)x)\operatorname{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^3e\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^2} - \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} + \frac{b(3c^2d-e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.36 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.08

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{-4bd^2\sqrt{1+\frac{d}{ex^2}}(-1+c^2x^2)\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right) + b(3c^2d-e)\,e\sqrt{1-\frac{1}{c^2x^2}}x^4\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}} \cdots}{12ce^2x \cdots}$$

```
[In] Integrate[(x^3*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (-4*b*d^2*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c^
2*x^2), -(d/(e*x^2))] + b*(3*c^2*d - e)*e*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1
- c^2*x^2]*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/
d)] - 2*x*(-1 + c^2*x^2)*(d + e*x^2)*(4*a*c*d + b*e*Sqrt[1 - 1/(c^2*x^2)]*x
 - 2*a*c*e*x^2 + 2*b*c*(2*d - e*x^2)*ArcSec[c*x]))/(12*c*e^2*x*(-1 + c^2*x^
2)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arcsec}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 1111, normalized size of antiderivative = 4.94

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*s
qrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d
^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 - 4*a*c
^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcsec(c*x))*s
qrt(e*x^2 + d))/(c^3*e^2), -1/24*(8*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2
- 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^
2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^
2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*s
qrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(2*a*c^3*e*x^2 - 4*a*c^
3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcsec(c*x))*sq
rt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*
e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(-e)
*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqr
t(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x^2 - 4
*a*c^3*d - sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcsec(c*x
))*sqrt(e*x^2 + d))/(c^3*e^2), -1/12*(4*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*
x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^
2*d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x
^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c
*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(2*a*c^3*e*x^2 - 4*a*c^3*d - sqrt(c^2*x^
2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c
^3*e^2)]
```

Sympy [F]

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^3(a + b\,\mathrm{asec}\,(cx))}{\sqrt{d + ex^2}}\,dx$$

```
[In] integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**3*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx) + a)x^3}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^3\,\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.133 $\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e} + \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} - \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

```
[Out] b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e/(c^2*x^2)
^(1/2)-b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(1/2)/(c^
2*x^2)^(1/2)+(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5344, 457, 132, 65, 223, 212, 12, 95, 210}

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e} + \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e\sqrt{c^2x^2}} - \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

```
[In] Int[(x*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/e + (b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*
x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(e*Sqrt[c^2*x^2]) - (b*x*ArcTanh[(Sqrt[
e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5344

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x\sqrt{-1+c^2x^2}}\,dx}{e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e} - \frac{(bcx)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{2e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2\sqrt{c^2x^2}} - \frac{(bcx)\text{Subst}\left(\int \frac{d}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e} - \frac{(bx)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{c\sqrt{c^2x^2}} - \frac{(bcdx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e} - \frac{(bx)\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{c\sqrt{c^2x^2}} - \frac{(bcdx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e} + \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} - \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e-c^2ex^2}{c^2d+e},1-c^2x^2\right)}{\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}}+b\sec^{-1}(cx)\right)}{e}$$

```
[In] Integrate[(x*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a - (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*AppellF1[1/2, -1/2, 1, 3
/2, (e - c^2*e*x^2)/(c^2*d + e), 1 - c^2*x^2])/Sqrt[(c^2*(d + e*x^2))/(c^2*
d + e)] + b*ArcSec[c*x]))/e
```

Maple [F]

$$\int \frac{x(a+b\,\text{arcsec}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 869, normalized size of antiderivative = 6.58

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \left[\frac{bc\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de+e^2)x^4-8\,(c^2d^2-de)x^2-4\,\sqrt{c^2x^2-1}\left((c^2d-e)x^2-2\,d\right)\sqrt{ex^2+d}\sqrt{-d}+8\,d^2}{x^4}\right)+b\sqrt{e}\log\left(8\,c^4e^2x^4+c^4d^2\right.}{}\right.$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)
*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d)
 + 8*d^2)/x^4) + b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4
*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(e) + e^2) + 4*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) + a*c))/(c*e
```

```
), 1/4*(2*b*c*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)
*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + b*sqr
t(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 -
4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e
^2) + 4*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) + a*c))/(c*e), 1/4*(b*c*sqrt(-d)*l
og(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^
2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 2*b
*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2
 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 4*sqrt(e*x^
2 + d)*(b*c*arcsec(c*x) + a*c))/(c*e), 1/2*(b*c*sqrt(d)*arctan(-1/2*sqrt(c^
2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (
c^2*d^2 - d*e)*x^2 - d^2)) + b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e
)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*
e - c*e^2)*x^2)) + 2*sqrt(e*x^2 + d)*(b*c*arcsec(c*x) + a*c))/(c*e)]
```

Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x(a+b\,\mathrm{asec}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] -(e*integrate((c^2*e*x^3*log(c) - e*x*log(c) + ((c^2*log(c) + c^2)*e*x^3 +
(c^2*d - e*log(c))*x)*e^(log(c*x + 1) + log(c*x - 1)) + (c^2*e*x^3 - e*x +
(c^2*e*x^3 - e*x)*e^(log(c*x + 1) + log(c*x - 1)))*log(x))/((c^2*e*x^2 + (c
^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e)*sqrt(e*x^2 + d)), x) - s
qrt(e*x^2 + d)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/e + sqrt(e*x^2 + d)*a
/e
```

Giac [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.134 $\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \mathrm{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])/(x*Sqrt[d + e*x^2]),x]
```

```
[Out] Defer[Int][(a + b*ArcSec[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{x\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{ex^2+dx}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [N/A]

Not integrable

Time = 7.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asec(c*x))/(x*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{x\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a + b\sec^{-1}(cx)}{x\sqrt{d + ex^2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{\sqrt{ex^2 + d}x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\sec^{-1}(cx)}{x\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

3.135 $\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] Defer[Int][(a + b*ArcSec[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcSec[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsec}\left(cx\right)}{x^3\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\sqrt{ex^2+d}x^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 27.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asec}\left(cx\right)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asec(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a + b\sec^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{\sqrt{ex^2 + dx^3}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\sec^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.136 $\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(x^2*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(x^2*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 32.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(x^2*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[(x^2*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\,\mathrm{arcsec}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arcsec(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 54.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\,\mathrm{asec}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**2*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)x^2}{\sqrt{ex^2 + d}} \, dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a + b \sec^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{x^2 \left(a + b \operatorname{acos}\left(\frac{1}{c x}\right)\right)}{\sqrt{e x^2 + d}} \, dx$$

```
[In] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.137 $\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\operatorname{Int}\left(\frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcSec[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcSec[c*x])/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\operatorname{arcsec}\left(cx\right)}{\sqrt{e\,x^{2}+d}}dx$$

```
[In] int((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\,\operatorname{arcsec}\left(cx\right)+a}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 15.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\sec^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\operatorname{asec}\left(cx\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b\sec^{-1}(cx)}{\sqrt{d + ex^2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a + b\sec^{-1}(cx)}{\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

3.138 $\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 246

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{b(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] -(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/d/x+b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2
)/d/(c^2*x^2)^(1/2)-b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+b
*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)
^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules

used = {270, 5346, 12, 486, 21, 434, 438, 437, 435, 432, 430}

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx} + \frac{bx\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} + \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d*Sqrt[c^2*x^2]) - (Sqrt[d + e*x^
2]*(a + b*ArcSec[c*x]))/(d*x) - (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*
EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*S
qrt[1 + (e*x^2)/d]) + (b*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d
]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]
*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
```

```
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}+\frac{(bcx)\int\frac{\sqrt{d+ex^2}}{dx^2\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}+\frac{(bcx)\int\frac{\sqrt{d+ex^2}}{x^2\sqrt{-1+c^2x^2}}\,dx}{d\sqrt{c^2x^2}}\\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}-\frac{(bcx)\int\frac{-e+c^2ex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}\\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}-\frac{(bcex)\int\frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}\\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}\\
&\quad -\frac{(bc^3x)\int\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{d\sqrt{c^2x^2}}+\frac{(bc(c^2d+e)\,x)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}\\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}\\
&\quad -\frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}+\frac{\left(bc(c^2d+e)\,x\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{d+ex^2}}\\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}\\
&\quad -\frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\\
&\quad +\frac{\left(bc(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

$$= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{dx}$$
$$- \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{b(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.58

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(-a+bc\sqrt{1-\frac{1}{c^2x^2}}x-b\sec^{-1}(cx)\right)}{dx} - \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)\left|-\frac{c^2d}{e}\right.\right)}{d\sqrt{-\frac{e}{d}}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x - b*ArcSec[c*x]))/(d*x)
- (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[
-(e/d)]*x], -((c^2*d)/e)])/(d*Sqrt[-(e/d)]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2
])
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{x^2\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx =$$
$$-\frac{\left(bcd\,\mathrm{arcsec}\,(cx)-\sqrt{c^2x^2-1}bcd+acd\right)\sqrt{ex^2+d}-\left(bc^4dxE(\arcsin\,(cx)\,|-\frac{e}{c^2d})-(bc^4d+be)xF(\mathrm{arcsi}\right.}{cd^2x}$$

[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] -((b*c*d*arcsec(c*x) - sqrt(c^2*x^2 - 1)*b*c*d + a*c*d)*sqrt(e*x^2 + d) - (b*c^4*d*x*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (b*c^4*d + b*e)*x*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x)

Sympy [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\mathrm{asec}\,(cx)}{x^2\sqrt{d + ex^2}}\,dx$$

[In] integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(1/2),x)

[Out] Integral((a + b*asec(c*x))/(x**2*sqrt(d + e*x**2)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{\sqrt{ex^2 + d}x^2}\,dx$$

[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.139 $\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 362

$$\begin{aligned}
&\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx\\
&=\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}+\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}\\
&\quad-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x}\\
&\quad-\frac{bc^2(2c^2d-5e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\\
&\quad+\frac{2b(c^2d-3e)\,(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

```
[Out] -1/3*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arcsec(c*x))*(e*x^2
+d)^(1/2)/d^2/x+1/9*b*c*(2*c^2*d-5*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2
/(c^2*x^2)^(1/2)+1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^2)^
(1/2)-1/9*b*c^2*(2*c^2*d-5*e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1
)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(
1/2)+2/9*b*(c^2*d-3*e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^
2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d
)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {277, 270, 5346, 12, 594, 597, 538, 438, 437, 435, 432, 430}

$$\begin{aligned}&\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx\\&=\frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3}\\&+\frac{2bx\sqrt{1-c^2x^2}(c^2d-3e)\,(c^2d+e)\,\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\\&-\frac{bc^2x\sqrt{1-c^2x^2}(2c^2d-5e)\,\sqrt{d+ex^2}E\left(\arcsin(cx)\,|-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\\&+\frac{bc\sqrt{c^2x^2-1}(2c^2d-5e)\,\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}+\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}\end{aligned}$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*c*(2*c^2*d - 5*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d^2*Sqrt[c^2*x^
2]) + (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*x^2*Sqrt[c^2*x^2]) - (S
qrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b
*ArcSec[c*x]))/(3*d^2*x) - (b*c^2*(2*c^2*d - 5*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[
d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[c^2*x^2]*Sqrt[
-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (2*b*(c^2*d - 3*e)*(c^2*d + e)*x*Sqrt[
1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(9*d
^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} \\
&\quad + \frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{3d^2x^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{x^4\sqrt{-1+c^2x^2}}\,dx}{3d^2\sqrt{c^2x^2}} \\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} \\
&\quad + \frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} + \frac{(bcx)\int \frac{d(2c^2d-5e)+(c^2d-6e)ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} + \frac{(bcx)\int \frac{d(c^2d-6e)e-c^2d(2c^2d-5e)ex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^3\sqrt{c^2x^2}}$$

$$= \frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} - \frac{(bc^3(2c^2d-5e)\,x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{9d^2\sqrt{c^2x^2}} + \frac{(2bc(c^2d-3e)\,(c^2d+e)\,x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{c^2x^2}}$$

$$= \frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} - \frac{\left(bc^3(2c^2d-5e)\,x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} + \frac{\left(2bc(c^2d-3e)\,(c^2d+e)\,x\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{9d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} - \frac{\left(bc^3(2c^2d-5e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(2bc(c^2d-3e)\,(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$
\begin{aligned}
&= \frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} \\
&\quad - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{3d^2x} \\
&\quad - \frac{bc^2(2c^2d-5e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{2b(c^2d-3e)\,(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.52 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.69

$$
\begin{aligned}
&\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx \\
&= \frac{\sqrt{d+ex^2}\left(bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2-5ex^2)-3a(d-2ex^2)-3b(d-2ex^2)\sec^{-1}(cx)\right)}{9d^2x^3} \\
&\quad - \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(2c^2d-5e)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\,\middle|-\frac{e}{c^2d}\right)+2(-c^4d^2+2c^2de+3e^2)\,\text{Elliptic}\right.}{9\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 - 5*e*x^2) -
 3*a*(d - 2*e*x^2) - 3*b*(d - 2*e*x^2)*ArcSec[c*x]))/(9*d^2*x^3) - ((I/9)*b
*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d - 5*e)*Ellip
ticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(-(c^4*d^2) + 2*c^2*d*e + 3
*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*Sq
rt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\ \text{arcsec}\,(cx)}{x^4\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.54

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$= \frac{\left(6\,acdex^2-3\,acd^2+3\left(2\,bcdex^2-bcd^2\right)\operatorname{arcsec}\left(cx\right)+\left(bcd^2+\left(2\,bc^3d^2-5\,bcde\right)x^2\right)\sqrt{c^2x^2-1}\right)\sqrt{ex^2+d}+}{9\,}$$

```
[In] integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*((6*a*c*d*e*x^2 - 3*a*c*d^2 + 3*(2*b*c*d*e*x^2 - b*c*d^2)*arcsec(c*x) +
 (b*c*d^2 + (2*b*c^3*d^2 - 5*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 +
d) + ((2*b*c^6*d^2 - 5*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d)) -
 (2*b*c^6*d^2 - (5*b*c^4 - b*c^2)*d*e - 6*b*e^2)*x^3*elliptic_f(arcsin(c*x)
, -e/(c^2*d)))*sqrt(-d))/(c*d^3*x^3)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{asec}\left(cx\right)}{x^4\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asec(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\sec^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{\sqrt{ex^2 + dx^4}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.140 $\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 1006

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx$$
$$= \frac{8bce^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{15d^3\sqrt{c^2x^2}} - \frac{4bce(2c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^3\sqrt{c^2x^2}}$$
$$+ \frac{bc(8c^4d^2+3c^2de-2e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{25dx^4\sqrt{c^2x^2}}$$
$$- \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^2x^2\sqrt{c^2x^2}} + \frac{bc(4c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^2x^2\sqrt{c^2x^2}}$$
$$- \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3}$$
$$- \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} - \frac{8bc^2e^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{4bc^2e(2c^2d+e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{45d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{bc^2(8c^4d^2+3c^2de-2e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{bc^2(8c^2d-e)(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$
$$- \frac{8bc^2e(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{45d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$
$$+ \frac{8be^2(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] -1/5*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/d/x^5+4/15*e*(a+b*arcsec(c*x))*(e*x^
2+d)^(1/2)/d^2/x^3-8/15*e^2*(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/d^3/x+8/15*b*
c*e^2*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)-4/45*b*c*e*(2*c
^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)+1/75*b*c*(8*c
^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/
2)+1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^4/(c^2*x^2)^(1/2)-4/45*b*
c*e*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2)^(1/2)+1/75*b*c*(4*c
^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2)^(1/2)-8/15*b*c^
2*e^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/
d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+4/45*b*c^2*e*(2*c^2
*d+e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/
```

```
d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-1/75*b*c^2*(8*c^4*d
^2+3*c^2*d*e-2*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e
*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/75*
b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^
(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/
2)-8/45*b*c^2*e*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
+8/15*b*e^2*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*
(1+e*x^2/d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules

used = {277, 270, 5346, 12, 6874, 486, 597, 538, 438, 437, 435, 432, 430, 21, 434}

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx$$
$$= -\frac{8be^2x\sqrt{1-c^2x^2}\sqrt{ex^2+d}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)c^2}{15d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}$$
$$+\frac{4be(2dc^2+e)\,x\sqrt{1-c^2x^2}\sqrt{ex^2+d}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)c^2}{45d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}$$
$$-\frac{b(8d^2c^4+3dec^2-2e^2)\,x\sqrt{1-c^2x^2}\sqrt{ex^2+d}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)c^2}{75d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}$$
$$+\frac{b(8c^2d-e)\,(dc^2+e)\,x\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)c^2}{75d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{ex^2+d}}$$
$$-\frac{8be(dc^2+e)\,x\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)c^2}{45d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{ex^2+d}}$$
$$+\frac{8be^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}c}{15d^3\sqrt{c^2x^2}}-\frac{4be(2dc^2+e)\sqrt{c^2x^2-1}\sqrt{ex^2+d}c}{45d^3\sqrt{c^2x^2}}$$
$$+\frac{b(8d^2c^4+3dec^2-2e^2)\sqrt{c^2x^2-1}\sqrt{ex^2+d}c}{75d^3\sqrt{c^2x^2}}$$
$$-\frac{4be\sqrt{c^2x^2-1}\sqrt{ex^2+d}c}{45d^2x^2\sqrt{c^2x^2}}+\frac{b(4dc^2+e)\sqrt{c^2x^2-1}\sqrt{ex^2+d}c}{75d^2x^2\sqrt{c^2x^2}}$$
$$+\frac{b\sqrt{c^2x^2-1}\sqrt{ex^2+d}c}{25dx^4\sqrt{c^2x^2}}-\frac{8e^2\sqrt{ex^2+d}(a+b\sec^{-1}(cx))}{15d^3x}$$
$$+\frac{4e\sqrt{ex^2+d}(a+b\sec^{-1}(cx))}{15d^2x^3}-\frac{\sqrt{ex^2+d}(a+b\sec^{-1}(cx))}{5dx^5}$$
$$+\frac{8be^2(dc^2+e)\,x\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{15d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{ex^2+d}}$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^6*Sqrt[d + e*x^2]),x]
```

```
[Out] (8*b*c*e^2*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(15*d^3*Sqrt[c^2*x^2]) - (4*
b*c*e*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(45*d^3*Sqrt[c^2*x^
2]) + (b*c*(8*c^4*d^2 + 3*c^2*d*e - 2*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^
2])/(75*d^3*Sqrt[c^2*x^2]) + (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(25*d
*x^4*Sqrt[c^2*x^2]) - (4*b*c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(45*d^2*
x^2*Sqrt[c^2*x^2]) + (b*c*(4*c^2*d + e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
/(75*d^2*x^2*Sqrt[c^2*x^2]) - (Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(5*d*x^
5) + (4*e*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))/(15*d^2*x^3) - (8*e^2*Sqrt[d
 + e*x^2]*(a + b*ArcSec[c*x]))/(15*d^3*x) - (8*b*c^2*e^2*x*Sqrt[1 - c^2*x^2
```

```
]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(15*d^3*Sqrt[c^2*x^
2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (4*b*c^2*e*(2*c^2*d + e)*x*Sqr
t[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(45*d^
3*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*c^2*(8*c^4*d^2
 + 3*c^2*d*e - 2*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[
c*x], -(e/(c^2*d))])/(75*d^3*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x
^2)/d]) + (b*c^2*(8*c^2*d - e)*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*
x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(75*d^2*Sqrt[c^2*x^2]*Sqrt[-1
 + c^2*x^2]*Sqrt[d + e*x^2]) - (8*b*c^2*e*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*S
qrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(45*d^2*Sqrt[c^2*x
^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]) + (8*b*e^2*(c^2*d + e)*x*Sqrt[1 - c
^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(15*d^3*S
qrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5}+\frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} \\
&\quad -\frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x}-\frac{(bcx)\int\frac{\sqrt{d+ex^2}(-3d^2+4dex^2-8e^2x^4)}{15d^3x^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5}+\frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} \\
&\quad -\frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x}-\frac{(bcx)\int\frac{\sqrt{d+ex^2}(-3d^2+4dex^2-8e^2x^4)}{x^6\sqrt{-1+c^2x^2}}\,dx}{15d^3\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} - \frac{(bcx)\int\left(-\frac{3d^2\sqrt{d+ex^2}}{x^6\sqrt{-1+c^2x^2}} + \frac{4de\sqrt{d+ex^2}}{x^4\sqrt{-1+c^2x^2}} - \frac{8e^2\sqrt{d+ex^2}}{x^2\sqrt{-1+c^2x^2}}\right)dx}{15d^3\sqrt{c^2x^2}}$$

$$= -\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} + \frac{(bcx)\int\frac{\sqrt{d+ex^2}}{x^6\sqrt{-1+c^2x^2}}\,dx}{5d\sqrt{c^2x^2}} - \frac{(4bcex)\int\frac{\sqrt{d+ex^2}}{x^4\sqrt{-1+c^2x^2}}\,dx}{15d^2\sqrt{c^2x^2}} + \frac{(8bce^2x)\int\frac{\sqrt{d+ex^2}}{x^2\sqrt{-1+c^2x^2}}\,dx}{15d^3\sqrt{c^2x^2}}$$

$$= \frac{8bce^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{15d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{25dx^4\sqrt{c^2x^2}} - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^2x^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} - \frac{(bcx)\int\frac{-4c^2d-e-3c^2ex^2}{x^4\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{25d\sqrt{c^2x^2}} + \frac{(4bcex)\int\frac{-2c^2d-e-c^2ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{45d^2\sqrt{c^2x^2}} - \frac{(8bce^2x)\int\frac{-e+c^2ex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{15d^3\sqrt{c^2x^2}}$$

$$= \frac{8bce^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{15d^3\sqrt{c^2x^2}} - \frac{4bce(2c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{25dx^4\sqrt{c^2x^2}} - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^2x^2\sqrt{c^2x^2}} + \frac{bc(4c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^2x^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} - \frac{(bcx)\int\frac{-8c^4d^2-3c^2de+2e^2-c^2e(4c^2d+e)x^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{75d^2\sqrt{c^2x^2}} + \frac{(4bcex)\int\frac{-c^2de+c^2e(2c^2d+e)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{45d^3\sqrt{c^2x^2}} - \frac{(8bce^3x)\int\frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\,dx}{15d^3\sqrt{c^2x^2}}$$

$$
\begin{aligned}
&= \frac{8bce^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{15d^3\sqrt{c^2x^2}} - \frac{4bce(2c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^3\sqrt{c^2x^2}} \\
&\quad + \frac{bc(8c^4d^2+3c^2de-2e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{25dx^4\sqrt{c^2x^2}} \\
&\quad - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^2x^2\sqrt{c^2x^2}} + \frac{bc(4c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^2x^2\sqrt{c^2x^2}} \\
&\quad - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} \\
&\quad - \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} - \frac{(bcx)\int \frac{-c^2de(4c^2d+e)+c^2e(8c^4d^2+3c^2de-2e^2)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{75d^3\sqrt{c^2x^2}} \\
&\quad - \frac{(8bc^3e^2x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{15d^3\sqrt{c^2x^2}} - \frac{(8bc^3e(c^2d+e)\,x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{45d^2\sqrt{c^2x^2}} \\
&\quad + \frac{(8bce^2(c^2d+e)\,x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{15d^3\sqrt{c^2x^2}} + \frac{(4bc^3e(2c^2d+e)\,x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{45d^3\sqrt{c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{8bce^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{15d^3\sqrt{c^2x^2}} - \frac{4bce(2c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^3\sqrt{c^2x^2}} \\
&\quad + \frac{bc(8c^4d^2+3c^2de-2e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{25dx^4\sqrt{c^2x^2}} \\
&\quad - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^2x^2\sqrt{c^2x^2}} + \frac{bc(4c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^2x^2\sqrt{c^2x^2}} \\
&\quad - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} \\
&\quad - \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} + \frac{(bc^3(8c^2d-e)(c^2d+e)\,x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{75d^2\sqrt{c^2x^2}} \\
&\quad - \frac{(bc^3(8c^4d^2+3c^2de-2e^2)\,x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{75d^3\sqrt{c^2x^2}} - \frac{\left(8bc^3e^2x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} \\
&\quad + \frac{\left(4bc^3e(2c^2d+e)\,x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{45d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} \\
&\quad - \frac{\left(8bc^3e(c^2d+e)\,x\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{45d^2\sqrt{c^2x^2}\sqrt{d+ex^2}} \\
&\quad + \frac{\left(8bce^2(c^2d+e)\,x\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{15d^3\sqrt{c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

$$= \frac{8bce^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{15d^3\sqrt{c^2x^2}} - \frac{4bce(2c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^3\sqrt{c^2x^2}}$$
$$+ \frac{bc(8c^4d^2+3c^2de-2e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^3\sqrt{c^2x^2}}$$
$$+ \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{25dx^4\sqrt{c^2x^2}} - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^2x^2\sqrt{c^2x^2}}$$
$$+ \frac{bc(4c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^2x^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5}$$
$$+ \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} - \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x}$$
$$- \frac{\left(bc^3(8c^4d^2+3c^2de-2e^2)\,x\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{75d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$
$$- \frac{\left(8bc^3e^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{\left(4bc^3e(2c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{45d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{\left(bc^3(8c^2d-e)\,(c^2d+e)\,x\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{75d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}$$
$$- \frac{\left(8bc^3e(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{45d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$
$$+ \frac{\left(8bce^2(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$
\begin{aligned}
&= \frac{8bce^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{15d^3\sqrt{c^2x^2}} - \frac{4bce(2c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^3\sqrt{c^2x^2}} \\
&\quad + \frac{bc(8c^4d^2+3c^2de-2e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{25dx^4\sqrt{c^2x^2}} \\
&\quad - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^2x^2\sqrt{c^2x^2}} + \frac{bc(4c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^2x^2\sqrt{c^2x^2}} \\
&\quad - \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} \\
&\quad - \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} - \frac{8bc^2e^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{4bc^2e(2c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{45d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{8bc^2e(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\mathrm{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{45d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&\quad + \frac{8be^2(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\mathrm{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&\quad - \frac{\left(bc^3(8c^4d^2+3c^2de-2e^2)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{75d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{\left(bc^3(8c^2d-e)\,(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{75d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{8bce^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{15d^3\sqrt{c^2x^2}} - \frac{4bce(2c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^3\sqrt{c^2x^2}} \\
&+ \frac{bc(8c^4d^2+3c^2de-2e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{25dx^4\sqrt{c^2x^2}} \\
&- \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{45d^2x^2\sqrt{c^2x^2}} + \frac{bc(4c^2d+e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d^2x^2\sqrt{c^2x^2}} \\
&- \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{5dx^5} + \frac{4e\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^2x^3} \\
&- \frac{8e^2\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{15d^3x} - \frac{8bc^2e^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{4bc^2e(2c^2d+e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{45d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&- \frac{bc^2(8c^4d^2+3c^2de-2e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{bc^2(8c^2d-e)(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{8bc^2e(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{45d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&+ \frac{8be^2(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{15d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.64 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.33

$$
\begin{aligned}
&\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx \\
&= \frac{\sqrt{d+ex^2}\left(-15a(3d^2-4dex^2+8e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(94e^2x^4-dex^2(17+31c^2x^2)+3d^2(3+4c^2x^2+8\right.}{225d^3x^5} \\
&- \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(24c^4d^2-31c^2de+94e^2)E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)-(24c^6d^3-19c^4d^2e\right.}{225\sqrt{-c^2}d^3\sqrt{1-c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^6*Sqrt[d + e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-15*a*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4) + b*c*Sqrt[1 - 1/(c
^2*x^2)]*x*(94*e^2*x^4 - d*e*x^2*(17 + 31*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 +
```

```
 8*c^4*x^4)) - 15*b*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4)*ArcSec[c*x]))/(225*d^3*
x^5) - ((I/225)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24*
c^4*d^2 - 31*c^2*d*e + 94*e^2)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*
d))] - (24*c^6*d^3 - 19*c^4*d^2*e + 77*c^2*d*e^2 + 120*e^3)*EllipticF[I*Arc
Sinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^3*Sqrt[1 - c^2*x^2]*Sqrt[
d + e*x^2])
```

Maple [F]

$$\int \frac{a + b \operatorname{arcsec}(cx)}{x^6 \sqrt{e\,x^2 + d}}dx$$

```
[In] int((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.29

$$\int \frac{a + b\sec^{-1}(cx)}{x^6\sqrt{d + ex^2}}\,dx =$$
$$-\frac{\left(120\,acde^2x^4 - 60\,acd^2ex^2 + 45\,acd^3 + 15\,(8\,bcde^2x^4 - 4\,bcd^2ex^2 + 3\,bcd^3)\operatorname{arcsec}(cx) - (9\,bcd^3 + (24\,bc^5\right.}{}$$

```
[In] integrate((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/225*((120*a*c*d*e^2*x^4 - 60*a*c*d^2*e*x^2 + 45*a*c*d^3 + 15*(8*b*c*d*e^
2*x^4 - 4*b*c*d^2*e*x^2 + 3*b*c*d^3)*arcsec(c*x) - (9*b*c*d^3 + (24*b*c^5*d
^3 - 31*b*c^3*d^2*e + 94*b*c*d*e^2)*x^4 + (12*b*c^3*d^3 - 17*b*c*d^2*e)*x^2
)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((24*b*c^8*d^3 - 31*b*c^6*d^2*e + 94
*b*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (24*b*c^8*d^3 - (31
*b*c^6 - 12*b*c^4)*d^2*e + (94*b*c^4 - 17*b*c^2)*d*e^2 + 120*b*e^3)*x^5*ell
iptic_f(arcsin(c*x), -e/(c^2*d))*sqrt(-d))/(c*d^4*x^5)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{x^6\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x**6/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*asec(c*x))/(x**6*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{\sqrt{ex^2+d}x^6}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^6/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/(sqrt(e*x^2 + d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{cx}\right)}{x^6\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^6*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x^6*(d + e*x^2)^(1/2)), x)
```

3.141 $\int \frac{x^5\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 252

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}\,(a+b\sec^{-1}(cx))}{3e^3} - \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{b(9c^2d-e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{c^2x^2}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x))/e^3-8/3*b*c*d^(3/2)*x*arctan((e*x^2+d
)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)+1/6*b*(9*c^2*d-e)*x*
arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(5/2)/(c^2*x^2)^
(1/2)-d^2*(a+b*arcsec(c*x))/e^3/(e*x^2+d)^(1/2)-2*d*(a+b*arcsec(c*x))*(e*x^
2+d)^(1/2)/e^3-1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e^2/(c^2*x^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules

used = {272, 45, 5346, 12, 1629, 163, 65, 223, 212, 95, 210}

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}\left(a+b\sec^{-1}(cx)\right)}{3e^3} - \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{bx(9c^2d-e)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}}$$

```
[In] Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -1/6*(b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(c*e^2*Sqrt[c^2*x^2]) - (d^2*
(a + b*ArcSec[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*Ar
cSec[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]))/(3*e^3) - (8*b*c*
d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*e^3*Sqrt
[c^2*x^2]) + (b*(9*c^2*d - e)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqr
t[d + e*x^2])])/(6*c^2*e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{(bcx)\int \frac{-8d^2-4dex^2+e^2x^4}{3e^3x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{(bcx)\int \frac{-8d^2-4dex^2+e^2x^4}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3e^3\sqrt{c^2x^2}}$$

$$= -\frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{(bcx)\text{Subst}\left(\int \frac{-8d^2-4dex+e^2x^2}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6e^3\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{(bx)\text{Subst}\left(\int \frac{-8c^2d^2e-\frac{1}{2}(9c^2d-e)e^2x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6ce^4\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} + \frac{(4bcd^2x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{3e^3\sqrt{c^2x^2}} + \frac{(b(9c^2d-e)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{12ce^2\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} + \frac{(8bcd^2x)\operatorname{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{(b(9c^2d-e)\,x)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{6c^3e^2\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{(b(9c^2d-e)\,x)\operatorname{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^3e^2\sqrt{c^2x^2}}$$

$$= -\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\sec^{-1}(cx))}{3e^3} - \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{b(9c^2d-e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.48 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.05

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{-16bd^2\sqrt{1+\frac{d}{ex^2}}(-1+c^2x^2)\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+b(9c^2d-e)\,e\sqrt{1}}{}$$

```
[In] Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (-16*b*d^2*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c
^2*x^2), -(d/(e*x^2))] + b*(9*c^2*d - e)*e*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1
 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)
/d)] - 2*x*(-1 + c^2*x^2)*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) + 2*a*c*
(8*d^2 + 4*d*e*x^2 - e^2*x^4) + 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcSec[
c*x]))/(12*c*e^3*x*(-1 + c^2*x^2)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 1483, normalized size of antiderivative = 5.88

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] [-1/24*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(8*c^4
*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2
 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 16*(b*c^
3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) - 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c
^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arcsec(c*x) - (b
*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^
3*d*e^3), -1/24*(32*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*arctan(-1/2*sqrt(c^
2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (
c^2*d^2 - d*e)*x^2 - d^2)) + (9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x
^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)
*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt
(e) + e^2) - 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + 2*(b*c^3
*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arcsec(c*x) - (b*c*e^2*x^2 + b*c*
d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), -1/12*(
(9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^
2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^
4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 8*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d
)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2
*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) -
2*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*
b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arcsec(c*x) - (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2
*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), -1/12*(16*(b*c^3*d*e
*x^2 + b*c^3*d^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 -
2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (
```

```
9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2
*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4
 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 -
 16*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arcsec(c*
x) - (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x
^2 + c^3*d*e^3)]
```

Sympy [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^5(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**5*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^5}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.142 $\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}}-\frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}$$

```
[Out] -b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(3/2)/(c^2*x^2)
^(1/2)+2*b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e^
2/(c^2*x^2)^(1/2)+d*(a+b*arcsec(c*x))/e^2/(e*x^2+d)^(1/2)+(a+b*arcsec(c*x))
*(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {272, 45, 5346, 12, 587, 163, 65, 223, 212, 95, 210}

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}+\frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e^2\sqrt{c^2x^2}}-\frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}$$

```
[In] Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (d*(a + b*ArcSec[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Arc
Sec[c*x]))/e^2 + (2*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 +
```

```
c^2*x^2])])/(e^2*Sqrt[c^2*x^2]) - (b*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2]
)/(c*Sqrt[d + e*x^2])])/(e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}-\frac{(bcx)\int\frac{2d+ex^2}{e^2x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}-\frac{(bcx)\int\frac{2d+ex^2}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{e^2\sqrt{c^2x^2}}\\
&= \frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2}-\frac{(bcx)\text{Subst}\left(\int\frac{2d+ex}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e^2\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} - \frac{(bcdx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{e^2\sqrt{c^2x^2}} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} - \frac{(2bcdx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} - \frac{(bx)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{ce\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} - \frac{(bx)\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{ce\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\sec^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^2} + \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} - \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{\frac{2bd\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cx} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x^3\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{\sqrt{1-c^2x^2}}}{2e^2\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*b*d*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^
2))])/(c*x) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 + (e*x^2)/d]*AppellF1
[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2] + 2*(2*d + e*x^2
)*(a + b*ArcSec[c*x]))/(2*e^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

[In] int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)

[Out] int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 1070, normalized size of antiderivative = 6.82

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")

[Out] [1/4*((b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 2*(b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(a*c*e*x^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), 1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(a*c*e*x^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), 1/2*((b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + (b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 2*(a*c*e*x^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), 1/2*(2*(b*c*e*x^2 + b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(a*c*e*x^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arcsec(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2)]

Sympy [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3(a+b\operatorname{asec}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**3*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^3}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3\,\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.143 $\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}$$

```
[Out] -b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/d^(1/2)/(c^2*x^2
)^(1/2)+(-a-b*arcsec(c*x))/e/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5344, 457, 95, 210}

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}$$

```
[In] Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSec[c*x])/(e*Sqrt[d + e*x^2])) - (b*c*x*ArcTan[Sqrt[d + e*x^2]/
(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*e*Sqrt[c^2*x^2])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5344

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bcx)\int\frac{1}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{e\sqrt{c^2x^2}}\\
&= -\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bcx)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e\sqrt{c^2x^2}}\\
&= -\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bcx)\text{Subst}\left(\int\frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}}\\
&= -\frac{a+b\sec^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\frac{-b\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)-2cx(a+b\sec^{-1}(cx))}{2cex\sqrt{d+ex^2}}$$

```
[In] Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (-(b*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))
]) - 2*c*x*(a + b*ArcSec[c*x]))/(2*c*e*x*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x(a+b\,\text{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.54

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\left[-\frac{(bex^2+bd)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de+e^2)x^4-8\,(c^2d^2-de)x^2-4\,\sqrt{c^2x^2-1}((c^2d-e)x^2-2\,d)\sqrt{ex^2+d}}{x^4}\right.}{4\,(de^2x^2+d^2e)}\right.$$
$$\left.-\frac{(bex^2+bd)\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}((c^2d-e)x^2-2\,d)\sqrt{ex^2+d}\sqrt{d}}{2\,(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)+2\,\sqrt{ex^2+d}(bd\,\text{arcsec}\,(cx)+ad)}{2\,(de^2x^2+d^2e)}\right]$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*e*x^2 + b*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*(b*d*arcsec(c*x) + a*d))/(d
*e^2*x^2 + d^2*e), -1/2*((b*e*x^2 + b*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 -
 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2
 - d*e)*x^2 - d^2)) + 2*sqrt(e*x^2 + d)*(b*d*arcsec(c*x) + a*d))/(d*e^2*x^2
 + d^2*e)]
```

Sympy [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x(a+b\operatorname{asec}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] -(sqrt(e*x^2 + d)*e*integrate((c^2*e*x^3*log(c) - e*x*log(c) + ((c^2*log(c)
 - c^2)*e*x^3 - (c^2*d + e*log(c))*x)*e^(log(c*x + 1) + log(c*x - 1)) + (c^
2*e*x^3 - e*x + (c^2*e*x^3 - e*x)*e^(log(c*x + 1) + log(c*x - 1)))*log(x))/
((c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c^2*d*e - e^2)*
x^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1)))*sqrt(e*x^2 + d)), x) + arctan(s
qrt(c*x + 1)*sqrt(c*x - 1)))*b/(sqrt(e*x^2 + d)*e) - a/(sqrt(e*x^2 + d)*e)
```

Giac [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.144 $\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\operatorname{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(3/2)),x]
[Out] Defer[Int][(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.75 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \mathrm{arcsec}\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x),
 x)
```

Sympy [N/A]

Not integrable

Time = 82.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asec}\,(cx)}{x\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x/(e*x**2+d)**(3/2),x)
[Out] Integral((a + b*asec(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

3.145 $\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsec}\left(cx\right)}{x^{3}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(3/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\,(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^3\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.146 $\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\ \mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^4*arcsec(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

Sympy [N/A]

Not integrable

Time = 144.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^4(a+b\operatorname{asec}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arcsec}(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^4\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.147 $\int \frac{x^2\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\ \mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^2*arcsec(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

Sympy [N/A]

Not integrable

Time = 35.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**2*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.148 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}}-\frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] x*(a+b*arcsec(c*x))/d/(e*x^2+d)^(1/2)-b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(
-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {197, 5336, 12, 432, 430}

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}}-\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSec[c*x]))/(d*Sqrt[d + e*x^2]) - (b*x*Sqrt[1 - c^2*x^2]*Sqrt[1
 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-
1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 5336

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x]
- Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{1}{d\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}} \\
&= \frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}} \\
&= \frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{\left(bcx\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{d+ex^2}} \\
&= \frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{\left(bcx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&= \frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\sec^{-1}(cx))}{d\sqrt{d+ex^2}} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\left(-c+c^3x^2\right)\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSec[c*x]))/(d*Sqrt[d + e*x^2]) - (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])
/(d*(-c + c^3*x^2)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arcsec}\,(cx)}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{(bex^2+bd)\sqrt{-d}F(\arcsin{(cx)}\mid-\frac{e}{c^2d}) + (bcdx\,\mathrm{arcsec}\,(cx)+acdx)\sqrt{ex^2+d}}{cd^2ex^2+cd^3}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b*e*x^2 + b*d)*sqrt(-d)*elliptic_f(arcsin(c*x), -e/(c^2*d)) + (b*c*d*x*ar
csec(c*x) + a*c*d*x)*sqrt(e*x^2 + d))/(c*d^2*e*x^2 + c*d^3)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\operatorname{asec}(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral((a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{b\operatorname{arcsec}(cx)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x^2)^(3/2),x)
[Out] int((a + b*acos(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

3.149 $\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 274

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx &= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}} \\ &\quad - \frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\ &\quad + \frac{b(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

```
[Out] (-a-b*arcsec(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arcsec(c*x))/d^2/(e*x^2+d
)^(1/2)+b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)-b*c^2*x*E
llipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*
x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+b*(c^2*d+2*e)*x*EllipticF(c*
x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2
)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules

used = {277, 197, 5346, 12, 597, 538, 438, 437, 435, 432, 430}

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = -\frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}} + \frac{bx\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} + \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[c^2*x^2]) - (a + b*ArcSe
c[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcSec[c*x]))/(d^2*Sqrt[d + e
*x^2]) - (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x],
-(e/(c^2*d))])/(d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) +
 (b*(c^2*d + 2*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[
c*x], -(e/(c^2*d))])/(d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
```

```
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{-d-2ex^2}{d^2x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{-d-2ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{c^2x^2}}\\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}\\
&\quad-\frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{-2de+c^2dex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^3\sqrt{c^2x^2}}\\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}}\\
&\quad-\frac{(bc^3x)\int\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{d^2\sqrt{c^2x^2}}+\frac{(bc(c^2d+2e)\,x)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{c^2x^2}}\\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}}\\
&\quad-\frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}+\frac{\left(bc(c^2d+2e)\,x\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}\\
&= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}-\frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}}\\
&\quad-\frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}}-\frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\\
&\quad+\frac{\left(bc(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

$$= \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\sec^{-1}(cx))}{d^2\sqrt{d+ex^2}}$$
$$- \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{b(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.77

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)-a(d+2ex^2)-b(d+2ex^2)\sec^{-1}(cx)}{d^2x\sqrt{d+ex^2}}$$
$$- \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)-(c^2d+2e)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right),-\frac{e}{c^2d}\right)\right)}{\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - a*(d + 2*e*x^2) - b*(d + 2*e*x^2
)*ArcSec[c*x])/(d^2*x*Sqrt[d + e*x^2]) - (I*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqr
t[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] -
(c^2*d + 2*e)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2
]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\text{arcsec}\,(cx)}{x^2\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.69

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx =$$
$$-\frac{\left(2\,acdex^2+acd^2+\left(2\,bcdex^2+bcd^2\right)\operatorname{arcsec}\left(cx\right)-\left(bcdex^2+bcd^2\right)\sqrt{c^2x^2-1}\right)\sqrt{ex^2+d}-\left(\left(bc^4dex^3+bc\right.\right.}{cd^3ex^3+cd^4}$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -((2*a*c*d*e*x^2 + a*c*d^2 + (2*b*c*d*e*x^2 + b*c*d^2)*arcsec(c*x) - (b*c*d
*e*x^2 + b*c*d^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((b*c^4*d*e*x^3 + b*
c^4*d^2*x)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((b*c^4*d*e + 2*b*e^2)*x^3
 + (b*c^4*d^2 + 2*b*d*e)*x)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/
(c*d^3*e*x^3 + c*d^4*x)
```

Sympy [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\operatorname{asec}\left(cx\right)}{x^2\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asec(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\operatorname{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.150 $\int \frac{a+b\sec^{-1}(cx)}{x^4(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 701

$$
\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx &= \frac{2bc(c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^3\sqrt{c^2x^2}} \\
&- \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}} \\
&- \frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} \\
&- \frac{2bc^2(c^2d-e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{bc^2(2c^2d-e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{8be^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[Out] 1/3*(-a-b*arcsec(c*x))/d/x^3/(e*x^2+d)^(1/2)+4/3*e*(a+b*arcsec(c*x))/d^2/x/
(e*x^2+d)^(1/2)+8/3*e^2*x*(a+b*arcsec(c*x))/d^3/(e*x^2+d)^(1/2)+2/9*b*c*(c^
2*d-e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)-4/3*b*c*e*(c^2
*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)+1/9*b*c*(c^2*x^2-1)^(1/2)
```

```
*(e*x^2+d)^(1/2)/d^2/x^2/(c^2*x^2)^(1/2)-2/9*b*c^2*(c^2*d-e)*x*EllipticE(c*
x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)/
(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+4/3*b*c^2*e*x*EllipticE(c*x,(-e/c^2/d)^
(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(
1/2)/(1+e*x^2/d)^(1/2)+1/9*b*c^2*(2*c^2*d-e)*x*EllipticF(c*x,(-e/c^2/d)^(1/
2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1
/2)/(e*x^2+d)^(1/2)-4/3*b*c^2*e*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2
+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)
^(1/2)-8/3*b*e^2*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*
x^2/d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {277, 197, 5346, 12, 6874, 432, 430, 491, 597, 538, 438, 437, 435, 507}

$$\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx &= \frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} \\
&-\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}-\frac{8be^2x\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} \\
&-\frac{2bc^2x\sqrt{1-c^2x^2}(c^2d-e)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} \\
&+\frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} \\
&+\frac{bc^2x\sqrt{1-c^2x^2}(2c^2d-e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} \\
&-\frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} \\
&+\frac{2bc\sqrt{c^2x^2-1}(c^2d-e)\sqrt{d+ex^2}}{9d^3\sqrt{c^2x^2}} \\
&-\frac{4bce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}}+\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}}
\end{aligned}$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

```
[Out] (2*b*c*(c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d^3*Sqrt[c^2*x^2]
) - (4*b*c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3*d^3*Sqrt[c^2*x^2]) + (b
*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d^2*x^2*Sqrt[c^2*x^2]) - (a + b*A
```

```
rcSec[c*x])/(3*d*x^3*Sqrt[d + e*x^2]) + (4*e*(a + b*ArcSec[c*x]))/(3*d^2*x*
Sqrt[d + e*x^2]) + (8*e^2*x*(a + b*ArcSec[c*x]))/(3*d^3*Sqrt[d + e*x^2]) -
(2*b*c^2*(c^2*d - e)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c
*x], -(e/(c^2*d))])/(9*d^3*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2
)/d]) + (4*b*c^2*e*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x
], -(e/(c^2*d))])/(3*d^3*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/
d]) + (b*c^2*(2*c^2*d - e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Elliptic
F[ArcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[
d + e*x^2]) - (4*b*c^2*e*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[
ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d
+ e*x^2]) - (8*b*e^2*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcS
in[c*x], -(e/(c^2*d))])/(3*d^3*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*
x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} \\
&\quad+\frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{-d^2+4dex^2+8e^2x^4}{3d^3x^4\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} \\
&\quad+\frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{-d^2+4dex^2+8e^2x^4}{x^4\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^3\sqrt{c^2x^2}} \\
&= -\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}}+\frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} \\
&\quad-\frac{(bcx)\int\left(\frac{8e^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}-\frac{d^2}{x^4\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}+\frac{4de}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\right)dx}{3d^3\sqrt{c^2x^2}} \\
&= -\frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}}+\frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} \\
&\quad+\frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}}+\frac{(bcx)\int\frac{1}{x^4\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d\sqrt{c^2x^2}} \\
&\quad-\frac{(4bcex)\int\frac{1}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^2\sqrt{c^2x^2}}-\frac{(8bce^2x)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^3\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{(bcx)\int\frac{-2(c^2d-e)-c^2ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{c^2x^2}} + \frac{(4bcex)\int\frac{c^2ex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^3\sqrt{c^2x^2}} - \frac{\left(8bce^2x\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^3\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{2bc(c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^3\sqrt{c^2x^2}} - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{(bcx)\int\frac{-c^2de+2c^2(c^2d-e)ex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^3\sqrt{c^2x^2}} + \frac{(4bc^3e^2x)\int\frac{x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^3\sqrt{c^2x^2}} - \frac{\left(8bce^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{2bc(c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^3\sqrt{c^2x^2}} - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{8be^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} - \frac{(2bc^3(c^2d-e)x)\int\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{9d^3\sqrt{c^2x^2}} + \frac{(bc^3(2c^2d-e)x)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{c^2x^2}} + \frac{(4bc^3ex)\int\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{3d^3\sqrt{c^2x^2}} - \frac{(4bc^3ex)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^2\sqrt{c^2x^2}}$$

$$
\begin{aligned}
&= \frac{2bc(c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^3\sqrt{c^2x^2}} - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}} \\
&+ \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} \\
&+ \frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{8be^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{\left(2bc^3(c^2d-e)\,x\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{9d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} + \frac{\left(4bc^3ex\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} \\
&+ \frac{\left(bc^3(2c^2d-e)\,x\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{9d^2\sqrt{c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{\left(4bc^3ex\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2bc(c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^3\sqrt{c^2x^2}} - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}} \\
&+ \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} \\
&+ \frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{8be^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{\left(2bc^3(c^2d-e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{9d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{\left(4bc^3ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{\left(bc^3(2c^2d-e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{\left(4bc^3ex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2bc(c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^3\sqrt{c^2x^2}} - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}} \\
&\quad + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{3dx^3\sqrt{d+ex^2}} + \frac{4e(a+b\sec^{-1}(cx))}{3d^2x\sqrt{d+ex^2}} \\
&\quad + \frac{8e^2x(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{2bc^2(c^2d-e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{9d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{bc^2(2c^2d-e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&\quad - \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&\quad - \frac{8be^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.45 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.40

$$
\int \frac{a+b\sec^{-1}(cx)}{x^4(d+ex^2)^{3/2}}\,dx = \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2-14ex^2)(d+ex^2)-3a(d^2-4dex^2-8e^2x^4)-3b(d^2-4d}{9d^3x^3\sqrt{d+ex^2}}
$$

$$
-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d-7e)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\middle|-\frac{e}{c^2d}\right)+(-2c^4d^2+13c^2de+24e^2)\,\text{EllipticF}}{9\sqrt{-c^2}d^3\sqrt{1-c^2x^2}\sqrt{d+ex^2}}
$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 - 14*e*x^2)*(d + e*x^2) - 3*a
*(d^2 - 4*d*e*x^2 - 8*e^2*x^4) - 3*b*(d^2 - 4*d*e*x^2 - 8*e^2*x^4)*ArcSec[c
*x])/(9*d^3*x^3*Sqrt[d + e*x^2]) - ((I/9)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[
1 + (e*x^2)/d]*(2*c^2*d*(c^2*d - 7*e)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(
e/(c^2*d))] + (-2*c^4*d^2 + 13*c^2*d*e + 24*e^2)*EllipticF[I*ArcSinh[Sqrt[-
c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^3*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arcsec}\left(cx\right)}{x^{4}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

[In] int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x)

[Out] int((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.47

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = \frac{\left(24\,acde^2x^4+12\,acd^2ex^2-3\,acd^3+3\left(8\,bcde^2x^4+4\,bcd^2ex^2-bcd^3\right)\mathrm{arcsec}\left(cx\right)+\left(\right.}{}$$

[In] integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="fricas")

[Out] 1/9*((24*a*c*d*e^2*x^4 + 12*a*c*d^2*e*x^2 - 3*a*c*d^3 + 3*(8*b*c*d*e^2*x^4 + 4*b*c*d^2*e*x^2 - b*c*d^3)*arcsec(c*x) + (b*c*d^3 + 2*(b*c^3*d^2*e - 7*b*c*d*e^2)*x^4 + (2*b*c^3*d^3 - 13*b*c*d^2*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) + (2*((b*c^6*d^2*e - 7*b*c^4*d*e^2)*x^5 + (b*c^6*d^3 - 7*b*c^4*d^2*e)*x^3)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((2*b*c^6*d^2*e - (14*b*c^4 - b*c^2)*d*e^2 - 24*b*e^3)*x^5 + (2*b*c^6*d^3 - (14*b*c^4 - b*c^2)*d^2*e - 24*b*d*e^2)*x^3)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^4*e*x^5 + c*d^5*x^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = \text{Timed out}$$

[In] integrate((a+b*asec(c*x))/x**4/(e*x**2+d)**(3/2),x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^4\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x^4*(d + e*x^2)^(3/2)), x)
```

3.151 $\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 244

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{bx\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}}$$

```
[Out] -1/3*d^2*(a+b*arcsec(c*x))/e^3/(e*x^2+d)^(3/2)-b*x*arctanh(e^(1/2)*(c^2*x^2
-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(5/2)/(c^2*x^2)^(1/2)+8/3*b*c*x*arctan((e*x^
2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e^3/(c^2*x^2)^(1/2)+2*d*(a+b*
arcsec(c*x))/e^3/(e*x^2+d)^(1/2)-1/3*b*c*d*x*(c^2*x^2-1)^(1/2)/e^2/(c^2*d+e
)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+(a+b*arcsec(c*x))*(e*x^2+d)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules

used = {272, 45, 5346, 12, 1628, 163, 65, 223, 212, 95, 210}

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{d^2(a+b\sec^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}} - \frac{bcdx\sqrt{c^2x^2-1}}{3e^2\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*d*x*Sqrt[-1 + c^2*x^2])/(e^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e
*x^2]) - (d^2*(a + b*ArcSec[c*x]))/(3*e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b*
ArcSec[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]))
/e^3 + (8*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])
])/(3*e^3*Sqrt[c^2*x^2]) - (b*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqr
t[d + e*x^2])])/(e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
```

```
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\sec^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\
&\quad +\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}-\frac{(bcx)\int \frac{8d^2+12dex^2+3e^2x^4}{3e^3x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d^2(a+b\sec^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\
&\quad +\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}-\frac{(bcx)\int \frac{8d^2+12dex^2+3e^2x^4}{x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e^3\sqrt{c^2x^2}} \\
&= -\frac{d^2(a+b\sec^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} \\
&\quad -\frac{(bcx)\text{Subst}\left(\int \frac{8d^2+12dex+3e^2x^2}{x\sqrt{-1+c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e^3\sqrt{c^2x^2}} \\
&= -\frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\
&\quad +\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}+\frac{(bcx)\text{Subst}\left(\int \frac{-4d^2(c^2d+e)-\frac{3}{2}de(c^2d+e)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3de^3\left(c^2d+e\right)\sqrt{c^2x^2}} \\
&= -\frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}-\frac{d^2(a+b\sec^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\
&\quad +\frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3}-\frac{(4bcdx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^3\sqrt{c^2x^2}} \\
&\quad -\frac{(bcx)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e^2\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} - \frac{(8bcdx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{(bx)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{ce^2\sqrt{c^2x^2}}$$

$$= -\frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{(bx)\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{ce^2\sqrt{c^2x^2}}$$

$$= -\frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\sec^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\sec^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\sec^{-1}(cx))}{e^3} + \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.30 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-\frac{2bcde\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)}{c^2d+e} + 2a(8d^2+12dex^2+3e^2x^4) + \frac{b(d+ex^2)\left(8d\sqrt{1+\frac{d}{ex^2}}\text{AppellF1}\left(1\right.\right.}{}}{}$$

```
[In] Integrate[(x^5*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] ((-2*b*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d + e) + 2*a*(8*d^2
+ 12*d*e*x^2 + 3*e^2*x^4) + (b*(d + e*x^2)*(8*d*Sqrt[1 + d/(e*x^2)]*AppellF
1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (3*c^2*e*Sqrt[1 - 1/(c^2*x^2
)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])
/Sqrt[1 - c^2*x^2]))/(c*x) + 2*b*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*ArcSec[c*
x])/(6*e^3*(d + e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

[In] int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)

[Out] int(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(206) = 412$.

Time = 0.49 (sec) , antiderivative size = 2123, normalized size of antiderivative = 8.70

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Too large to display}$$

[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 8*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*arcsec(c*x) - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), 1/12*(16*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*arcsec(c*x) - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), 1/6*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 4*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b

```
*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c
^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e
)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 2*(8*a*c^3*d^3 + 8*a*
c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2
+ (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^
2*e + b*c*d*e^2)*x^2)*arcsec(c*x) - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^
2 - 1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4
 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), 1/6*(8*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d
*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*
sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*
x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^
4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x
^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c
*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*
e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 + (8*b*c^3*d^3 + 8*b*
c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)
*arcsec(c*x) - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 +
d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 + c
*d*e^5)*x^2)]
```

Sympy [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^5(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**5*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**5*(a + b*asec(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcsec}\,(cx)+a)x^5}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^5*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsec(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

[In] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)

[Out] int((x^5*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)

3.152 $\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcx\sqrt{-1+c^2x^2}}{3e(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3\sqrt{d}e^2\sqrt{c^2x^2}}$$

```
[Out] 1/3*d*(a+b*arcsec(c*x))/e^2/(e*x^2+d)^(3/2)-2/3*b*c*x*arctan((e*x^2+d)^(1/2
)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/d^(1/2)/(c^2*x^2)^(1/2)+(-a-b*arcsec(c*x))
/e^2/(e*x^2+d)^(1/2)+1/3*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(c^2*x^2)^(1/2
)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {272, 45, 5346, 12, 587, 157, 95, 210}

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3\sqrt{d}e^2\sqrt{c^2x^2}} + \frac{bcx\sqrt{c^2x^2-1}}{3e\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x*Sqrt[-1 + c^2*x^2])/(3*e*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2])
+ (d*(a + b*ArcSec[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcSec[c*x])/(
```

```
e^2*Sqrt[d + e*x^2]) - (2*b*c*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c
^2*x^2])])/(3*Sqrt[d]*e^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-2d-3ex^2}{3e^2x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-2d-3ex^2}{x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e^2\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\text{Subst}\left(\int \frac{-2d-3ex}{x\sqrt{-1+c^2x}(d+ex)^{3/2}}\,dx, x, x^2\right)}{6e^2\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{3e(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\text{Subst}\left(\int \frac{d(c^2d+e)}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{3de^2(c^2d+e)\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{3e(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{3e^2\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{3e(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\sec^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(2bcx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\sec^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\sec^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3\sqrt{d}e^2\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)}{c^2d+e} + a(2d+3ex^2) + \frac{b\sqrt{1+\frac{d}{ex^2}}(d+ex^2)\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cx} + b(2d+3ex^2)\sec^{-1}(cx)}{3e^2\left(d+ex^2\right)^{3/2}}$$

```
[In] Integrate[(x^3*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
[Out] -1/3*(-((b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d + e)) + a*(2*d +
 3*e*x^2) + (b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(
c^2*x^2), -(d/(e*x^2))])/(c*x) + b*(2*d + 3*e*x^2)*ArcSec[c*x])/(e^2*(d + e
*x^2)^(3/2))
```

Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(137) = 274.

Time = 0.39 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.07

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de+\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}\,((c^2d-e)x^2-2\,d)\sqrt{ex^2+d}\sqrt{d}}{2\,(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)+(2\,a\cdots}{3\,(c^2d^4e^2+d^3e^3+\cdots}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt
(-d) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)
*x^2 + (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arcsec(c*x
) - (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^4*e^
2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2), -
1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*
d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*s
qrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (2*a*c^
2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 + (2*b*c^2*d^3 + 2*b*d^2*
e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arcsec(c*x) - (b*d*e^2*x^2 + b*d^2*e)*sq
rt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d
*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

Sympy [F]

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x^3(a + b\operatorname{asec}(cx))}{(d + ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*asec(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^3\,\left(a+b\operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^3*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.153 $\int \frac{x\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcx\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\,\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\sec^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}$$

```
[Out] 1/3*(-a-b*arcsec(c*x))/e/(e*x^2+d)^(3/2)-1/3*b*c*x*arctan((e*x^2+d)^(1/2)/d
^(1/2)/(c^2*x^2-1)^(1/2))/d^(3/2)/e/(c^2*x^2)^(1/2)-1/3*b*c*x*(c^2*x^2-1)^(
1/2)/d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5344, 457, 98, 95, 210}

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{a+b\sec^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{c^2x^2}} - \frac{bcx\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}\,(c^2d+e)\,\sqrt{d+ex^2}}$$

```
[In] Int[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*x*Sqrt[-1 + c^2*x^2])/(d*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2
]) - (a + b*ArcSec[c*x])/(3*e*(d + e*x^2)^(3/2)) - (b*c*x*ArcTan[Sqrt[d + e
*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*d^(3/2)*e*Sqrt[c^2*x^2])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5344

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\sec^{-1}(cx)}{3e\,(d+ex^2)^{3/2}}+\frac{(bcx)\int\frac{1}{x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e\sqrt{c^2x^2}}$$

$$=-\frac{a+b\sec^{-1}(cx)}{3e\,(d+ex^2)^{3/2}}+\frac{(bcx)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6de\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3de\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\sec^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \frac{x(a+b\sec^{-1}(cx))}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{-\frac{b\sqrt{1+\frac{d}{ex^2}}\left(d+ex^2\right)\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cdex} + \frac{2\left(-a-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}x\left(d+ex^2\right)}{d\left(c^2d+e\right)}-b\sec^{-1}(cx)\right)}{e}}{6\left(d+ex^2\right)^{3/2}}$$

```
[In] Integrate[(x*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
[Out] (-((b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2),
 -(d/(e*x^2))])/(c*d*e*x)) + (2*(-a - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e
*x^2))/(d*(c^2*d + e)) - b*ArcSec[c*x]))/e)/(6*(d + e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{x(a+b\ \text{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(114) = 228.

Time = 0.36 (sec) , antiderivative size = 571, normalized size of antiderivative = 4.14

$$\int \frac{x(a+b\sec^{-1}(cx))}{\left(d+ex^2\right)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de+\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}\left((c^2d-e)x^2-2\,d\right)\sqrt{ex^2+d}\sqrt{d}}{2\,(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)+2\,(\cdots}{6\,(c^2d^5e+d^4e^2+(c^2d^3e^3+d^2e^4)x^4+2\,(\cdots}$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
 d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqr
t(-d) + 8*d^2)/x^4) + 4*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arcsec
(c*x) + (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^
5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2
), -1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
+ b*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*
d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 2*(
a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arcsec(c*x) + (b*d*e^2*x^2 + b*
d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*
e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(a + b\sec^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x(a + b\operatorname{asec}(cx))}{(d + ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*asec(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a + b\sec^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.154 $\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx=\text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcSec[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arcsec}\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a+b\sec^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{(ex^2+d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2
*e*x^3 + d^3*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asec(c*x))/x/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

```
[In] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

3.155 $\int \frac{a+b\sec^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\operatorname{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcSec[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \mathrm{arcsec}\left(cx\right)}{x^3\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2
*e*x^5 + d^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*asec(c*x))/x**3/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sec^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^3\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.156 $\int \frac{x^6 (a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^6 (a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx = \text{Int}\left(\frac{x^6 (a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}}, x\right)$$

```
[Out] Unintegrable(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^6 (a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx = \int \frac{x^6 (a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$$

```
[In] Int[(x^6*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^6*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^6 (a+b \sec^{-1}(cx))}{(d+ex^2)^{5/2}} dx$$

Mathematica [N/A]

Not integrable

Time = 14.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

```
[In] Integrate[(x^6*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^6*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^6(a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^6*arcsec(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4
 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**6*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^6(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^6\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

```
[In] int((x^6*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^6*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.157 $\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\text{Int}\left(\frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

```
[In] Int[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

```
[In] Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^4*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\,\operatorname{arcsec}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arcsec}(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^4*arcsec(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4
 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^4\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

```
[In] int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^4*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.158 $\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 276

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcx^2\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\,\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\sec^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{3de\,(c^2d+e)\,\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] 1/3*x^3*(a+b*arcsec(c*x))/d/(e*x^2+d)^(3/2)-1/3*b*c*x^2*(c^2*x^2-1)^(1/2)/d
/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+1/3*b*c^2*x*EllipticE(c*x,(-e/c^
2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/e/(c^2*d+e)/(c^2*x^2)^(1/2
)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-1/3*b*x*EllipticF(c*x,(-e/c^2/d)^(1/2
))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/e/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/
2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules

used = {270, 5346, 12, 482, 434, 438, 437, 435, 432, 430}

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{x^3(a+b\sec^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} - \frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\,(c^2d+e)\sqrt{\frac{ex^2}{d}+1}} - \frac{bcx^2\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}\,(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*x^2*Sqrt[-1 + c^2*x^2])/(d*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x
^2]) + (x^3*(a + b*ArcSec[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (b*c^2*x*Sqrt[1
- c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*(c^
2*d + e)*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*x*Sqrt[
1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d
*e*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
```

```
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{x^3(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{x^2}{3d\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{x^3(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{x^2}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3d\sqrt{c^2x^2}}$$

$$= -\frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\,dx}{3d\left(c^2d+e\right)\sqrt{c^2x^2}}$$

$$= -\frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3de\sqrt{c^2x^2}} + \frac{(bc^3x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{3de\left(c^2d+e\right)\sqrt{c^2x^2}}$$

$$= -\frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3de\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\left(bcx\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3de\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{3de\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bcx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3de\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3de\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{x^2\left(a(c^2d+e)\,x-bc\sqrt{1-\frac{1}{c^2x^2}}(d+ex^2)+b(c^2d+e)\,x\sec^{-1}(cx)\right)}{3d\,(c^2d+e)\,(d+ex^2)^{3/2}} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|-\frac{c^2d}{e}\right)}{3d\sqrt{-\frac{e}{d}}\,(c^2d+e)\,\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^2*(a + b*ArcSec[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (x^2*(a*(c^2*d + e)*x - b*c*Sqrt[1 - 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d +
e)*x*ArcSec[c*x]))/(3*d*(c^2*d + e)*(d + e*x^2)^(3/2)) + (b*c*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], -((c^2*d)/
e)])/(3*d*Sqrt[-(e/d)]*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^2(a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.04

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{((bc^3d^2e+bcde^2)x^3\,\mathrm{arcsec}\,(cx)+(ac^3d^2e+acde^2)x^3-(bcde^2x^3+bcd^2ex)\sqrt{c^2x^2}}{}$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((b*c^3*d^2*e + b*c*d*e^2)*x^3*arcsec(c*x) + (a*c^3*d^2*e + a*c*d*e^2)
*x^3 - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - (
(b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x), -
e/(c^2*d)) - (b*c^4*d^3 + (b*c^4*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^4*d^
2*e + b*d*e^2)*x^2)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c^3*d^5
*e + c*d^4*e^2 + (c^3*d^3*e^3 + c*d^2*e^4)*x^4 + 2*(c^3*d^4*e^2 + c*d^3*e^3
)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x^
2*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x
^2 + d)), x)
```

Giac [F]

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\sec^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^2*(a + b*acos(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.159 $\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 296

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\sec^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{2bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] 1/3*x*(a+b*arcsec(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arcsec(c*x))/d^2/(e*x^
2+d)^(1/2)+1/3*b*c*e*x^2*(c^2*x^2-1)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(e
*x^2+d)^(1/2)-1/3*b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2
)*(e*x^2+d)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/
d)^(1/2)-2/3*b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^
2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules

used = {198, 197, 5336, 12, 541, 538, 438, 437, 435, 432, 430}

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b\sec^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} - \frac{2bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\,(c^2d+e)\sqrt{\frac{ex^2}{d}+1}} + \frac{bcex^2\sqrt{c^2x^2-1}}{3d^2\sqrt{c^2x^2}\,(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*ArcSec[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*e*x^2*Sqrt[-1 + c^2*x^2])/(3*d^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*
x^2]) + (x*(a + b*ArcSec[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSe
c[c*x]))/(3*d^2*Sqrt[d + e*x^2]) - (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*(c^2*d + e)*Sqrt[c^2*x^2]*S
qrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*x*Sqrt[1 - c^2*x^2]*Sqrt[1 +
(e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*Sqrt[c^2*x^2]*Sqrt[
-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 5336

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x]
- Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
```

```
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{3d+2ex^2}{3d^2\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{x(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{3d+2ex^2}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3d^2\sqrt{c^2x^2}}\\
&= \frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{x(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}}\\
&\quad+\frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{d(3c^2d+2e)+c^2dex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^3\left(c^2d+e\right)\sqrt{c^2x^2}}\\
&= \frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{x(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}}\\
&\quad-\frac{(2bcx)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^2\sqrt{c^2x^2}}-\frac{\left(bc^3x\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}}\\
&= \frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{x(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}}\\
&\quad-\frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}-\frac{\left(2bcx\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}\\
&= \frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{x(a+b\sec^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}}\\
&\quad+\frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}}-\frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\\
&\quad-\frac{\left(2bcx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

$$= \frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\sec^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\sec^{-1}(cx))}{3d^2\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{2bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.84

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x\left(bce\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)+a(c^2d+e)(3d+2ex^2)+b(c^2d+e)(3d+2ex^2)\sec^{-1}(c\right.}{3d^2(c^2d+e)(d+ex^2)^{3/2}} - \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\middle|-\frac{e}{c^2d}\right)+2(c^2d+e)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right),-\frac{e}{c^2d}\right)\right)}{3\sqrt{-c^2}d^2(c^2d+e)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (x*(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) + a*(c^2*d + e)*(3*d + 2*e*x^
2) + b*(c^2*d + e)*(3*d + 2*e*x^2)*ArcSec[c*x]))/(3*d^2*(c^2*d + e)*(d + e*
x^2)^(3/2)) - ((I/3)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d
*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(c^2*d + e)*EllipticF
[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*(c^2*d + e)*Sqrt[
1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\text{arcsec}\,(cx)}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.18

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{(2\,(ac^3d^2e+acde^2)x^3+3\,(ac^3d^3+acd^2e)x+(2\,(bc^3d^2e+bcde^2)x^3+3\,(bc^3d^3+bcd$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*(a*c^3*d^2*e + a*c*d*e^2)*x^3 + 3*(a*c^3*d^3 + a*c*d^2*e)*x + (2*(b
*c^3*d^2*e + b*c*d*e^2)*x^3 + 3*(b*c^3*d^3 + b*c*d^2*e)*x)*arcsec(c*x) + (b
*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) + ((b*c^4*d*
e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x), -e/(c^2*d)
) - (((b*c^4 - 3*b*c^2)*d*e^2 - 2*b*e^3)*x^4 + (b*c^4 - 3*b*c^2)*d^3 - 2*b*
d^2*e + 2*((b*c^4 - 3*b*c^2)*d^2*e - 2*b*d*e^2)*x^2)*elliptic_f(arcsin(c*x)
, -e/(c^2*d)))*sqrt(-d))/(c^3*d^6 + c*d^5*e + (c^3*d^4*e^2 + c*d^3*e^3)*x^4
 + 2*(c^3*d^5*e + c*d^4*e^2)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*asec(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\sec^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(a
rctan(sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2
+ d)), x)
```

Giac [F]

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^{5/2}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec^{-1}(cx)}{(d + ex^2)^{5/2}}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(d + e*x^2)^(5/2),x)
[Out] int((a + b*acos(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

3.160 $\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 631

$$\begin{aligned}\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{5/2}}\,dx =& -\frac{bce\sqrt{-1+c^2x^2}}{d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} \\ &- \frac{4bce^2x^2\sqrt{-1+c^2x^2}}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^3(c^2d+e)\sqrt{c^2x^2}} \\ &- \frac{a+b\sec^{-1}(cx)}{dx(d+ex^2)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2(d+ex^2)^{3/2}} - \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} \\ &+ \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\ &- \frac{bc^2(c^2d+2e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\ &+ \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\ &+ \frac{8bex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

```
[Out] (-a-b*arcsec(c*x))/d/x/(e*x^2+d)^(3/2)-4/3*e*x*(a+b*arcsec(c*x))/d^2/(e*x^2
+d)^(3/2)-8/3*e*x*(a+b*arcsec(c*x))/d^3/(e*x^2+d)^(1/2)-b*c*e*(c^2*x^2-1)^(
1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)-4/3*b*c*e^2*x^2*(c^2*x^2
-1)^(1/2)/d^3/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+b*c*(c^2*d+2*e)*(c^
2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2*d+e)/(c^2*x^2)^(1/2)+4/3*b*c^2*e*x*
EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(c^2
*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-b*c^2*(c^2*d+2*e)
```

```
*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^3/(
c^2*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+b*c^2*x*Ellipt
icF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2
)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)+8/3*b*e*x*EllipticF(c*x,(-e/c^2/d
)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^3/(c^2*x^2)^(1/2)/(c^2*x^2-
1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {277, 198, 197, 5346, 12, 6874, 425, 21, 438, 437, 435, 483, 597, 538, 432, 430, 482, 434}

$$\begin{aligned}
\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{5/2}}\,dx &= -\frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\sec^{-1}(cx))}{3d^2(d+ex^2)^{3/2}}\\
&-\frac{a+b\sec^{-1}(cx)}{dx(d+ex^2)^{3/2}}+\frac{8bex\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\\
&-\frac{bc^2x\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\,(c^2d+e)\sqrt{\frac{ex^2}{d}+1}}\\
&+\frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}\,(c^2d+e)\sqrt{\frac{ex^2}{d}+1}}\\
&+\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\\
&-\frac{4bce^2x^2\sqrt{c^2x^2-1}}{3d^3\sqrt{c^2x^2}\,(c^2d+e)\sqrt{d+ex^2}}\\
&+\frac{bc\sqrt{c^2x^2-1}(c^2d+2e)\sqrt{d+ex^2}}{d^3\sqrt{c^2x^2}\,(c^2d+e)}-\frac{bce\sqrt{c^2x^2-1}}{d^2\sqrt{c^2x^2}\,(c^2d+e)\sqrt{d+ex^2}}
\end{aligned}$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

```
[Out] -((b*c*e*Sqrt[-1 + c^2*x^2])/(d^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2]
)) - (4*b*c*e^2*x^2*Sqrt[-1 + c^2*x^2])/(3*d^3*(c^2*d + e)*Sqrt[c^2*x^2]*Sq
rt[d + e*x^2]) + (b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d^
3*(c^2*d + e)*Sqrt[c^2*x^2]) - (a + b*ArcSec[c*x])/(d*x*(d + e*x^2)^(3/2))
- (4*e*x*(a + b*ArcSec[c*x]))/(3*d^2*(d + e*x^2)^(3/2)) - (8*e*x*(a + b*Arc
Sec[c*x]))/(3*d^3*Sqrt[d + e*x^2]) + (4*b*c^2*e*x*Sqrt[1 - c^2*x^2]*Sqrt[d
+ e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d^3*(c^2*d + e)*Sqrt[c^2*
x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*c^2*(c^2*d + 2*e)*x*Sqrt[
1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d^3*(c^
```

```
2*d + e)*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (b*c^2*x*S
qrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/
(d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]) + (8*b*e*x*Sqrt[1 -
c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d^3*S
qrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{a+b\sec^{-1}(cx)}{dx\,(d+ex^2)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2\,(d+ex^2)^{3/2}} - \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-3d^2-12dex^2-8e^2x^4}{3d^3x^2\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{dx\,(d+ex^2)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2\,(d+ex^2)^{3/2}} - \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-3d^2-12dex^2-8e^2x^4}{x^2\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3d^3\sqrt{c^2x^2}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{dx\,(d+ex^2)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2\,(d+ex^2)^{3/2}} - \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{(bcx)\int \left(-\frac{12de}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} - \frac{3d^2}{x^2\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}} - \frac{8e^2x^2}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\right)dx}{3d^3\sqrt{c^2x^2}}$$

$$= -\frac{a+b\sec^{-1}(cx)}{dx\,(d+ex^2)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2\,(d+ex^2)^{3/2}} - \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{1}{x^2\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{d\sqrt{c^2x^2}} + \frac{(4bcex)\int \frac{1}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{d^2\sqrt{c^2x^2}} + \frac{(8bce^2x)\int \frac{x^2}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3d^3\sqrt{c^2x^2}}$$

$$= -\frac{bce\sqrt{-1+c^2x^2}}{d^2\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{4bce^2x^2\sqrt{-1+c^2x^2}}{3d^3\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\sec^{-1}(cx)}{dx\,(d+ex^2)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2\,(d+ex^2)^{3/2}} - \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{c^2d+2e-c^2ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\,(c^2d+e)\sqrt{c^2x^2}} + \frac{(4bcex)\int \frac{c^2d+c^2ex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^3\,(c^2d+e)\sqrt{c^2x^2}} - \frac{(8bce^2x)\int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\,dx}{3d^3\,(c^2d+e)\sqrt{c^2x^2}}$$

$$= -\frac{bce\sqrt{-1+c^2x^2}}{d^2\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{4bce^2x^2\sqrt{-1+c^2x^2}}{3d^3\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^3\,(c^2d+e)\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{dx\,(d+ex^2)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2\,(d+ex^2)^{3/2}} - \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \frac{(8bcex)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^3\sqrt{c^2x^2}} + \frac{(bcx)\int \frac{-c^2de-c^2e(c^2d+2e)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^3\,(c^2d+e)\sqrt{c^2x^2}} - \frac{(8bc^3ex)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{3d^3\,(c^2d+e)\sqrt{c^2x^2}} + \frac{(4bc^3ex)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{d^3\,(c^2d+e)\sqrt{c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{bce\sqrt{-1+c^2x^2}}{d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{4bce^2x^2\sqrt{-1+c^2x^2}}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} \\
&\quad + \frac{bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^3(c^2d+e)\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{dx(d+ex^2)^{3/2}} \\
&\quad - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2(d+ex^2)^{3/2}} - \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \frac{(bc^3x)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{c^2x^2}} \\
&\quad - \frac{(bc^3(c^2d+2e)x)\int\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{d^3(c^2d+e)\sqrt{c^2x^2}} - \frac{\left(8bc^3ex\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} \\
&\quad + \frac{\left(4bc^3ex\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} + \frac{\left(8bcex\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^3\sqrt{c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bce\sqrt{-1+c^2x^2}}{d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{4bce^2x^2\sqrt{-1+c^2x^2}}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} \\
&\quad + \frac{bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^3(c^2d+e)\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{dx(d+ex^2)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2(d+ex^2)^{3/2}} \\
&\quad - \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} - \frac{\left(bc^3(c^2d+2e)x\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} \\
&\quad - \frac{\left(8bc^3ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{\left(4bc^3ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad + \frac{\left(bc^3x\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d^2\sqrt{c^2x^2}\sqrt{d+ex^2}} \\
&\quad + \frac{\left(8bcex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bce\sqrt{-1+c^2x^2}}{d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{4bce^2x^2\sqrt{-1+c^2x^2}}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} \\
&+ \frac{bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^3(c^2d+e)\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{dx(d+ex^2)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2(d+ex^2)^{3/2}} \\
&- \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{8bex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&- \frac{\left(bc^3(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bce\sqrt{-1+c^2x^2}}{d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{4bce^2x^2\sqrt{-1+c^2x^2}}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} \\
&+ \frac{bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^3(c^2d+e)\sqrt{c^2x^2}} - \frac{a+b\sec^{-1}(cx)}{dx(d+ex^2)^{3/2}} - \frac{4ex(a+b\sec^{-1}(cx))}{3d^2(d+ex^2)^{3/2}} \\
&- \frac{8ex(a+b\sec^{-1}(cx))}{3d^3\sqrt{d+ex^2}} + \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&- \frac{bc^2(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^3(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&+ \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} \\
&+ \frac{8bex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.59 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.51

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{5/2}}\,dx = \frac{-a(c^2d+e)(3d^2+12dex^2+8e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(3c^2d(d+ex^2)+e(3}{3d^3(c^2d+e)x(d+ex^2)^{3/2}}$$
$$-\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(3c^2d+2e)E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-(3c^4d^2+11c^2de+8e^2)\,\text{EllipticF}\,(i}{3\sqrt{-c^2}d^3(c^2d+e)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

```
[Out] (-(a*(c^2*d + e)*(3*d^2 + 12*d*e*x^2 + 8*e^2*x^4)) + b*c*Sqrt[1 - 1/(c^2*x^
2)]*x*(d + e*x^2)*(3*c^2*d*(d + e*x^2) + e*(3*d + 2*e*x^2)) - b*(c^2*d + e)
*(3*d^2 + 12*d*e*x^2 + 8*e^2*x^4)*ArcSec[c*x])/(3*d^3*(c^2*d + e)*x*(d + e*
x^2)^(3/2)) - ((I/3)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d
*(3*c^2*d + 2*e)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (3*c^4*
d^2 + 11*c^2*d*e + 8*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))])
)/(Sqrt[-c^2]*d^3*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\text{arcsec}\,(cx)}{x^2\,(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.14 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.85

$$\int \frac{a+b\sec^{-1}(cx)}{x^2(d+ex^2)^{5/2}}\,dx =$$
$$-\frac{(3\,ac^3d^4+3\,acd^3e+8\,(ac^3d^2e^2+acde^3)x^4+12\,(ac^3d^3e+acd^2e^2)x^2+(3\,bc^3d^4+3\,bcd^3e+8\,(bc^3d^2e^2+b}{}$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*a*c^3*d^4 + 3*a*c*d^3*e + 8*(a*c^3*d^2*e^2 + a*c*d*e^3)*x^4 + 12*(
a*c^3*d^3*e + a*c*d^2*e^2)*x^2 + (3*b*c^3*d^4 + 3*b*c*d^3*e + 8*(b*c^3*d^2*
```

```
e^2 + b*c*d*e^3)*x^4 + 12*(b*c^3*d^3*e + b*c*d^2*e^2)*x^2)*arcsec(c*x) - (3
*b*c^3*d^4 + 3*b*c*d^3*e + (3*b*c^3*d^2*e^2 + 2*b*c*d*e^3)*x^4 + (6*b*c^3*d
^3*e + 5*b*c*d^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - (((3*b*c^6*
d^2*e^2 + 2*b*c^4*d*e^3)*x^5 + 2*(3*b*c^6*d^3*e + 2*b*c^4*d^2*e^2)*x^3 + (3
*b*c^6*d^4 + 2*b*c^4*d^3*e)*x)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((3*b*
c^6*d^2*e^2 + (2*b*c^4 + 9*b*c^2)*d*e^3 + 8*b*e^4)*x^5 + 2*(3*b*c^6*d^3*e +
 (2*b*c^4 + 9*b*c^2)*d^2*e^2 + 8*b*d*e^3)*x^3 + (3*b*c^6*d^4 + (2*b*c^4 + 9
*b*c^2)*d^3*e + 8*b*d^2*e^2)*x)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-
d))/((c^3*d^5*e^2 + c*d^4*e^3)*x^5 + 2*(c^3*d^6*e + c*d^5*e^2)*x^3 + (c^3*d
^7 + c*d^6*e)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*asec(c*x))/x**2/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)/((e*x^2 + d)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec^{-1}(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^2\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x^2*(d + e*x^2)^(5/2)), x)
```

3.161 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 589

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) \, dx =$$

$$-\frac{be\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$-\frac{be^2\left(e(5+m)^2+3c^2d\left(42+13m+m^2\right)\right)x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$-\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}$$

$$+\frac{3d^2e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}$$

$$+\frac{3de^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\sec^{-1}(cx)\right)}{f^7(7+m)}$$

$$-\frac{b\left(\frac{c^6d^3(2+m)(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638m+179m^2+22m^3+m^4\right)\right)}{(3+m)(5+m)(7+m)}\right)x(f}{c^5f(1+m)(2+m)(4+m)(6+m)\sqrt{c^2x^2}\sqrt{-1+c}}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arcsec(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arcsec(c
*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arcsec(c*x))/f^5/(5+m)+e^3*(f*x)^(7
+m)*(a+b*arcsec(c*x))/f^7/(7+m)-b*(c^6*d^3*(2+m)*(4+m)*(6+m)/(1+m)+e*(1+m)*
(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+1
79*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2
+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^5/f/(1+m)/(2+m)/(4+m)/(6+
m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)-b*e*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m
)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*x*(f*x)^(1+m)*(
c^2*x^2-1)^(1/2)/c^5/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)/(c^2*x^2)^(1
```

```
/2)-b*e^2*(e*(5+m)^2+3*c^2*d*(m^2+13*m+42))*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)
/c^3/f^3/(4+m)/(5+m)/(6+m)/(7+m)/(c^2*x^2)^(1/2)-b*e^3*x*(f*x)^(5+m)*(c^2*x
^2-1)^(1/2)/c/f^5/(6+m)/(7+m)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {276, 5346, 1823, 1281, 470, 372, 371}

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) dx = \frac{d^3(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)} + \frac{3d^2e(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)} + \frac{3de^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)} + \frac{e^3(fx)^{m+7}\left(a+b\sec^{-1}(cx)\right)}{f^7(m+7)} - \frac{be^3x\sqrt{c^2x^2-1}(fx)^{m+5}}{cf^5(m+6)(m+7)\sqrt{c^2x^2}} - \frac{be^2x\sqrt{c^2x^2-1}(fx)^{m+3}\left(3c^2d(m^2+13m+42)+e(m+5)^2\right)}{c^3f^3(m+4)(m+5)(m+6)(m+7)\sqrt{c^2x^2}} - \frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1}\left(\frac{e\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^6(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)}+\frac{d^3}{(m+1)^2} \cdots}{f\sqrt{c^2x^2}\sqrt{c^2x^2-1}} - \frac{bex\sqrt{c^2x^2-1}(fx)^{m+1}\left(3c^4d^2(m^4+22m^3+179m^2+638m+840)+3c^2de(m+3)^2\left(m^2+13m+42\right) \cdots}{c^5f(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)\sqrt{c^2x^2}}$$

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSec[c*x]),x]
```

```
[Out] -((b*e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*
c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*x*(f*x)^(1 + m)*Sqrt[-1 + c
^2*x^2])/(c^5*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^
2])) - (b*e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*x*(f*x)^(3 + m)*Sqr
t[-1 + c^2*x^2])/(c^3*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]) -
(b*e^3*x*(f*x)^(5 + m)*Sqrt[-1 + c^2*x^2])/(c*f^5*(6 + m)*(7 + m)*Sqrt[c^2*
x^2]) + (d^3*(f*x)^(1 + m)*(a + b*ArcSec[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x
)^(3 + m)*(a + b*ArcSec[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a +
b*ArcSec[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcSec[c*x]))/(f^
7*(7 + m)) - (b*c*(d^3/(1 + m)^2 + (e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(
3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^
4)))/(c^6*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)))*x*(f*x)^(1 + m)
*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(
f*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}+\frac{3d^2e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\sec^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad-\frac{(bcx)\int\frac{(fx)^m\left(\frac{d^3}{1+m}+\frac{3d^2ex^2}{3+m}+\frac{3de^2x^4}{5+m}+\frac{e^3x^6}{7+m}\right)}{\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)} \\
&\quad+\frac{3d^2e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\sec^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad-\frac{(bx)\int\frac{(fx)^m\left(\frac{c^2d^3(6+m)}{1+m}+\frac{3c^2d^2e(6+m)x^2}{3+m}+\frac{e^2\left(e(5+m)^2+3c^2d\left(42+13m+m^2\right)\right)x^4}{(5+m)(7+m)}\right)}{\sqrt{-1+c^2x^2}}\,dx}{c(6+m)\sqrt{c^2x^2}} \\
&= -\frac{be^2\left(e(5+m)^2+3c^2d\left(42+13m+m^2\right)\right)x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}} \\
&\quad-\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)} \\
&\quad+\frac{3d^2e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\sec^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad-\frac{(bx)\int\frac{(fx)^m\left(\frac{c^4d^3(4+m)(6+m)}{1+m}+\frac{e\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638m+179m^2+22m^3+m^4\right)\right)x^2}{(3+m)(5+m)(7+m)}\right)}{\sqrt{-1+c^2x^2}}\,dx}{c^3(4+m)(6+m)\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$
$$-\frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$
$$-\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}$$
$$+\frac{3d^2e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}$$
$$+\frac{3de^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\sec^{-1}(cx)\right)}{f^7(7+m)}$$
$$-\frac{\left(b\left(\frac{c^4d^3(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^2(2+m)(3+m)(5+m)(7+m)}\right.\right.}{c^3(4+m)(6+m)\sqrt{c^2x^2}}$$

$$= -\frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$
$$-\frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$
$$-\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}$$
$$+\frac{3d^2e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}$$
$$+\frac{3de^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\sec^{-1}(cx)\right)}{f^7(7+m)}$$
$$-\frac{\left(b\left(\frac{c^4d^3(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^2(2+m)(3+m)(5+m)(7+m)}\right.\right.}{c^3(4+m)(6+m)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

$$= -\frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$-\frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$-\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}$$

$$+\frac{3d^2e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}$$

$$+\frac{3de^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\sec^{-1}(cx)\right)}{f^7(7+m)}$$

$$-\frac{b\left(\frac{c^6d^3}{(1+m)^2}+\frac{e\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638m+179m^2+22m^3+m^4\right)\right)}{(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}\right)x(fx)^{1+m}\sqrt{}}{c^5f\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.68

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx$$

$$= x(fx)^m\left(\frac{ad^3}{1+m}+\frac{3ad^2ex^2}{3+m}+\frac{3ade^2x^4}{5+m}+\frac{ae^3x^6}{7+m}+\frac{bd^3\sec^{-1}(cx)}{1+m}+\frac{3bd^2ex^2\sec^{-1}(cx)}{3+m}\right.$$
$$+\frac{3bde^2x^4\sec^{-1}(cx)}{5+m}+\frac{be^3x^6\sec^{-1}(cx)}{7+m}$$
$$+\frac{bcd^3\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}}$$
$$+\frac{3bcd^2e\sqrt{1-\frac{1}{c^2x^2}}\,x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2\sqrt{1-c^2x^2}}$$
$$+\frac{3bcde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2\sqrt{1-c^2x^2}}$$
$$\left.+\frac{bce^3\sqrt{1-\frac{1}{c^2x^2}}\,x^7\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7+m}{2},\frac{9+m}{2},c^2x^2\right)}{(7+m)^2\sqrt{1-c^2x^2}}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSec[c*x]),x]
```

```
[Out] x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5 +
 m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcSec[c*x])/(1 + m) + (3*b*d^2*e*x^2*Ar
```

```
cSec[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcSec[c*x])/(5 + m) + (b*e^3*x^6*ArcSe
c[c*x])/(7 + m) + (b*c*d^3*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, (
1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) + (3*b*c*d^2*e
*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2
*x^2])/((3 + m)^2*Sqrt[1 - c^2*x^2]) + (3*b*c*d*e^2*Sqrt[1 - 1/(c^2*x^2)]*x
^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^2*Sqrt[1
 - c^2*x^2]) + (b*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x^7*Hypergeometric2F1[1/2, (7
 + m)/2, (9 + m)/2, c^2*x^2])/((7 + m)^2*Sqrt[1 - c^2*x^2]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 (a+b\,\mathrm{arcsec}\,(cx))\,dx$$

[In] `int((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x)`

[Out] `int((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x)`

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^3(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\,dx$$

[In] `integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x, algorithm="fricas")`

[Out] `integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsec(c*x))*(f*x)^m, x)`

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right)\,dx=\text{Timed out}$$

[In] `integrate((f*x)**m*(e*x**2+d)**3*(a+b*asec(c*x)),x)`

[Out] `Timed out`

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^3 (b\operatorname{arcsec}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x, algorithm="maxima")
```

```
[Out] a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*x
^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((b*e^3*f^m*m^3 + 9*b*e
^3*f^m*m^2 + 23*b*e^3*f^m*m + 15*b*e^3*f^m)*x^7 + 3*(b*d*e^2*f^m*m^3 + 11*b
*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m + 21*b*d*e^2*f^m)*x^5 + 3*(b*d^2*e*f^m*m^
3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2*e*f^m*m + 35*b*d^2*e*f^m)*x^3 + (b*d^3*f^
m*m^3 + 15*b*d^3*f^m*m^2 + 71*b*d^3*f^m*m + 105*b*d^3*f^m)*x)*x^m*arctan(sq
rt(c*x + 1)*sqrt(c*x - 1)) - (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*integrat
e(-(b*d^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + (b*e^3*f^m*m^3 + 9*b*e^3*f^m*m^2 + 2
3*b*e^3*f^m*m + 15*b*e^3*f^m)*x^6 + 71*b*d^3*f^m*m + 105*b*d^3*f^m + 3*(b*d
*e^2*f^m*m^3 + 11*b*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m + 21*b*d*e^2*f^m)*x^4
+ 3*(b*d^2*e*f^m*m^3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2*e*f^m*m + 35*b*d^2*e*f
^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(m^4 + 16*m^3 - (c^2*m^4 + 16*c^2*
m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m + 105), x))/(m
^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^3 (b\operatorname{arcsec}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsec(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\sec^{-1}(cx)\right) \, dx = \int (f\,x)^m \left(e\,x^2+d\right)^3 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((f*x)^m*(d + e*x^2)^3*(a + b*acos(1/(c*x))),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)^3*(a + b*acos(1/(c*x))), x)
```

3.162 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 374

$$\begin{aligned}
&\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx \\
&= -\frac{be(e(3+m)^2+2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}} \\
&\quad -\frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}}+\frac{d^2(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)} \\
&\quad +\frac{2de(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}+\frac{e^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)} \\
&\quad -\frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right))\,x(fx)^{1+m}\sqrt{1-\cdots}}{c^3f(1+m)^2(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}\sqrt{-1+c^2x\cdots}}
\end{aligned}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arcsec(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arcsec(c*x
))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arcsec(c*x))/f^5/(5+m)-b*(c^4*d^2*(2+m)*(
3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20)))*x*(f*x)^(1+m)*
hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^3/f/(1
+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)-b*e*(e*(3+m
)^2+2*c^2*d*(m^2+9*m+20))*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c^3/f/(4+m)/(5+m)
/(m^2+5*m+6)/(c^2*x^2)^(1/2)-b*e^2*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)/c/f^3/(4
+m)/(5+m)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {276, 5346, 12, 1281, 470, 372, 371}

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right) dx = \frac{d^2(fx)^{m+1}\left(a+b\sec^{-1}(cx)\right)}{f(m+1)} + \frac{2de(fx)^{m+3}\left(a+b\sec^{-1}(cx)\right)}{f^3(m+3)} + \frac{e^2(fx)^{m+5}\left(a+b\sec^{-1}(cx)\right)}{f^5(m+5)} - \frac{be^2x\sqrt{c^2x^2-1}(fx)^{m+3}}{cf^3(m+4)(m+5)\sqrt{c^2x^2}} - \frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1}\left(\frac{e(2c^2d(m^2+9m+20)+e(m+3)^2)}{c^4(m+2)(m+3)(m+4)(m+5)}+\frac{d^2}{(m+1)^2}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{f\sqrt{c^2x^2}\sqrt{c^2x^2-1}} - \frac{bex\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d(m^2+9m+20)+e(m+3)^2\right)}{c^3f(m+2)(m+3)(m+4)(m+5)\sqrt{c^2x^2}}$$

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] -((b*e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2))*x*(f*x)^(1 + m)*Sqrt[-1 + c
^2*x^2])/(c^3*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*Sqrt[c^2*x^2])) - (b*e^2*x*
(f*x)^(3 + m)*Sqrt[-1 + c^2*x^2])/(c*f^3*(4 + m)*(5 + m)*Sqrt[c^2*x^2]) + (
d^2*(f*x)^(1 + m)*(a + b*ArcSec[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(
a + b*ArcSec[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcSec[c*x]))
/(f^5*(5 + m)) - (b*c*(d^2/(1 + m)^2 + (e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m
+ m^2)))/(c^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)))*x*(f*x)^(1 + m)*Sqrt[1 - c^
2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*Sqrt[c^2*x
^2]*Sqrt[-1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d^2(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}+\frac{2de(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{e^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)} \\
&\quad-\frac{(bcx)\int\frac{(fx)^m\left(d^2\left(15+8m+m^2\right)+2de\left(5+6m+m^2\right)x^2+e^2\left(3+4m+m^2\right)x^4\right)}{(1+m)(3+m)(5+m)\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}
\end{aligned}
$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)} - \frac{(bcx)\int \frac{(fx)^m\left(d^2(15+8m+m^2)+2de(5+6m+m^2)x^2+e^2(3+4m+m^2)x^4\right)}{\sqrt{-1+c^2x^2}}\,dx}{(15+23m+9m^2+m^3)\sqrt{c^2x^2}}$$

$$= -\frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)} - \frac{(bx)\int \frac{(fx)^m\left(c^2d^2(3+m)(4+m)(5+m)+e(1+m)\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x^2\right)}{\sqrt{-1+c^2x^2}}\,dx}{c(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{c^2x^2}}$$

$$= -\frac{be\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} - \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)} - \frac{\left(b(-c^4d^2(2+m)(3+m)(4+m)(5+m)-e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right))x\right)\int}{c^3(2+m)(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{c^2x^2}}$$

$$= -\frac{be\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} - \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)} - \frac{\left(b(-c^4d^2(2+m)(3+m)(4+m)(5+m)-e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right))x\sqrt{}\right.}{c^3(2+m)(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

$$= -\frac{be\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} - \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\sec^{-1}(cx)\right)}{f^5(5+m)} - \frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right))x(fx)^{1}}{c^3f(1+m)(2+m)(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.78

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right)\,dx$$
$$= x(fx)^m \left(\frac{ad^2}{1+m}+\frac{2adex^2}{3+m}+\frac{ae^2x^4}{5+m}+\frac{bd^2\sec^{-1}(cx)}{1+m}+\frac{2bdex^2\sec^{-1}(cx)}{3+m} + \frac{be^2x^4\sec^{-1}(cx)}{5+m}+\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}} + \frac{2bcde\sqrt{1-\frac{1}{c^2x^2}}\,x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2\sqrt{1-c^2x^2}} + \frac{bce^2\sqrt{1-\frac{1}{c^2x^2}}\,x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2\sqrt{1-c^2x^2}}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSec[c*x]),x]
```

```
[Out] x*(f*x)^m*((a*d^2)/(1 + m) + (2*a*d*e*x^2)/(3 + m) + (a*e^2*x^4)/(5 + m) +
(b*d^2*ArcSec[c*x])/(1 + m) + (2*b*d*e*x^2*ArcSec[c*x])/(3 + m) + (b*e^2*x^
4*ArcSec[c*x])/(5 + m) + (b*c*d^2*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1
[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) + (2*b*
c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2
, c^2*x^2])/((3 + m)^2*Sqrt[1 - c^2*x^2]) + (b*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*
x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^2*Sqrt[
1 - c^2*x^2]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^2 \left(a+b\,\mathrm{arcsec}\left(cx\right)\right)dx$$

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^2 (b\,\mathrm{arcsec}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arcsec(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right) \, dx = \int (fx)^m (a+b\,\mathrm{asec}\,(cx)) \left(d+ex^2\right)^2 \, dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*asec(c*x)),x)
[Out] Integral((f*x)**m*(a + b*asec(c*x))*(d + e*x**2)**2, x)
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^2 (b\,\mathrm{arcsec}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d
^2/(f*(m + 1)) + (((b*e^2*f^m*m^2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^5 + 2*(b
*d*e*f^m*m^2 + 6*b*d*e*f^m*m + 5*b*d*e*f^m)*x^3 + (b*d^2*f^m*m^2 + 8*b*d^2*
f^m*m + 15*b*d^2*f^m)*x)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)) - (m^3 + 9
*m^2 + 23*m + 15)*integrate(-(b*d^2*f^m*m^2 + 8*b*d^2*f^m*m + (b*e^2*f^m*m^
2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^4 + 15*b*d^2*f^m + 2*(b*d*e*f^m*m^2 + 6*
b*d*e*f^m*m + 5*b*d*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(m^3 - (c^2
*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 +
9*m^2 + 23*m + 15)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^2 (b\,\mathrm{arcsec}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\sec^{-1}(cx)\right) \, dx = \int (f\,x)^m \left(e\,x^2+d\right)^2 \left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((f*x)^m*(d + e*x^2)^2*(a + b*acos(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^2*(a + b*acos(1/(c*x))), x)
```

3.163 $\int (fx)^m (d+ex^2) (a+b\sec^{-1}(cx)) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}
&\int (fx)^m (d+ex^2) (a+b\sec^{-1}(cx)) \, dx \\
&= -\frac{bex^{2+m}\sqrt{-1+c^2x^2}}{c(6+5m+m^2)\sqrt{c^2x^2}} + \frac{dx^{1+m}(a+b\sec^{-1}(cx))}{1+m} + \frac{ex^{3+m}(a+b\sec^{-1}(cx))}{3+m} \\
&\quad + \frac{b(e(1+m)^2+c^2d(2+m)(3+m))\,x^{2+m}\sqrt{-1+c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{3+m}{2},c^2x^2\right)}{c(1+m)^2(2+m)(3+m)\sqrt{c^2x^2}}
\end{aligned}$$

```
[Out] d*x^(1+m)*(a+b*arcsec(c*x))/(1+m)+e*x^(3+m)*(a+b*arcsec(c*x))/(3+m)-b*e*x^(
2+m)*(c^2*x^2-1)^(1/2)/c/(m^2+5*m+6)/(c^2*x^2)^(1/2)+b*(e*(1+m)^2+c^2*d*(2+
m)*(3+m))*x^(2+m)*hypergeom([1, 1+1/2*m],[3/2+1/2*m],c^2*x^2)*(c^2*x^2-1)^(
1/2)/c/(1+m)^2/(2+m)/(3+m)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 5346, 12, 470, 372, 371}

$$\begin{aligned}
&\int (fx)^m (d+ex^2) (a+b\sec^{-1}(cx)) \, dx \\
&= \frac{d(fx)^{m+1}(a+b\sec^{-1}(cx))}{f(m+1)} + \frac{e(fx)^{m+3}(a+b\sec^{-1}(cx))}{f^3(m+3)} \\
&\quad - \frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1}\left(\frac{e}{c^2(m+2)(m+3)}+\frac{d}{(m+1)^2}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{f\sqrt{c^2x^2}\sqrt{c^2x^2-1}} \\
&\quad - \frac{bex\sqrt{c^2x^2-1}(fx)^{m+1}}{cf(m^2+5m+6)\sqrt{c^2x^2}}
\end{aligned}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] -((b*e*x*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c*f*(6 + 5*m + m^2)*Sqrt[c^2*x^
2])) + (d*(f*x)^(1 + m)*(a + b*ArcSec[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)
*(a + b*ArcSec[c*x]))/(f^3*(3 + m)) - (b*c*(d/(1 + m)^2 + e/(c^2*(2 + m)*(3
 + m)))*x*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2])/(f*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5346

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
```

```
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{d(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} - \frac{(bcx)\int \frac{(fx)^m\left(d(3+m)+e(1+m)x^2\right)}{(1+m)(3+m)\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{d(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}-\frac{(bcx)\int \frac{(fx)^m\left(d(3+m)+e(1+m)x^2\right)}{\sqrt{-1+c^2x^2}}\,dx}{(3+4m+m^2)\sqrt{c^2x^2}}$$

$$= -\frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf(6+5m+m^2)\sqrt{c^2x^2}}+\frac{d(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)}+\frac{\left(bc\left(-\frac{e(1+m)^2}{c^2(2+m)}-d(3+m)\right)x\right)\int \frac{(fx)^m}{\sqrt{-1+c^2x^2}}\,dx}{(3+4m+m^2)\sqrt{c^2x^2}}$$

$$= -\frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf(6+5m+m^2)\sqrt{c^2x^2}}+\frac{d(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} + \frac{\left(bc\left(-\frac{e(1+m)^2}{c^2(2+m)}-d(3+m)\right)x\sqrt{1-c^2x^2}\right)\int \frac{(fx)^m}{\sqrt{1-c^2x^2}}\,dx}{(3+4m+m^2)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

$$= -\frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf(6+5m+m^2)\sqrt{c^2x^2}}+\frac{d(fx)^{1+m}\left(a+b\sec^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\sec^{-1}(cx)\right)}{f^3(3+m)} - \frac{bc\left(\frac{e(1+m)^2}{c^2(2+m)}+d(3+m)\right)x(fx)^{1+m}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f(1+m)(3+4m+m^2)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.95

$$\int (fx)^m\left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = x(fx)^m\left(\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}} + \frac{\frac{(3+m)\left(d(3+m)+e(1+m)x^2\right)\left(a+b\sec^{-1}(cx)\right)}{1+m}+\frac{bce\sqrt{1-\frac{1}{c^2x^2}}\,x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{\sqrt{1-c^2x^2}}}{(3+m)^2}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcSec[c*x]),x]
[Out] x*(f*x)^m*((b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) + (((3 + m)*(d*(3 + m)
+ e*(1 + m)*x^2)*(a + b*ArcSec[c*x]))/(1 + m) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)
]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/Sqrt[1 - c^2*x
^2])/(3 + m)^2)
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right) \left(a+b\,\mathrm{arcsec}\left(cx\right)\right) dx$$

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\, dx = \int (ex^2+d)(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\, dx = \int (fx)^m \left(a+b\,\mathrm{asec}\,(cx)\right) \left(d+ex^2\right)\, dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*asec(c*x)),x)
[Out] Integral((f*x)**m*(a + b*asec(c*x))*(d + e*x**2), x)
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\sec^{-1}(cx)\right)\, dx = \int (ex^2+d)(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + (((b*e*f^m*m + b*
e*f^m)*x^3 + (b*d*f^m*m + 3*b*d*f^m)*x)*x^m*arctan(sqrt(c*x + 1)*sqrt(c*x -
 1)) - (m^2 + 4*m + 3)*integrate((b*d*f^m*m + 3*b*d*f^m + (b*e*f^m*m + b*e*
f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2
- m^2 - 4*m - 3), x))/(m^2 + 4*m + 3)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\sec^{-1}(cx)\right)\,dx = \int (f\,x)^m\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*acos(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*acos(1/(c*x))), x)
```

3.164 $\int \frac{(fx)^m(a+b\sec^{-1}(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \operatorname{Int}\left(\frac{(fx)^m\,(a+b\sec^{-1}(cx))}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{d+ex^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{d+ex^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(fx)^m (a+b\sec^{-1}(cx))}{d+ex^2}\,dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\,\mathrm{arcsec}\,(cx))}{e\,x^2+d}dx$$

```
[In] int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x)
[Out] int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 39.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(fx)^m (a+b\,\mathrm{asec}\,(cx))}{d+ex^2}\,dx$$

```
[In] integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d),x)
[Out] Integral((f*x)**m*(a + b*asec(c*x))/(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{d + ex^2} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{ex^2 + d} \, dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{d + ex^2} \, dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{ex^2 + d} \, dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{d + ex^2} \, dx = \int \frac{(f\,x)^m \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2 + d} \, dx$$

```
[In] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2),x)
[Out] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2), x)
```

3.165 $\int \frac{(fx)^m(a+b\sec^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^2,x]
[Out] Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b \operatorname{arcsec}(cx))}{(e x^2 + d)^2} dx$$

```
[In] int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x)
[Out] int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(b \operatorname{arcsec}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m (a + b\sec^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(f\,x)^m \left(a + b \operatorname{acos}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^2} dx$$

```
[In] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^2, x)
```

3.166 $\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx=\text{Int}\left((fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx=\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$$

```
[In] Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
```

```
[Out] Defer[Int][(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

Rubi steps

$$\text{integral}=\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSec[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arcsec}\left(cx\right)\right) dx$$

```
[In] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} \left(b\,\mathrm{arcsec}\left(cx\right)+a\right) (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsec(c*x))*sqrt(e*x^2 + d)*(f*x
)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*asec(c*x)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int (ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int (ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\sec^{-1}(cx)\right) \, dx = \int (f\,x)^m\,\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*acos(1/(c*x))), x)
```

3.167 $\int (fx)^m \sqrt{d+ex^2} \left(a+b\sec^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b\sec^{-1}(cx)\right)\,dx = \text{Int}\left((fx)^m \sqrt{d+ex^2} \left(a+b\sec^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b\sec^{-1}(cx)\right)\,dx = \int (fx)^m \sqrt{d+ex^2} \left(a+b\sec^{-1}(cx)\right)\,dx$$

```
[In] Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
[Out] Defer[Int][(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

Rubi steps

$$\text{integral} = \int (fx)^m \sqrt{d+ex^2} \left(a+b\sec^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int (fx)^m\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx$$

```
[In] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]),x]
[Out] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSec[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m\sqrt{e\,x^2+d}\,(a+b\,\mathrm{arcsec}\,(cx))\,dx$$

```
[In] int((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 52.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx=\int (fx)^m\,(a+b\,\mathrm{asec}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)**(1/2)*(a+b*asec(c*x)),x)
[Out] Integral((f*x)**m*(a + b*asec(c*x))*sqrt(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(1/2)*(a+b*arcsec(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\sec^{-1}(cx)\right)\,dx = \int (f\,x)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*acos(1/(c*x))), x)
```

3.168 $\int \frac{(fx)^m(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\operatorname{Int}\left(\frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\left(a+b\sec^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(fx)^m (a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m \left(a+b \operatorname{arcsec}\left(cx\right)\right)}{\sqrt{e\,x^2+d}}dx$$

```
[In] int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arcsec}(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arcsec(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 22.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(fx)^m \left(a+b\operatorname{asec}\left(cx\right)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*asec(c*x))/sqrt(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arcsec(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.169 $\int \frac{(fx)^m(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \operatorname{Int}\left(\frac{(fx)^m\,(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcSec[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m (a+b\,\mathrm{arcsec}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
[Out] int((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m (a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arcsec(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

Sympy [N/A]

Not integrable

Time = 140.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(fx)^m\,(a+b\,\mathrm{asec}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)**m*(a+b*asec(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral((f*x)**m*(a + b*asec(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\,(a+b\sec^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arcsec(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m \left(a+b\sec^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int(((f*x)^m*(a + b*acos(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.170 $\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 401

$$\begin{aligned}\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx &= \frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{7b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} \\ &\quad + \frac{13b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{3b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} \\ &\quad + \frac{b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}\,(a+b\sec^{-1}(cx))}{2c^{12}} \\ &\quad + \frac{(1-c^4x^4)^{3/2}\,(a+b\sec^{-1}(cx))}{3c^{12}} \\ &\quad - \frac{(1-c^4x^4)^{5/2}\,(a+b\sec^{-1}(cx))}{10c^{12}} \\ &\quad - \frac{4b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\end{aligned}$$

```
[Out] 1/3*(-c^4*x^4+1)^(3/2)*(a+b*arcsec(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b*
arcsec(c*x))/c^12-7/90*b*(c^2*x^2+1)^(3/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c
^2/x^2)^(1/2)+13/150*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2
/x^2)^(1/2)-3/70*b*(c^2*x^2+1)^(7/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2
)^(1/2)+1/90*b*(c^2*x^2+1)^(9/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2)^(1
/2)-4/15*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^
2)^(1/2)+4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2)^(
1/2)-1/2*(a+b*arcsec(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {272, 45, 5354, 12, 6853, 6874, 862, 52, 65, 214, 797}

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{(1-c^4x^4)^{5/2}\,(a+b\sec^{-1}(cx))}{10c^{12}} + \frac{(1-c^4x^4)^{3/2}\,(a+b\sec^{-1}(cx))}{3c^{12}} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}} - \frac{4b\sqrt{1-c^2x^2}\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{15c^{13}x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{b\sqrt{1-c^2x^2}(c^2x^2+1)^{9/2}}{90c^{13}x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{3b\sqrt{1-c^2x^2}(c^2x^2+1)^{7/2}}{70c^{13}x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{13b\sqrt{1-c^2x^2}(c^2x^2+1)^{5/2}}{150c^{13}x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{7b\sqrt{1-c^2x^2}(c^2x^2+1)^{3/2}}{90c^{13}x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{4b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{15c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

```
[In] Int[(x^11*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[1 - 1/(c^2*x^2)]*x)
 - (7*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(90*c^13*Sqrt[1 - 1/(c^2*x^2
)]*x) + (13*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(150*c^13*Sqrt[1 - 1/(
c^2*x^2)]*x) - (3*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(7/2))/(70*c^13*Sqrt[1
- 1/(c^2*x^2)]*x) + (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(9/2))/(90*c^13*Sqrt
[1 - 1/(c^2*x^2)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcSec[c*x]))/(2*c^12) + (
(1 - c^4*x^4)^(3/2)*(a + b*ArcSec[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(a
 + b*ArcSec[c*x]))/(10*c^12) - (4*b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*
x^2]])/(15*c^13*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 5354

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcSec[c*x], v, x] - Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}}-\frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}}-\frac{b\int\frac{\sqrt{1-c^4x^4}(-8-4c^4x^4-3c^8x^8)}{30c^{12}\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}}-\frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}}-\frac{b\int\frac{\sqrt{1-c^4x^4}(-8-4c^4x^4-3c^8x^8)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{30c^{13}}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}}-\frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}}-\frac{\left(b\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{1-c^4x^4}(-8-4c^4x^4-3c^8x^8)}{x\sqrt{1-c^2x^2}}dx}{30c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}}-\frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}}+\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-c^4x^2}(8+4c^4x^2+3c^8x^4)}{x\sqrt{1-c^2x}}dx,x,x^2\right)}{60c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}}-\frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}}+\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\left(\frac{8\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}+\frac{4c^4x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}+\frac{3c^8x^3\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\right)dx,x,x^2\right)}{60c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}} + \frac{(2b\sqrt{1-c^2x^2})\,\mathrm{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{(b\sqrt{1-c^2x^2})\,\mathrm{Subst}\left(\int \frac{x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\,dx,x,x^2\right)}{15c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{(b\sqrt{1-c^2x^2})\,\mathrm{Subst}\left(\int \frac{x^3\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\,dx,x,x^2\right)}{20c^5\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}} + \frac{(2b\sqrt{1-c^2x^2})\,\mathrm{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx,x,x^2\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{(b\sqrt{1-c^2x^2})\,\mathrm{Subst}\left(\int x\sqrt{1+c^2x}\,dx,x,x^2\right)}{15c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{(b\sqrt{1-c^2x^2})\,\mathrm{Subst}\left(\int x^3\sqrt{1+c^2x}\,dx,x,x^2\right)}{20c^5\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}} + \frac{(2b\sqrt{1-c^2x^2})\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{(b\sqrt{1-c^2x^2})\,\mathrm{Subst}\left(\int \left(-\frac{\sqrt{1+c^2x}}{c^2}+\frac{(1+c^2x)^{3/2}}{c^2}\right)dx,x,x^2\right)}{15c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{(b\sqrt{1-c^2x^2})\,\mathrm{Subst}\left(\int \left(-\frac{\sqrt{1+c^2x}}{c^6}+\frac{3(1+c^2x)^{3/2}}{c^6}-\frac{3(1+c^2x)^{5/2}}{c^6}+\frac{(1+c^2x)^{7/2}}{c^6}\right)dx,x,x^2\right)}{20c^5\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{7b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{13b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{3b\sqrt{1-c^2x^2}(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}} + \frac{\left(4b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{15c^{15}\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{7b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{13b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{3b\sqrt{1-c^2x^2}(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}(a+b\sec^{-1}(cx))}{10c^{12}} - \frac{4b\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{-105a\sqrt{1-c^4x^4}(8+4c^4x^4+3c^8x^8) + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(768+36c^2x^2+78c^4x^4+5c^6x^6+35c^8x^8)}{-1+c^2x^2} - 105b\sqrt{1-c^4x^4}(8}{3150c^{12}}$$

```
[In] Integrate[(x^11*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x^6 + 35
*c^8*x^8))/(-1 + c^2*x^2) - 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*
x^8)*ArcSec[c*x] + 840*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x)/Sqrt[1 - c^4*x^
4]])/(3150*c^12)
```

Maple [F]

$$\int \frac{x^{11}(a + b\ \mathrm{arcsec}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

[In] int(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)

[Out] int(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.59

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{(35\,bc^8x^8 + 5\,bc^6x^6 + 78\,bc^4x^4 + 36\,bc^2x^2 + 768\,b)\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1} - 840\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4}}{\sqrt{c^2x^2-}}\right.}{}$$

[In] integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")

[Out] 1/3150*((35*b*c^8*x^8 + 5*b*c^6*x^6 + 78*b*c^4*x^4 + 36*b*c^2*x^2 + 768*b)*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1) - 840*(b*c^2*x^2 - b)*arctan(sqrt(-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) - 105*(3*a*c^10*x^10 - 3*a*c^8*x^8 + 4*a*c^6*x^6 - 4*a*c^4*x^4 + 8*a*c^2*x^2 + (3*b*c^10*x^10 - 3*b*c^8*x^8 + 4*b*c^6*x^6 - 4*b*c^4*x^4 + 8*b*c^2*x^2 - 8*b)*arcsec(c*x) - 8*a)*sqrt(-c^4*x^4 + 1))/(c^14*x^2 - c^12)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

[In] integrate(x**11*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^{11}}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sq
rt(-c^4*x^4 + 1)/c^12) - 1/30*(30*c^12*integrate((30*sqrt(c*x + 1)*c^10*x^1
1*log(c) + (3*c^8*x^9 + 4*c^6*x^7 + 3*(10*c^10*log(c) + c^10)*x^11 + 4*c^4*
x^5 + 8*c^2*x^3 + 8*x)*e^(3/2*log(c*x + 1) + log(c*x - 1)) + 30*(c^10*x^11*
e^(3/2*log(c*x + 1) + log(c*x - 1)) + sqrt(c*x + 1)*c^10*x^11)*log(x))/((c^
10*e^(2*log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + c^10*e^(log(c*x
+ 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x) + (3*c^8*x^8 + 4*c^4*x^4
+ 8)*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(c*x + 1)*sq
rt(c*x - 1)))*b/c^12
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^11*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^{11}\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((x^11*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((x^11*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.171 $\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\begin{aligned}\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx &= \frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}x}\\ &\quad+\frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8}\\ &\quad+\frac{(1-c^4x^4)^{3/2}\,(a+b\sec^{-1}(cx))}{6c^8}\\ &\quad-\frac{b\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x}\end{aligned}$$

```
[Out] 1/6*(-c^4*x^4+1)^(3/2)*(a+b*arcsec(c*x))/c^8-1/18*b*(c^2*x^2+1)^(3/2)*(-c^2
*x^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)+1/30*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+
1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/3*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x
^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)+1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^
(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/2*(a+b*arcsec(c*x))*(-c^4*x^4+1)^(1/2)/c^
8
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {272, 45, 5354, 12, 6853, 6874, 862, 52, 65, 214, 797}

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{(1-c^4x^4)^{3/2}\,(a+b\sec^{-1}(cx))}{6c^8} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} - \frac{b\sqrt{1-c^2x^2}\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{3c^9x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{b\sqrt{1-c^2x^2}(c^2x^2+1)^{5/2}}{30c^9x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{b\sqrt{1-c^2x^2}(c^2x^2+1)^{3/2}}{18c^9x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{3c^9x\sqrt{1-\frac{1}{c^2x^2}}}$$

```
[In] Int[(x^7*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(3*c^9*Sqrt[1 - 1/(c^2*x^2)]*x) - (
b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(18*c^9*Sqrt[1 - 1/(c^2*x^2)]*x) +
 (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(30*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
 - (Sqrt[1 - c^4*x^4]*(a + b*ArcSec[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/2)*(
a + b*ArcSec[c*x]))/(6*c^8) - (b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2
]])/(3*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 5354

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcSec[c*x], v, x] - Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} - \frac{b\int \frac{(-2-c^4x^4)\sqrt{1-c^4x^4}}{6c^8\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{c}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} - \frac{b\int \frac{(-2-c^4x^4)\sqrt{1-c^4x^4}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{6c^9}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{(-2-c^4x^4)\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}\,dx}{6c^9\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}(2+c^4x^2)}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{12c^9\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{2\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}} + \frac{c^4x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\right)dx, x, x^2\right)}{12c^9\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{6c^9\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\,dx, x, x^2\right)}{12c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx, x, x^2\right)}{6c^9\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int x\sqrt{1+c^2x}\,dx, x, x^2\right)}{12c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{6c^9\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{\sqrt{1+c^2x}}{c^2} + \frac{(1+c^2x)^{3/2}}{c^2}\right)dx, x, x^2\right)}{12c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{3c^{11}\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\sec^{-1}(cx))}{6c^8} - \frac{b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.59

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{-15a\sqrt{1-c^4x^4}(2+c^4x^4)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(28+c^2x^2+3c^4x^4)}{-1+c^2x^2}-15b\sqrt{1-c^4x^4}(2+c^4x^4)\sec^{-1}(cx)+30b\,\mathrm{arc}}{90c^8}$$

```
[In] Integrate[(x^7*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[
1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c^2*x^2) - 15*b*Sqrt[1 - c^4
*x^4]*(2 + c^4*x^4)*ArcSec[c*x] + 30*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x)/S
qrt[1 - c^4*x^4]])/(90*c^8)
```

Maple [F]

$$\int \frac{x^7(a+b\,\mathrm{arcsec}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

```
[In] int(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.68

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{(3\,bc^4x^4+bc^2x^2+28\,b)\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}-30\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right)-15\,(ac^6x^6-ac^4x^4+2}{90\,(c^{10}x^2-c^8)}$$

```
[In] integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/90*((3*b*c^4*x^4 + b*c^2*x^2 + 28*b)*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1)
 - 30*(b*c^2*x^2 - b)*arctan(sqrt(-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) - 15*(a*
c^6*x^6 - a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^6*x^6 - b*c^4*x^4 + 2*b*c^2*x^2 -
2*b)*arcsec(c*x) - 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 - c^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x**7*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arcsec}\,(cx)+a)x^7}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) - 1/6*(6*sqrt(c
^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8*integrate((6*sqrt(c*x + 1)*c^6
*x^7*log(c) + (c^4*x^5 + (6*c^6*log(c) + c^6)*x^7 + 2*c^2*x^3 + 2*x)*e^(3/2
*log(c*x + 1) + log(c*x - 1)) + 6*(c^6*x^7*e^(3/2*log(c*x + 1) + log(c*x -
1)) + sqrt(c*x + 1)*c^6*x^7)*log(x))/((c^6*e^(2*log(c*x + 1) + log(c*x - 1)
 + 1/2*log(-c*x + 1)) + c^6*e^(log(c*x + 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*
x^2 + 1)), x) - (c^8*x^8 + c^4*x^4 - 2)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1))
)*b/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^7(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^7*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a + b\sec^{-1}(cx))}{\sqrt{1 - c^4x^4}}\,dx = \int \frac{x^7\,\left(a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1 - c^4\,x^4}}\,dx$$

```
[In] int((x^7*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^7*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.172 $\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 126

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} - \frac{bx\arctan\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{-1+c^2x^2}}\right)}{2c^3\sqrt{c^2x^2}}$$

```
[Out] -1/2*b*x*arctan((-c^4*x^4+1)^(1/2)/(c^2*x^2-1)^(1/2))/c^3/(c^2*x^2)^(1/2)-1
/2*(a+b*arcsec(c*x))*(-c^4*x^4+1)^(1/2)/c^4+1/2*b*x*(-c^4*x^4+1)^(1/2)/c^3/
(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {267, 5354, 12, 1586, 1266, 862, 52, 65, 214}

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} - \frac{b\sqrt{1-c^2x^2}\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}$$

```
[In] Int[(x^3*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(2*c^5*Sqrt[1 - 1/(c^2*x^2)]*x) - (
Sqrt[1 - c^4*x^4]*(a + b*ArcSec[c*x]))/(2*c^4) - (b*Sqrt[1 - c^2*x^2]*ArcTa
nh[Sqrt[1 + c^2*x^2]])/(2*c^5*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1586

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Dist[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(
x^mn*e)))^FracPart[q]))/x^(mn*FracPart[q]), Int[x^(m + mn*q)*(1 + d*(1/(x^m
n*e)))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && E
qQ[n2, -2*mn] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n2]
```

Rule 5354

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcSec[c*x], v, x] - Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} - \frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{c} \\
&= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} + \frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{2c^5} \\
&= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} + \frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}\,dx}{2c^5\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx, x, x^2\right)}{4c^5\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= -\frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx, x, x^2\right)}{4c^5\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= \frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} \\
&\quad + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx, x, x^2\right)}{4c^5\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$= \frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} + \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{2c^7\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\sec^{-1}(cx))}{2c^4} - \frac{b\sqrt{1-c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{2c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{\frac{\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}x-ac^2x^2\right)\sqrt{1-c^4x^4}}{-1+c^2x^2} - b\sqrt{1-c^4x^4}\sec^{-1}(cx) + b\arctan\left(\frac{c\sqrt{1-\frac{1}{c^2x^2}}x}{\sqrt{1-c^4x^4}}\right)}{2c^4}$$

```
[In] Integrate[(x^3*(a + b*ArcSec[c*x]))/Sqrt[1 - c^4*x^4],x]
[Out] (((a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x - a*c^2*x^2)*Sqrt[1 - c^4*x^4])/(-1 + c^
2*x^2) - b*Sqrt[1 - c^4*x^4]*ArcSec[c*x] + b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)
]*x)/Sqrt[1 - c^4*x^4]])/(2*c^4)
```

Maple [F]

$$\int \frac{x^3(a+b\,\text{arcsec}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

```
[In] int(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
[Out] int(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.99

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}b - (bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right) - \sqrt{-c^4x^4+1}(ac^2x^2+(bc^2x^2-b)\,\text{arcsec}\,(cx) -}{2\,(c^6x^2-c^4)}$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1)*b - (b*c^2*x^2 - b)*arctan(sqrt(-
c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) - sqrt(-c^4*x^4 + 1)*(a*c^2*x^2 + (b*c^2*x^
2 - b)*arcsec(c*x) - a))/(c^6*x^2 - c^4)
```

Sympy [F]

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{\sqrt{1 - c^4x^4}}\,dx = \int \frac{x^3(a + b\operatorname{asec}(cx))}{\sqrt{-(cx - 1)(cx + 1)(c^2x^2 + 1)}}\,dx$$

```
[In] integrate(x**3*(a+b*asec(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asec(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)),
 x)
```

Maxima [F]

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{\sqrt{1 - c^4x^4}}\,dx = \int \frac{(b\operatorname{arcsec}(cx) + a)x^3}{\sqrt{-c^4x^4 + 1}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^4*integrate((2*sqrt(c*x + 1)*c^2*x^3*log(c) + ((2*c^2*log(c) + c^
2)*x^3 + x)*e^(3/2*log(c*x + 1) + log(c*x - 1)) + 2*(c^2*x^3*e^(3/2*log(c*x
 + 1) + log(c*x - 1)) + sqrt(c*x + 1)*c^2*x^3)*log(x))/(sqrt(c^2*x^2 + 1)*(
c^2*e^(2*log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + c^2*e^(log(c*x
+ 1) + 1/2*log(-c*x + 1)))), x) + sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x
 + 1)*arctan(sqrt(c*x + 1)*sqrt(c*x - 1)))*b/c^4 - 1/2*sqrt(-c^4*x^4 + 1)*a
/c^4
```

Giac [F]

$$\int \frac{x^3(a + b\sec^{-1}(cx))}{\sqrt{1 - c^4x^4}}\,dx = \int \frac{(b\operatorname{arcsec}(cx) + a)x^3}{\sqrt{-c^4x^4 + 1}}\,dx$$

```
[In] integrate(x^3*(a+b*arcsec(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\sec^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((x^3*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^3*(a + b*acos(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.173 $\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \text{Int}\left(\frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

```
[Out] Defer[Int][(a + b*ArcSec[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcSec[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\,\operatorname{arcsec}\left(cx\right)}{x\sqrt{-c^{4}x^{4}+1}}dx$$

```
[In] int((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x)
[Out] int((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\operatorname{arcsec}\left(cx\right)+a}{\sqrt{-c^{4}x^{4}+1}x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arcsec(c*x) + a)/(c^4*x^5 - x), x)
```

Sympy [N/A]

Not integrable

Time = 11.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\operatorname{asec}\left(cx\right)}{x\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^{2}x^{2}+1\right)}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x/(-c**4*x**4+1)**(1/2),x)
[Out] Integral((a + b*asec(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*inte
grate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*
sqrt(-c*x + 1)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsec(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\sec^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
```

```
[Out] int((a + b*acos(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

3.174 $\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \operatorname{Int}\left(\frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

```
[In] Int[(a + b*ArcSec[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

```
[Out] Defer[Int][(a + b*ArcSec[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

```
[In] Integrate[(a + b*ArcSec[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcSec[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\ \mathrm{arcsec}\left(cx\right)}{x^5\sqrt{-c^4x^4+1}}dx$$

```
[In] int((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
[Out] int((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arcsec}\left(cx\right)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arcsec(c*x) + a)/(c^4*x^9 - x^5), x)
```

Sympy [N/A]

Not integrable

Time = 82.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{a+b\sec^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\mathrm{asec}\left(cx\right)}{x^5\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

```
[In] integrate((a+b*asec(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
[Out] Integral((a + b*asec(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
), x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.27

$$\int \frac{a + b\sec^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x^5}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) + 2
*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(arctan(sqrt(c*x + 1)*sqrt(c*x - 1)
)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^5), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a + b\sec^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\mathrm{arcsec}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x^5}\,dx$$

```
[In] integrate((a+b*arcsec(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arcsec(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a + b\sec^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}}\,dx = \int \frac{a + b\,\mathrm{acos}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1 - c^4\,x^4}}\,dx$$

```
[In] int((a + b*acos(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
[Out] int((a + b*acos(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                 end if
             else #result contains complex but optimal is not
                 if debug then
                         print("result contains complex but optimal is not");
                 fi;
                 return "C","Result contains complex when optimal does not.";
             fi;
         else # result do not contain complex
              # this assumes optimal do not as well. No check is needed here.
             if debug then
                    print("result do not contain complex, this assumes optimal do not as well")
             fi;
             if leaf_count_result<=2*leaf_count_optimal then
                 if debug then
                     print("leaf_count_result<=2*leaf_count_optimal");
                 fi;
                 return "A"," ";
             else
                 if debug then
                     print("leaf_count_result>2*leaf_count_optimal");
                 fi;
                 return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_count
             fi;
         fi;
     else #ExpnType(result) > ExpnType(optimal)
         if debug then
             print("ExpnType(result) > ExpnType(optimal)");
         fi;
         return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
     fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```